Study on the Maximum Thickness of Superposition Throttle Slices of Shock Absorber

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Abstract: The governing differential equation for bending deformation of a single throttle slice is introduced first, and based on its solution satisfying required boundary conditions, the formula and coefficient for bending deformation of a single slice is obtained through equivalency transformation. Then, the formula for equivalent thickness of the throttle multi-slice, the stress coefficient and the thickness coefficient of slice are defined, the maximum thickness of multi-slices is studied. Based on these results, a new scheme for designing maximum thickness of multi-slices according to its nominal thickness is suggested. Followed is a practical example for design and stress analysis of a multi-slice throttle, together with its numerical simulation by ANSYS, and damping characteristic test. The compared results show that the design scheme of maximum thickness of multi-slices is effective and the formulas for equivalent thickness and maximum stress of the throttle multi-slice are accurate enough. It has practical value for the damper design.

Key words: Shock absorber; Maximum thickness multi-slices design; Stress analysis; Damping characteristic

1  INTRODUCTION

The throttle slices in damper is the key part that affect the damping characteristics. The thickness of throttle slice and acreage of constant opening throttle holes have important influence to the damper damping characteristics[1,2]. Practically, the throttle slice in damper is not single, but multi-slices superposition, for the damping characteristic of damper, at the same time, for the manufacture and the stress of throttle slices[3].

However, the throttle slice was single at designing. If the throttle slice been single as designed, the stress would go beyond the stress limit. Therefore, it is the very important that how to design multi-slice instead of one slice, in condition of the equivalent thickness of the throttle multi-slice being equal to design thickness, the damping characteristic of damper dos not change, but the stress on the multi-slice reduce. At same equivalent thickness and damping characteristic[4], the advantages of superposition throttle multi-slice are

1) The damping characteristic is same to one throttle slice designed. That is, damper with multi-slices being the same damping force to damper with the one throttle slice.

2) Reduces the stress on the throttle slices, improves the condition of force, increases reliability.

3) The thickness of throttle slice is standard, so, adapted to batch manufacture, the price depressed.

So, it is necessary that to study the multi-slice design, to research the equivalent thickness, stress of multi-slices, and the maximum thickness of multi-slices, for to design the damper that satisfied the practicality request.

2  BENDING DEFORMATION OF THE THROTTLE SLICE
2.1 Differential equation of the throttle slice bending deformation

Fig. 1 is the mechanics model of elastic throttle-slice. The boundary conditions of throttle slice are fixation restriction at the inner radius, and free restriction at the outside radius. To the throttle slice, the inner radius is $r_a$ (fixing dimension is considered), the outside radius is $r_b$, the thickness is $h$, the pressure is $q$, and the bending deformation at radius $r$ is $f_r$.

Building the bipolar coordinates by the throttle slice circle center. Being symmetrical to the $z$ axis, the load and the structure, and being small deformation the bending deformation of throttle slice in damper, according to the elasticity mechanics principles[5], the differential equation of elastic throttle slice deformation is established as

$$D \frac{d^2}{dr^2} \left( \frac{d}{r} \frac{df}{dr} \right) + \frac{d}{r} \left( \frac{df}{dr} \right) = q$$

(1)

Where, $D = \frac{E h^3}{12(1-\mu)}$, $r$ is any radius of throttle slice, \( r \in [r_a, r_b] \); $E$ is the elasticity material coefficient of throttle slice; $\mu$ is the Possion ratio. By formula of $D$, the value of $D$ could be calculated.

The solution of the differential equation of elastic throttle slice deformation is

$$f_r = C_1 \ln r + C_2 r^2 \ln r + C_3 r^2 + C_4 + f_r^*$$

(2)

Where, $C_1 \ln r + C_2 r^2 \ln r + C_3 r^2 + C_4$ is the odd general solution of the differential equation $f_r^*$ is the special solution of the differential equation.

2.2 Bending deformation coefficient of throttle slice

$C_1$, $C_2$, $C_3$ and $C_4$ can be defined by the boundary conditions. Analyzed the solution of the differential equation, it is found that the each item in the solution has the common factor of $q/h^3$. Therefore, equivalency transformation of the solution made, the formula about the $q/h^3$ can be got, the other remainder factors, for example, the material elasticity coefficient $E$, the inner radius $r_a$, the outside radius $r_b$, and the radius $r$ at where the bending deformation was computed of throttle slice, turned into a coefficient $G_r$. The bending deformation accurate analytic formula of throttle slice at any radius $r$ was written briefly as[6]

$$f_r = G_r \frac{q}{h^3}$$

(3)

$G_r$ is an inherent characteristic coefficient of throttle slice bending deformation, reflecting the throttle slice’s capability of bending deformation. Related with inner radius $r_a$, outside radius $r_b$, and the material elasticity coefficient $E$ of throttle slice, but not related with the pressure on the throttle slice and thickness of throttle slice. The unit of $G_r$ is $m^3 N^{-1}$.

The $G_r$ is the function of radius $r$. The curve of $G_r$ vs. radius $r$ as shown in Fig. 2.

2.3 Deformation at any radius $r$

With the same method, the coefficients $G_r$, and the bending deformations those at any radius $r$ of throttle slice can be calculated by the computation programme, the bending deformation at any radius $r$ is $f_r = G_r q / h^3$.

At any radius $r$ of throttle slice, the values of bending deformation are shown in the table 1.

<table>
<thead>
<tr>
<th>$r / 10^{-3}m$</th>
<th>5.0</th>
<th>5.5</th>
<th>6.0</th>
<th>6.5</th>
<th>7.0</th>
<th>7.5</th>
<th>8.0</th>
<th>8.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_r / 10^3m$</td>
<td>0.00</td>
<td>0.426</td>
<td>1.50</td>
<td>2.97</td>
<td>4.67</td>
<td>6.47</td>
<td>8.30</td>
<td>10.1</td>
</tr>
</tbody>
</table>

The curve of bending deformation vs. radius $r$ is shown in Fig. 3.
3 MULTI-SLICES EQUIVALENT THICKNESS

The structure sketch of damper throttle was shown in Fig.4.

Superposition throttle slice of unequal thickness could be taken as the springs paralleled that have the equal length, but unequal elasticity coefficient. So, the deformations of multi-slices are equal, the forces on multi-slices are unequal. The composition of forces on multi-slices is equal to the total force. According to the bending deformation formula of throttle slice (3) , it can be obtained as

\[ G \frac{q_1}{h_1} + G \frac{q_2}{h_2} + \cdots + G \frac{q_n}{h_n} = G \frac{q}{h_d} \]  

(4)

That is \[ \frac{q_1}{h_1} = \frac{q_2}{h_2} = \cdots = \frac{q_n}{h_n} = \frac{q}{h_d} \]

For \( q_1 + q_2 + \cdots + q_n = q \) , So, the equivalent thickness is written as

\[ h_d = \sqrt[3]{h_1^3 + h_2^3 + \cdots + h_n^3} \]  

(5)

It is known that relations of the thickness of multi-slices with the equivalent thickness is

(1) The three power of the equivalent thickness is the sum of three power of each superposition slice thickness.

(2) The equivalent thickness of unequal thickness multi-slice is larger than the maximum thickness, i.e. \( h_d > \max[h_i] \)

(3) The equivalent thickness is smaller the sum of all superposition throttle slice.

(4) If the maximum thickness of one superposition throttle slice is much larger the other superposition slice, the equivalent thickness of total superposition slice is close to the maximum thickness of maximum throttle slice, the equivalent thickness could be instead of with the maximum thickness of superposition throttle slice.

4 STRESS ON MULTI-SLICES

4.1 Stress on single slice

The stress on single slice can be written as[7]

\[ \sigma_r = A_7 \frac{r^2 q}{h^2}, \quad \sigma_x = A_8 \frac{r^2 q}{h^3}, \quad \sigma_k = B_7 \frac{r^2 q}{h^2}. \]  

(6)

Where, \( A_7 \) is the maximum stress coefficient at radius \( r \) orientation, \( A_8 \) at axis \( z \), \( B_7 \) at circle \( \theta \).

4.2 Stress on multi-slices

The thickness of each superposition throttle slices is \( h_1, h_2, h_3, \ldots, h_n \). Because the deformations of multi-slices are equal, according formula (4), it can be written as follows

\[ \frac{q_1}{h_1} = \frac{q_2}{h_2} = \cdots = \frac{q_n}{h_n} = \frac{q}{h_d} \]  

(7)

Where

\[ h_i \ (i \in [1,2,\ldots,n]) \] is the thickness of each superposition slices, and \( h < h_i \)

It is known that the pressure \( q_i \) on the superposition slice \( h_i \) is proportion with the three power of the thickness \( h_i \). By the formula of maximum stress, the stress on the superposition slice \( h_i \) may be written as follows

\[ \sigma_{i_{max}} = A_7 \frac{r^2 q_i}{h_i^3} = A_8 \frac{r^2 h_i q_i}{h_i^3} = A_7 \frac{r^2 h_i q}{h_i^3} \]  

(8)

The thickness coefficient of superposition slice \( h_i \) is defined as \( k_{h_i} \), i.e. \( k_{h_i} = h_i/h \), \( h \) is the thickness of single slice designed. So, the formula (8) can be written as

\[ \sigma_{i_{max}} = A_7 \frac{r^2 q}{k_{h_i} h^2} \]  

The maximum stress on single slice designed is \( \sigma_{i_{max}} \), therefore, the maximum on superposition slice \( h_i \) can be written as follows
$$\sigma_{\text{max}} = \frac{\sigma_{\text{max}}}{k_h}$$

The stress coefficient of superposition slice $h_i$ is defined as $k_{\sigma_i}$, so, it can be written as

$$k_{\sigma_i} = k_h$$

(9)

It can be known that the stress coefficient of superposition slice is equal to the thickness coefficient, i.e. $k_{\sigma_i} = k_h > 1$.

Therefore, the stress on the each superposition slice can written in the form

$$\sigma_{1\text{max}} = \frac{\sigma_{\text{max}}}{k_h}, \sigma_{2\text{max}} = \frac{\sigma_{\text{max}}}{k_{h_2}}, \ldots, \sigma_{n\text{max}} = \frac{\sigma_{\text{max}}}{k_{h_n}}$$

(10)

Through the analysis above, it can be known that the stress on the each superposition slice is related with the thickness of superposition slices. The stress on the thick slice is larger than the thin slice’s.

When the stress on the single throttle slice $h$ designed, the throttle slice must be designed to multi-slices. The maximum thickness of superposition slices can be written as follows

$$h_{\text{max}} = \frac{h}{k_{\sigma_i}} = \frac{h[\sigma]}{\sigma_{\text{max}}}$$

(11)

5 DESIGNING EXAMPLE OF MAXIMUM THICKNESS OF MULTI-SLICES

For example, the throttle slice in a damper, the outside radius $b_r$, the inner radius $a_r$, the ratio of $r_b$ to $r_a$ is $r_b/r_a$, the elasticity coefficient of throttle slice material is $E$, the stress limit is $[\sigma]$, the press of valve opening is $q$. With the machine design handbook, the maximum stress coefficient $A$ at radial orientation can be gotten.

5.1 Maximum stress on Single throttle slice designed

According to the maximum stress formal, the maximum stress $\sigma_{\text{max}}$ on the single throttle slice designed is

$$\sigma_{\text{max}} = A_7 \frac{r_a^2 q}{b^2} > [\sigma]$$

It is known that the maximum stress $\sigma_{\text{max}}$ on the single throttle slice designed is larger than the $[\sigma]$. So, the throttle slice must be designed into multi-slices, according the superposition throttle slices equal thickness rule.

5.2 Maximum thickness of multi-slices

According to the maximum stress limit, the stress coefficient can be calculated, so the maximum thickness of multi-slices is

$$h_{\text{max}} = \frac{h}{k_{\sigma_i}} = h[\sigma]/\sigma_{\text{max}}$$

Chosen the maximum thickness $h_i$ of superposition slices, according the request of $h_i \leq h_{\text{max}}$

5.3 Number of maximum thickness multi-slices

According the equivalent thickness formula of same thickness superposition slice and the maximum thickness chosen above. The number of maximum thickness multi-slices can be computed as

$$n_i = \frac{h_{\text{d}}}{h_i} = \frac{h^3}{h_i^3}$$

Get the integral $n_i$ by the value calculated by above formula.

5.4 The thickness and number of other superposition throttle slices

The thickness and the number of other superposition throttle slices can be computed according to the equivalent thickness rule and formula. The other superposition throttle slices are $h_2, n_2; h_3, n_3; h_4, n_4$.

5.5 Equivalent thickness of superposition throttle slices

According to the equivalent thickness formula. The equivalent thickness of superposition throttle slices is

$$h_{\text{d}} = \sqrt[3]{n_1 h_1^3 + n_2 h_2^3 + n_3 h_3^3 + n_4 h_4^3}$$

The error of equivalent thickness to the design thickness is

$$\Delta h = h - h_{\text{d}}$$

It is known that the equivalent thickness is very approach the design thickness, the error of equivalent thickness to the design thickness is very much less, by the appropriate thickness and number of superposition throttle slices.

6 COMPUTATION AND SIMULATION OF DEFORMATION AND STRESS

6.1 Deformation computation

For the damping characteristic of damper being affected by the bending deformation at valve mouth radius $r_b$, so, the bending deformation of single slice and multi-slices was computed.
With the method of the bending deformation coefficient $G_r$, the bending deformation of single throttle slice at valve mouth radius $r_k$ is

$$f_{ik} = G_r q / h_k^3$$

By the same way, the bending deformation of multi-slices is $f_{ik} = G_r q / h_d^3$

Where, $h$ is the thickness of single throttle slice, $h_d$ is the equivalent thickness of multi-slices.

The bending deformation curves of the single slice and multi-slices are shown in Fig.5

$$h$$ is the thickness of single throttle slice,
$$h_d$$ is the equivalent thickness of multi-slices.

6.2 Maximum stress computation on the multi-slices

The maximum stress on the multi-slices could be calculated with the coefficient of stress or thickness. The thickness coefficients of superposition slices are $k_{h1}$, $k_{h2}$, $k_{h3}$ and $k_{h4}$, therefore the maximum stress on the every superposition slice are

$$\sigma_{1max} = \sigma_{max} / k_{h1}, \; \sigma_{2max} = \sigma_{max} / k_{h2}$$
$$\sigma_{3max} = \sigma_{max} / k_{h3}, \; \sigma_{4max} = \sigma_{max} / k_{h4}$$

For $k_{h1}$, $k_{h2}$, $k_{h3}$ and $k_{h4}$ being not less 1, so, the maximum stress on each superposition slices is less than the stress on the single slice designed, and within the stress limit of throttle slice.

6.3 Statics simulation

Building the model of the throttle slice with the ANSYS software, the Statics simulation analysis that the bending deformation and stress simulation of throttle slice are carried out.

6.3.1 Deformation simulation

The bending deformation simulation results of single slice and multi-slices’s are uniform, shown as in Fig.6.

6.3.2 Maximum stress simulation

The maximum stress simulation results of single slice and multi-slices’s are same, shown as in Fig.7

It is known by the results simulated that the stress on the multi-slices is more less than the single slice, the result computed is consistent with the simulated, and the analysis method is exact and effective.

7 DAMPING CHARACTERISTIC TESTING

The damping characteristic of the dampers that with single throttle slice and multi-slices are tested, the damping characteristic curves are uniform, shown as in the Fig.8.
Fig.8 The indicator diagram of damper with single and multi-slices

It is known that the damping characteristic of damper with superposition throttle slices is the same to the single throttle slice.

8 CONCLUSIONS

The design rules and the stress computation method of the maximum thickness of superposition throttle slices have very important application value for the damper design, characteristic analysis, and the throttle slice maximum stress computation. By the throttle slice static simulation and damper damping characteristic test, it is known that:

1. With the method of bending deformation coefficient Gr, not only the bending deformation of throttle slice at any radius r could be calculated, the thickness of single throttle slice be designed, the maximum thickness of superposition throttle multi-slices be also designed, but also the damping characteristic of damper could analyzed.

2. When the stress on the single throttle slice going beyond the limit stress, the single slice could be instead with the superposition multi-slices.

3. The coefficients of thickness and stress of superposition throttle slices are equivalent, so, the maximum thickness of superposition throttle slices could be designed with the stress coefficient.

4. It was shown by the static simulation of the throttle slice that the stress computation methods of superposition throttle slices is accurate, the stress on the multi-slices is more less than the single slice designed thickness.

5. It was shown by the damping characteristic test of damper that the dampers with single slice designed thickness has the same damping characteristic to the multi-slices.

References:


