Mathematical Modeling and Simulation of Automotive Internal Combustion Engine

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Abstract- This paper presents a mathematical model of internal combustion engine dynamics namely air, fuel and rotational dynamics. Engine under study is naturally aspirated spark ignition four-stroke internal combustion engine with electronic fuel injection system. The purpose of model development is to provide a simple system level model for design of model-based control methods and fault diagnostics. Such type of model is known as mean value engine model. Novel features of this model are: constant volume cycle is used for approximation of combustion process; fittings equations and constants are avoided except only for estimation of frictional mean effective pressure. The model is verified at steady state with the data of engine test bed facility which results in 1-4% overall accuracy. Most of the models available in literature are specific to a certain brand or make because of their use of curve fittings, thus limiting their general use. Here a model is proposed which is not confined to a certain engine model; rather it is a generic one, thus having a significant research value.

Keywords: Engine Model, Automotive, Spark Ignition, Internal combustion engines, Engine test bed.

1. Introduction

The purpose of this paper is to develop a simple mathematical model of naturally aspirated SI four stroke IC engine, which captures the air, fuel and rotational dynamics and can be implemented using a simulation software. In this paper an engine model is presented in detail. The equations which represent throttle body, intake manifold (air and fuel dynamics) and torque generation sub-models are discussed. In this way it is tried to present a comprehensive system model. The look up tables are not considered, instead law of conservation of energy and mass for air flow model, and Otto cycle for analysis of in-cylinder dynamics are applied and thermodynamic relationships are utilized. The use of Newton's law is brought into play to calculate the torque and rotational dynamics. Fitting equations are avoided as much as possible but for calculation of friction mean effective pressure (fmep) an empirical equation of engine speed is applied. For validation purposes, electronic fuel injection is replaced with carburetor model because carbureted engine is available in the local heat engine lab. The graphical simulation software is used. This modeling tool is very flexible in nature so that electronic fuel injection subsystem model is easily replaced with carburetor model. So sub models can be decoupled as well as sub models can be easily coupled.

Mean value spark ignition four-Stroke engine models are found in the literature [1], [5], [6] and [10]. Hendricks *et al.* in their work [1] presented a nonlinear three state dynamic model. The model was fitted by using experimental data and simulation results. The model is simple and they used modular structure which works well with control system analysis and design tasks. However, the model contains a number of fitting parameters and constants. Crossley *etal.* in their work [5] developed a nonlinear mathematical engine model for incorporation into an overall vehicle driveline model. In the paper the throttle body and inlet airflow, engine pumping and torque generation were modeled as nonlinear algebraic relations based on experimental data whereas intake / exhaust manifold and rotational dynamics were modeled with differential equations. The algebraic equation used for calculation of engine torque is very complex, but it generates results rapidly. In [5] entire engine model was not developed as important fuel dynamics were not included at all and a lot of fitting equations represent engine model. In the paper by Weeks *et al.* [6] the engine model already developed by Moskwa was used in a larger control system. In the paper control components, sensors, actuators and controllers were used. In this engine model intake manifold dynamics contains air and fuel dynamics were presented with some additional detail. The emphasis was on control systems. This model is suitable for control analysis. In [6] look up tables were incorporated in the model. Lastly, Dobner in his paper [10] described a mathematical engine model for development of dynamic engine control. The model is formulated in a modular manner which connects physical processes, the carburetor, intake manifold, combustion and rotational dynamics. The engine was modeled in a discrete form. In [10] the modeling equations were not written in detail.

This paper is organized as following. Section 2 describes the schematic diagram, model description and the list of notations and variables used in model development. In section 3 throttle body sub-model is described. Section 4 and 5 details the air and fuel dynamics respectively. Torque generation equations are given in section 6. Section 7 consists of engine model simulation blocks namely throttle body, air dynamics, fuel dynamics, and torque generation. In section 8 simulation results are presented and discussed. Finally concluding remarks are given in section 9. Engine test bed specifications, obtained data in appendix and references are given at the end.

2. Engine Schematic

2.1 Schematic Diagram

Fig. 1. shows a schematic diagram of an internal combustion engine. The engine components: throttle body, intake manifold, cylinder, shaft and exhaust manifold can be seen. This diagram shows that air passes across the throttle valve and reaches the intake manifold. The fuel is injected on the air when it enters the cylinder. Diagram shows intake stroke only.

2.2 Model Description.

In this section the sub-models: throttle body, air dynamics, fuel dynamics, combustion process and rotational dynamics will be modeled. The engine model description is shown by a schematic in Fig. 2. The throttle body controls the air into the intake manifold where fuel is mixed with air. This mixture enters the cylinder where combustion takes place. The heat generated by combustion produces high pressure of gases which is converted into mechanical work by piston and crankshaft linkages. The brake torque is then found at crank shaft by subtracting the friction and pumping torque. The net torque is the difference of the brake torque and load torque.

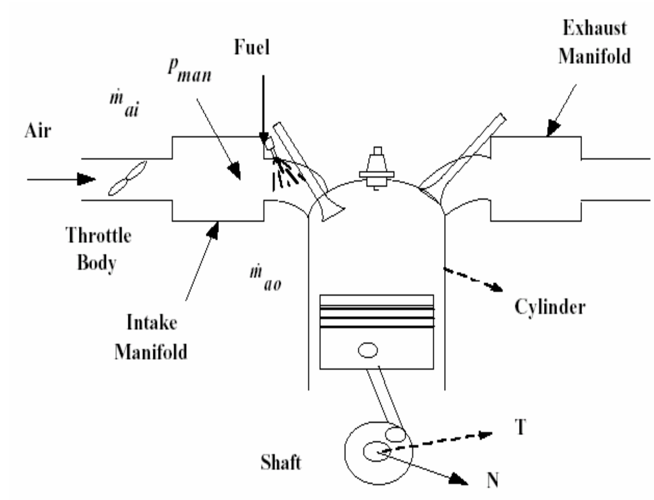

Fig. 1. Schematic Diagram of IC Engine

2.3 Nomenclature

The following variables, notations and abbreviations are used in the paper. All units are in SI system until and otherwise mentioned.

TDC	top dead center
IVC	Intake valve closing
mep	mean effective pressure
a	throttle angle (degrees)
A_E	effective throttle area (m^2)
A_f	effective area of fuel jet at exit (m^2)
c_1	speed-density constant, velocity
c_2	velocity of fluid
C_f	velocity of fuel at exit to the jet
C_D	throttle discharge coefficient
C_{Df}	fuel jet discharge coefficient
D	diameter of throttle pipe (m)
w_e	engine speed (rad / sec)
e	fuel split parameter
g	ratio of specific heat capacities
$\bar{g}$	fraction of fuel injected before IVC
J_e	inertia of engine
K	constant
$\dot{m}_a$	time rate of change of air mass flow
$\dot{m}_{ai}$	air flow rate into intake (Kg/sec)
$\dot{m}_{ao}$	air flow rate into cylinder (Kg/sec)
$\dot{m}_{fi}$	fuel flow rate at injector (Kg/sec)
$\dot{m}_{fo}$	fuel flow rate into cylinder (Kg/sec)
$\dot{m}_{ff2}$	fuel flow injected after IVC (Kg/sec)
$\dot{m}_{ff3}$	fuel flow injected before IVC (Kg/sec)

$\dot{m}_{fsl}$ fuel flow lagged by wall wetting (Kg/sec)
N engine speed (revolutions per minute)
p_f pressure of fuel pump at fuel jet (kPa)
p_a ambient air pressure (kPa)
p_{man} intake manifold pressure (kPa)
R universal gas constant (KJ/kgK)
r, r_1, r_2 density of air(kg/m^3)
r_a density of air
r_f density of fuel (kg/m^3)
T torque (N.m)
T_i indicated torque (N.m)
T_b brake torque (N.m)
T_p Pumping torque (N.m)
T_f friction torque(N.m)
t_f slow fuel time constant (sec)
T_{man} intake manifold temperature (k)
T_a ambient air temperature (k)
h_{vol} volumetric efficiency of engine
V_d engine displacement (m^3)
V_{man} intake manifold volume (m^3)
p_2, p_3 Engine cycle pressures at end of compression and combustion respectively (kPa)
T_2, T_3 Engine cycle Temperatures at end of compression and combustion respectively (K).
r compression ratio
C_p specific heat capacity at constant pressure ($KJ/kg.K$)
Q_{in} heat input by fuel combustion

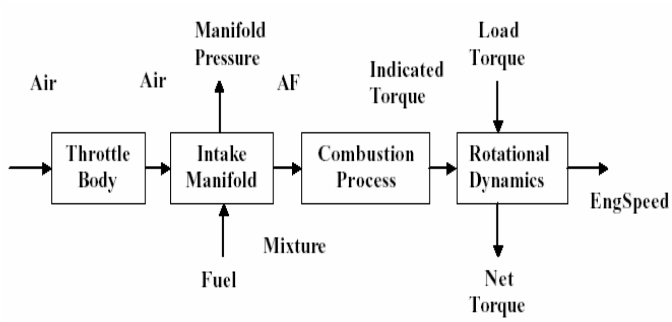

Fig. 2. Engine Model Description

3. Throttle Body Sub-model

In this sub-model the mass flow rate of air across the throttle is approximated as one dimensional compressible steady flow through converging nozzle. This flow can be expressed by the equation of continuity of mass flow [8],[9] as follows:

$$\dot{m}_{ai} = r_1 A_1 C_1 = r_2 A_2 C_2 . \tag{1}$$

The speed of air is calculated by using steady flow energy equation as follows:

$$C_2 = \sqrt{2 C_P T_a \left[1 - \left(\frac{p_{man}}{p_a} \right)^{\frac{g-1}{g}} \right]} , \tag{2}$$

Where the density of fluid at throttle is approximated by polytropic and perfect gas laws as follows:

$$r_2 = \left(\frac{p_{man}}{p_a} \right)^{\frac{1}{g}} \frac{p_a}{RT_a} , \tag{3}$$

The effective area of throttle is calculated by subtracting the elliptic area from throttle pipe area as follows:

$$A_E = (1 - \cos a) \frac{p D^2}{4} \tag{4}$$

The mass flow across the throttle is obtained by putting (2), (3), and (4) in (1) ,using relationship $C_p = \frac{g R}{g - 1}$ and simplifying as

$$\dot{m}_{ai} = \sqrt{\frac{2g}{(g-1)RT_a}} p_a C_D (1 - \cos a) \frac{p D^2}{4} \sqrt{\left(\frac{p_{man}}{p_a} \right)^{\frac{2}{g}} - \left(\frac{p_{man}}{p_a} \right)^{\frac{g+1}{g}}} \tag{5}$$

4. Air Dynamics Sub-model

The time rate of change of air mass flow in the finite volume of intake manifold is the difference between the air mass flow past the throttle plate and that which flows into the cylinder intake valves. It can be expressed in equation form as follows:

$$\dot{m}_a = \dot{m}_{ai} - \dot{m}_{ao} , \tag{6}$$

The air mass flow into the cylinder intake valves can be obtained simply using the speed density formula [3]. This formula can be written as:

$$\dot{m}_{ao} = \frac{N}{120} V_d \, r \, h_{vol} , \tag{7}$$

Now we find the rate of change of air mass in the intake manifold using perfect gas law by differentiating pressure and temperature with respect to time we get

$$\dot{m}_a = \frac{V_{man}}{RT_{man}} \dot{p}_{man} - \frac{V_{man} p_{man}}{RT_{man}^2} \dot{T}_{man} , \tag{8}$$

The state equation for manifold pressure is obtained by putting (7) and (8) in (6) as

$$\dot{p}_{man} = \left(\frac{\dot{T}_{man}}{T_{man}} - C_1 w h_{vol} \right) p_{man} + \frac{RT_{man}}{V_{man}} \dot{m}_{ai}, \qquad (9)$$

where $C_1 = V_d / (240 p V_{man})$.

5. Fuel Dynamics Sub-model

The fuel flow is incompressible flow through either by injection or carburetion system. The fuel flow can be modeled with help of Bernoulli's equation [7],[8]. The speed of flow is obtained as

$$C_f = \sqrt{2\left[\frac{p_a - p_{man}}{r_f} - gz \right]}, \qquad (10)$$

Where gz is potential energy of fuel mass at height z from a reference. As the mass flow rate of fuel can be expressed in the mathematical form as follows:

$$\dot{m}_f = A_f C_f r_f \qquad (11)$$

The mass flow rate of fuel is obtained by putting (10) in (11) as

$$\dot{m}_f = C_{Df} A_f \sqrt{2 r_f \left(p_a - p_{man} - r_f gz \right)}, \qquad (12)$$

In fuel injection system the height of nozzle does not matter, therefore, it can be neglected and fuel is sprayed with pumping pressure. Therefore (12) can be written as follows:

$$\dot{m}_{fi} = C_{Df} A_f \sqrt{2 r_f \left(p_f - p_{man} \right)}, \qquad (13)$$

The injected fuel flow mainly consists of two parts which are: a fast fuel flow and a slow fuel flow. The fast fuel is in the vapor form. It immediately becomes the part of flow into the cylinder; therefore, it is expressed with an algebraic equation. The slow fuel, in the form of a film, does not become the part of flow instantaneously because it evaporates with a time constant. As the evaporation is time developing process therefore it requires a differential equation for its expression. This evaporated fuel then combines with the fuel film flow and enters the cylinder. The fraction of the fuel flow which becomes vapor is defined as e (Epsilon) while the rest of fuel which strikes the manifold walls, valve stems etc becomes fuel film is $1-e$. The vapor fuel flow is further divided into two parts on the basis of intake valve closing (IVC) which are: before IVC and after IVC. The fraction of vapor fuel flow which is injected before IVC is defined as $\bar{g}$ while which is sprayed after IVC is $1-\bar{g}$. Using the symbols defined above the simplified model of fast fuel flow can be expressed in equation form as

$$\dot{m}_{ff3} = \dot{m}_{fi} e \bar{g}, \qquad (14)$$

$$\dot{m}_{ff2} = \dot{m}_{fi} e (1-\bar{g}), \qquad (15)$$

As evaporation is a time developing process; therefore, it requires a differential equation which can be derived as follows: The equation for rate of change of mass of fuel film can be written as:

$$\frac{d}{dt}(m_{film}) = (1-e)\dot{m}_{fi} - \dot{m}_{fsl}, \qquad (16)$$

$$(1-e)\dot{m}_{fi} - \dot{m}_{fsl} = (1-e)\dot{m}_{fi} - \frac{m_{film}}{t_f}, \qquad (17)$$

Taking time derivative on both sides of (17), we get

$$\frac{d}{dt}\left(\dot{m}_{fsl}\right) = \frac{1}{t_f}\frac{d}{dt}\left(m_{film}\right), \qquad (18)$$

we obtain following equation by putting the equation (16) in (18),:

$$\frac{d}{dt}\left(\dot{m}_{fsl}\right) = \frac{\dot{m}_{fi}(1-e) - \dot{m}_{fsl}}{t_f}. \qquad (19)$$

Now the value of fraction of vapor fuel is discussed. Its value is one when the fuel injection is completed before IVC. Otherwise, its value is ratio between the time from injection starting to intake valve closing and pulse width in the shape of crank shaft degrees.

In the first cycle only vapor part of fast fuel enters the cylinder. In the subsequent cycles all three types of fuel altogether enter the cylinder. The total fuel flow entering the cylinder can be expressed in the mathematical form as follows:

$$\dot{m}_{fo} = \dot{m}_{ff2} + \dot{m}_{ff3} + \dot{m}_{fsl}, \qquad (20)$$

6. Torque Generation Sub-Model

In this sub-model combustion process is modeled with the help of constant volume engine cycle. The mean effective pressure will be used for calculation of torque. The torque can be written in the mathematical form as:

$$T = \frac{1}{4p} V_d \, mep, \qquad (21)$$

The net work output for an Otto cycle can be found by subtracting work output from work input[7],

$$W = \frac{p_3 V_3 - p_4 V_4}{g_2 - 1} - \frac{p_2 V_2 - p_1 V_1}{g_1 - 1}, \qquad (22)$$

We find expression for mean effective pressure by using its basic definition as net work done per swept volume as

$$mep = \frac{(g_1-1)(p_3 - p_{ex}r) - (g_2-1)(p_2 - p_{man}r)}{(g_1-1)(g_2-1)(r-1)} \tag{23}$$

Where g_1 and g_2 are ratios of specific heats before and after combustion process. We get following equation after assuming ratio of specific heat capacities remains constant in whole process of combustion:

$$mep = \frac{p_{man}\, r \left(\frac{p_3}{p_2} - 1\right)\left[r^{g-1} - 1\right]}{(g-1)(r-1)} \quad , \tag{24}$$

Now we find the pressures and temperatures before and after combustion at TDC by using thermodynamic relationships as:

$$T_2 = T_{man}\left(\frac{V_c + V_d}{V_c}\right)^{g-1}, \quad p_2 = p_{man}\left(\frac{T_2}{T_{man}}\right)^{\frac{g}{g-1}}, \tag{25}$$

$$T_3 = T_2 + \frac{Q_{in}}{m_a C_V}, \qquad p_3 = p_2 \frac{T_3}{T_2}, \tag{26}$$

We find indicated torque multiplying (21) with cycle efficiency as

$$T_i = \left[1 - \left(\frac{1}{r}\right)^{g-1}\right] T \tag{27}$$

In suction and exhaust strokes torque is same quantity but in different directions. In the intake stroke pressure is same as p_{man}, and in exhaust stroke pressure can be assumed to be ambient standard pressure so

$$T_P = \frac{1}{4p} V_d \left(p_a - p_{man}\right), \tag{28}$$

and friction torque is calculated by using the equation in [4] as

$$T_f = \frac{1}{4p} V_d \left[10^4 \left(9.7 + 1.5\frac{N}{10^3} + 0.5\frac{N^2}{10^6}\right)\right], \tag{29}$$

Brake torque can be expressed in mathematical form as

$$T_b = T_i - T_p - T_f, \tag{30}$$

Now we can find the engine speed using (30) [11].

$$w = \int \frac{T_b}{J_e} dt, \tag{31}$$

7. Engine Model Simulations

The engine model consists of subsystem blocks which are: the throttle body, air dynamics, fuel dynamics, torque generation and engine block and can be summarized by following system of equations.

$$\dot{p}_{man} = \left(\frac{\dot{T}_{man}}{T_{man}} - C_1 w h_{vol}\right) p_{man} + \frac{RT_{man}}{V_{man}} \dot{m}_{ai} \tag{32}$$

$$\dot{m}_{fi} = C_{Df} A_f \sqrt{2 r_f \left(p_f - p_{man}\right)} \tag{33}$$

$$w = \int \frac{T_b}{J_e} dt \tag{34}$$

7.1 Throttle Body Block

The throttle body block calculates the total mass flow rate of air entering the intake manifold. In this sub-model following flow conditions in [1] have been used.

$$\frac{p_{man}}{p_a} \geq \left(\frac{2}{g+1}\right)^{\frac{g}{g-1}} \tag{35}$$

If the pressure ratio either equal to or greater than the critical value (32) then the mass flow rate of air at throttle is expressed as (5). Otherwise $\sqrt{\frac{g-1}{2g}\cdot\left(\frac{2}{g+1}\right)^{\frac{g+1}{g-1}}}$ will replace the square root portion of flow equation. Standard temperature and pressure corrections have been introduced as in [2] which can be applied through following expression

$$Correctedflow = flow \times \left(\sqrt{\frac{T_{amb}}{T_a}}\right) \frac{p_{amb}}{p_a} \tag{36}$$

Vacuum leaks of intake manifold have been considered. Leak size can be varied with respect to engine manifold conditions. In this model leak size of two percent of throttle area has been assumed. It is modeled with (5) as a repeating sequence.

Idle air control has been modeled with (5) in manner similar to the throttle body. In this model idle air effective area has been taken equal to five degree opening of the throttle plate. The total flow rate into the manifold is simply the sum of the throttle flow rate, idle air control and the flow due to any intake manifold leaks.

7.2 Air Dynamics Block

In this block, manifold pressure has been calculated by using (9). The mass flow rate of air entering the cylinder has been computed by using (7). Inputs to this block are mass of air flow past throttle which is output of throttle body block and engine speed which is output of engine block. In this way, this block gives pressure manifold, volumetric efficiency and mass flow rate of air entering the cylinder.

7.3 Torque Generation Block

In this block indicated torque has been first calculated. It then subtracts off the friction and pumping torque of the engine to get the brake torque. External load torque which to be subtracted if it is, but in this model the external load torque has been assumed to be zero

The mean effective pressure has been calculated from the combustion process by using (24). This theoretical mean effective pressure is multiplied by the Otto cycle efficiency to get the indicated mean effective pressure. Indicated torque is calculated by (27) and pumping torque is evaluated through (28) and friction torque is calculated by (29). Engine speed is found by (31).

8. Simulation Results

The simulation results of model of carbureted engine are presented. These results have been verified with the measured data taken from engine test bed facility at Faculty of Mechanical engineering (FME) at the GIKI Institute of engineering sciences and technology. The parameters, air mass flow rate, fuel mass flow rate, air-fuel ratio, and torque have been compared.

9. Conclusion

Mathematical model of internal combustion spark ignition four strokes engine consisting of sub-models which are: throttle body, air dynamics, fuel dynamics and torque generation models has been presented in this paper. The model has been implemented in graphical software as a dynamic continuous system. The simulation results show some of the potential uses of the model. The model may be used in many ways, explained in [6], as nonreal-time engine model, real-time engine model, embedded model, system model and subsystem model in a larger system. The purpose of the paper is not to achieve the best possible model of any specific engine but a mean value engine model for design of model-based control methods. Hendricks [1] states that control system design procedures are tolerant of small system modeling errors so an approximate model is satisfactory for engine control development application.

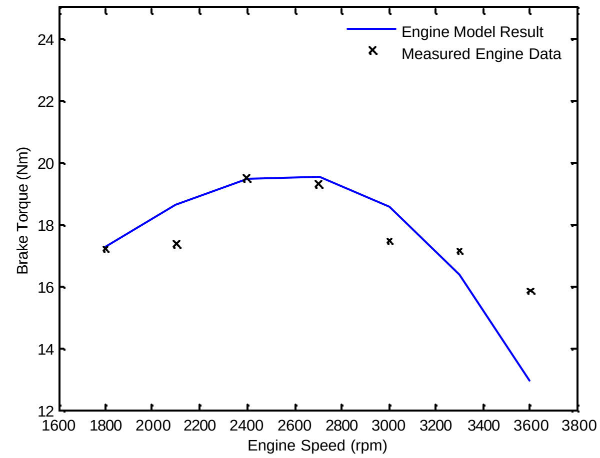

Fig. 3. Brake Torque

TABLE 1
ENGINE AIR FUEL RATIOS

Engine Speed (rpm)	Engine Mass Flow Rate of Air (Kg/sec.)	Engine Mass Flow Rate of Fuel (Kg/sec.)	Air-Fuel Ratio
1800	0.0084	0.000505	16.6205
2100	0.0094	0.000578	16.2686
2400	0.0108	0.000634	17.0239
2700	0.012	0.00066	18.1845
3000	0.0132	0.000752	17.5625
3300	0.015	0.000792	18.9441
3600	0.0151	0.000792	19.0704
Average			17.6678

TABLE 2
Model Air Fuel Ratios

Engine Speed (rpm)	Model Mass Flow Rate of Fuel (Kg/sec.)	Model Mass Flow Rate of Air (Kg/sec.)	Air-Fuel Ratio
1800	0.00058082	0.0066051	11.3720
2100	0.00058234	0.0077060	13.2328
2400	0.00058386	0.0088068	15.0837
2700	0.00058538	0.0099076	16.9250
3000	0.00058689	0.0110000	18.7428
3300	0.00058840	0.0121000	20.5642
3600	0.00058990	0.0132000	22.37661
Average			16.8996

TABLE 3
ENGINE CALCULATED PARAMETERS

S.No.	Torque (N.m)	Mass Flow Rate of Air (Kg/sec)	Fuel flow rate (gm/sec)	Air-Fuel Ratio
1	17.225	0.008359	0.5054	16.53946
2	17.375	0.0094	0.5778	16.23243
3	19.525	0.0108	0.6344	17.01094
4	19.325	0.0120	0.6599	18.25712
5	17.475	0.0132	0.7516	17.51980
6	17.15	0.0150	0.7918	18.94998
7	15.875	0.0151	0.7918	19.05858

TABLE 4
ENGINE MEASURED PARAMETERS

S.No.	Engine Speed (rpm)	Force (N)	Time for 30ml fuel (sec.)	Manometer Corrected Height (mm)
1	1800	68.9	42.74	13.50
2	2100	69.5	37.38	17.00
3	2400	78.1	34.05	22.50
4	2700	77.3	32.73	28.05
5	3000	69.9	28.74	33.50
6	3300	68.6	27.28	43.50
7	3600	63.5	27.28	44.00

APPENDIX –TEST BED MEASUREMENTS

Test bed type	Cussons P8160
Test at	GIK Institute (FME)
Ambient air temperature	298 K
Atmospheric pressure	101.325 kPa
Manometer zero error	+3
Sp. gravity of manometer fluid	1.88
Engine model	243430
Manufacturer	Briggs & Stratton
Type	Four stroke engine
Capacity	392 cc
Maximum power	7.46 kW at 3600 rpm
Maximum torque	22.7 Nm at 2400 rpm
Air flow orifice diameter	23 mm
Density of air	1.18 Kg/m^3
Density of petrol	720 Kg/ m^3

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