Probabilistic Seismic Hazard Assessment
without Source Characterization

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Abstract: - Conventional Probabilistic Seismic Hazard Assessment (PSHA) is difficult to apply in regions lacking sufficient information of the geological setting, active faults, and so forth. Also, for a site-specific PSHA, site effects arising from both crustal rock and overlying soil sediments are generally not assessed rigorously. This is of particular importance for those metropolitan cities having a significant proportion of reclaimed land, because the site-to-site variability of such site effects can be very large. The objective of this paper is to demonstrate an alternative procedure for constructing site-specific uniform hazard spectra (UHS), extended from a recently-developed Direct Amplitude-Based (DAB) approach. The method has a number of important advantages compared with conventional PSHA. Using the proposed approach, response spectral values have been computed for the whole period range of engineering interest, to form a set of site-specific UHS.

Key-Words: - probabilistic, seismic hazard, attenuation, recurrence, response spectrum, uniform hazard

1 Introduction

The most commonly adopted methodology for Probabilistic Seismic Hazard Assessment (PSHA) is that developed originally by Cornell [1]. The method incorporates the influence of all potential sources of earthquakes and their corresponding activity rates. The concept of a potential source of earthquakes plays a very important role in this methodology. A potential source of earthquakes, which can be in the form of a point, a fault, and area, is a location where future earthquakes may occur. To describe a potential source of earthquakes, one must decide its form, size, boundary, and the activity rates of earthquakes having various magnitudes.

Analytically, the effects of all earthquakes of different sizes, occurring at different locations within different earthquake sources and having various probabilities of occurrence are integrated into a single hazard curve that shows the probabilities of exceeding different levels of spectral response at the site during a specified period of time, as follows:

$$P[SV_T > z] = \sum_{i=1}^{N_s} \upsilon_i \int_{m=m_0}^{m=m_u} \int_{r=0}^{r=\infty} P[SV_T > z \mid m, r]$$

$$f_i(m) f_i(r) dr dm \qquad (1)$$

where $P[SV_T > z]$ is the probability of spectral velocity level SV_T (at natural period T) exceeding z; υ_i is the mean rate of occurrence of earthquakes between threshold and maximum magnitudes (m_0 and m_u) being considered in the i-th source; $P[SV_T > z \mid m, r]$ is the probability that the spectral

velocity level SV_T of a given earthquake with magnitude m and epicentral distance r will exceed z; $f_i(m)$ is the probability density function (PDF) of magnitude within the i-th source; $f_i(r)$ is the PDF of epicentral distance, describing the spatial distribution between the various locations within the i-th source; and N_s is the number of source zones being considered.

Employing Equation (1), the probability of exceedance (PE) of a certain shaking level can be obtained. In the past, the engineering design spectrum was constructed by scaling a standard spectral shape to the site-specific PGA and/or PGV [2]. But in recent decades, the concept of the uniform hazard spectrum (UHS) has become common practice for constructing design spectra. It computes the response spectral ordinates for a range of oscillator periods [3]. The advantage of it over the scaled-spectrum approach is that it represents a uniform hazard level for all spectral periods, and hence can describe the site-specific frequency content more accurately, without overestimating the response spectra for intermediate period ranges.

An alternative procedure has been developed in this paper for constructing site-specific UHS, extended from a recently-developed *Direct Amplitude-Based* (DAB) approach [4]. The response spectral attenuation relationship and the response-recurrence relationship have been presented in Sections 3 and 5, respectively. A generic analytical solution has been provided to avoid lengthy integration process. Using the proposed approach, and taking Hong Kong as a case study, response spectral values have been computed for the whole period range of engineering interest, to form a set of site-specific UHS (Section 6).

2   Difficulties in Cornell's Approach

The ability of Cornell's *PSHA* to incorporate both seismicity and geologic information has led to its widespread applications worldwide. Geologic information can be incorporated through the definition of sources, given that sufficient and reliable information are available. However, evidence has shown that the discrepancies arising from the use of different definitions of seismic source zones can be very large. Also, if a number of experts or expert groups are required to work independently to define the seismic source zones, large variations amongst individual expert judgments on defining seismic source zones, and in turn, large dispersion in the hazard calculations, can result. Moreover, these differences in areas such as western United States are smaller, as active surface faults are relatively well defined. So, the definition of seismic source zones would be even more difficult in most regions in the world, as detailed information of the geological settings, active faults, paleo-seismicity, crustal deformation, and so forth, are very difficult to obtain.

In view of these problems, Frankel [5] has developed a methodology for the (United States) national seismic hazard mapping program that would eliminate the need to define seismic source zones. For regions far from identified active faults, the probabilistic amplitude calculation was based on smoothed historical seismicity. The uncertainties associated with the historical catalog, such as the location error, could be reduced by smoothing the historical seismicity spatially to different length scales. However, the choice of the correlation distance c assumed for the Gaussian function in the smoothing process is highly subjective, and yet to be justified. The spatially-smoothed historical seismicity could be spread out if the assumed correlation distance c is too large, which could undoubtedly affect the precision of the hazard calculation, especially on the site-specific level.

Furthermore, the relationship between defined fault structures and the occurrence of earthquakes is still an open question. For example, the Tangshan earthquake (1976, China) and Northridge earthquake (1994, U.S.) did not occur on well-known faults. Cornell [1] also pointed out that, for some regions, it is not possible to correlate past activity with known geological structure. This is the case in most stable continental intra-plate areas, in which potential zones of weakness are widespread and seismicity is often diffuse, occurring in broad regional zones rather than along narrow well-defined faults. As a result, the identification of seismic sources is rather difficult and uncertain. The maximum possible earthquake magnitude for each source is usually determined based on historical records along with seismo-tectonic considerations, with a large element of uncertainty.

On the other hand, in conventional *PSHA*, the adopted ground motion attenuation model is spatially smoothed. As mentioned above, such models are unable to capture the site-to-site variability and event-specific characteristics.

3   Response Spectral Attenuation Relationships

Response spectral attenuation relationships are important components in seismic hazard assessment (SHA). Atkinson [6] pointed out that the future trend in SHA is to replace generic attenuation relationships

with more detailed models based on the physics of wave propagation, which would require multi-disciplinary knowledge, such as seismology, geology, and so forth. One promising way to achieve that is to employ the seismological model [7] using a stochastic method. A generic framework for developing earthquake response spectral attenuation relationships by the seismological modelling approach has been established in recent years for application in low to moderate seismic regions [8]. The approach has the ability to model separately the path and site effects (the latter arising from both upper rock crust and soil sediment) within one simple framework. Each event can be assigned a set of filter functions. For instance, the anelastic whole path attenuation can be characterised by an event-specific quality (Q_0) factor of the rock crust, along the travel path of the seismic wave. Furthermore, the approach can also handle the wave modification properties of different local bedrock conditions. The Fourier amplitude spectrum of displacement, $A(f)$, used in the seismological model, may be expressed as a product of a number of factors (filters):

$$A(f) = C\, M_0\, S(f)\, G\, An(f)\, P(f)\, V(f)\, F(f) \quad (2)$$

where C is a scaling factor, M_0 the seismic moment, $S(f)$ the source spectrum, G the geometric attenuation factor, $An(f)$ the anelastic whole path attenuation filter, $P(f)$ the local upper crust attenuation filter, $V(f)$ the local upper crust amplification filter, and $F(f)$ the soil site response transfer function.

The above factors can be classified into four classes, as defined in Table 1. The required input parameters for each class are also listed.

Table 1**:** Input parameters for Response Spectral Attenuation Modelling

Class	Factors	Input Parameters
Source	C $S(f)$	SWV, Density at Source Seismic Moment M_0
Path	G $An(f)$	Crustal Thickness D Source-site Distance r Crustal Quality Factor Q
Local	$P(f)$ $V(f)$	Crustal SWV and Density profiles Upper crust attenuation parameter κ
Site	$F(f)$	SWV, Density and Plasticity profiles of the soil layer

* For single-corner ω–squared source spectrum, it is also a function of stress drop $\Delta\sigma$.

It is emphasized that the soil site response factor $F(f)$ is not an invariant function for a specific site, as it depends on the level of soil damping and is, in turn, related to the shaking level at the soil-bedrock interface. Together with different resonant conditions, which are interactive effects arising between the earthquake scenario and site condition, this factor would vary for different scenario earthquake events. That represents a unique and distinctive feature of the proposed attenuation relationship, in contrast to commonly employed attenuation models that are spatially smoothed, and are unable to deal with individual local and site effects.

It is noted that source and path parameters are readily available from the global database, *e.g.* CRUST 2.0 [9], whilst the local and site-specific parameters can be obtained by local geotechnical and geophysical measurements. The process of allowing local measurements to be incorporated directly into the crustal modification factor has been summarized in Chandler *et al.* [10]. The event-specific crustal quality factor Q_0 (herein termed simply Q) can be obtained if Q contour map is available.

As with empirical attenuation relationships, there are invariably very large uncertainties associated with the simulations of the seismological model. Aleatory uncertainty can be obtained by stochastic simulation of the seismological model, given the associated *PDF* for each input parameter listed in Table 1. It can then be represented by the standard deviation of the *PDF* for the response prediction and is expected to be site-specific. Also, this can be varied with the levels of shaking, different natural periods, and so forth.

4 Theoretical Development

In light of the aforementioned difficulties concerning the conventional approach, a new method, *Direct-Amplitude Based* (DAB) approach has been developed recently [4], based on the Cornell's approach, using the idea of considering an infinite number of sources, *i.e.* $N_s \to \infty$ in Equation (1).

In effect, every finite point can be considered as a "source", assuming that there is no repetition of earthquake occurrence at any individual point. This is reasonable, if the size of a finite point tends to zero. If no earthquake occurs at the point i,

$$f_i(m) = 0 \qquad \text{for all } m \qquad (3)$$
$$f_i(r) = 0 \qquad \text{for all } r \qquad (4)$$
$$\upsilon_i = 0 \qquad\qquad (5)$$

If an earthquake [m_i, r_i] occurs at a particular point i,

$$\begin{cases} f_i(m) = 1 & m = m_i \\ f_i(m) = 0 & m \neq m_i \end{cases} \qquad (6)$$

$$\begin{cases} f_i(r) = 1 & r = r_i \\ f_i(r) = 0 & r \neq r_i \end{cases} \qquad (7)$$

$$v_i = 1 \tag{8}$$

Hence, $\quad P[SV_T > z] = \sum_{i=1}^{n_i} P[SV_T > z \mid m_i, r_i]. \tag{9}$

where n_i is the number of historical earthquake events considered in the earthquake catalog. Unlike Cornell's approach, in which the adopted attenuation relationship is spatially smoothed, in Equation (9), $P[SV_T > z \mid m_i, r_i]$ can actually be site-specific and event-specific, so that local geological features, soil site effects, and event-specific characteristic can all be included. Hence, the event-specific conditional probability can be replaced by $P[SV_T > z \mid \Delta_{T,i}]$, where $\Delta_{T,i}$ is a variable representing the event-specific and site-specific median spectral velocity at natural period T of a particular event $[m_i, r_i]$.

Considering a subset of earthquake events $\{m_{j,k}, r_{j,k}\}$ that has a proper subset of median spectral responses $\{\Delta_{T,j,k}\}$, within a narrow band of $[\Delta_{T,j} - \delta\Delta_{T,j}/2, \Delta_{T,j} + \delta\Delta_{T,j}/2]$, then,

$$P[SV_T > z] = \sum_{j=1}^{N_j} \sum_{k=1}^{\delta n_j} P[SV_T > z \mid \Delta_{T,j,k}]$$
$$= \sum_{j=1}^{N_j} P[SV_T > z \mid \Delta_{T,j}] \delta n_j \tag{10}$$

where N_j is the number of subsets; δn_j is the number of events in subset j; and k is the element number in subset j. On the other hand, it is possible that the earthquake catalogues are complete for different time periods at different magnitude or intensity levels. Hence, similar to the development of a magnitude-recurrence relationship in the conventional methodology, a set of response-recurrence relationships at a series of structural natural periods can be obtained, so that catalogues of different completeness criteria can be combined together by statistical methods. It is proposed that a doubly truncated exponential recurrence relationship should be employed, with the consideration of maximum ($\Delta_{T,\max}$) and minimum values ($\Delta_{T,\min}$). $\Delta_{T,\max}$ can be used to account for a large event that has not been observed historically. Determining $\Delta_{T,\max}$ would be similar to performing deterministic SHA. This is also similar to the definition of the maximum magnitude for each source in the Cornell's approach and the concept of characteristic earthquake in Frankel's smoothed seismicity approach. Hence, the full range of possible earthquakes that could generate strong ground shaking at the site can be captured. Furthermore, it is emphasised that the spectral response computed here can be both site-specific and event-specific using the aforementioned attenuation relationship. A set of PDF $f(\Delta_T)$ of the spectral

responses can be obtained by differentiating the cumulative distribution function (CDF), derived from the response-recurrence relationship. Details of the response-recurrence relationship are given in the following section.

Substituting $\quad \delta n_j = f(\Delta_{T,j}) \delta\Delta_{T,j} N(\Delta_{T,\min})\quad$ into Equation (10), where $N(\Delta_{T,\min})$ is the mean rate of the spectral response (Δ_T) exceeding $\Delta_{T,\min}$. If $\delta\Delta_{T,j} \to 0$, and $N_j \to \infty$, Equation (10) becomes

$$P[SV_T > z] = N(\Delta_{T,\min}) \int_{\Delta_{T,\min}}^{\Delta_{T,\max}} P[SV_T > z \mid \Delta_T] f(\Delta_T) d\Delta_T \tag{11}$$

In the derivation process, the cited problems associated with Cornell's PSHA methodology are avoided. More specifically, it is not necessary to characterize any seismic sources, because all events that significantly affect the site are included in the analysis, without considering the spatial distribution of seismicity. Moreover, the maximum magnitude can be defined in simpler terms, instead of assigning its value for each individual source. Actually, the proposed approach has also considered the geological information, when the attenuation relationships are obtained, but not for defining the sources.

In this study, spectral response is obtained by the computer program GENQKE [11], in which the structural damping ratio is user-specified. In other words, the UHS for different structural damping ratios can be computed.

Also, any event-specific characteristic that profoundly influences ground motion, such as directivity, near-fault displacements, and basin effects can be incorporated in the early stage of the numerical process. In other words, as these extra effects are considered in the attenuation relationship, the form of Equation (11) can still be adopted, without further adding a conditional probability term, such as, to account for the nonlinear site effects [12].

5 Response-Recurrence Relationships

For the doubly truncated exponential recurrence relationship for the logarithm of the spectral response amplitude, the CDF can be expressed as

$$F(\Delta_T) = \frac{\Delta_{T,\min}^{-b_T} - \Delta_T^{-b_T}}{\Delta_{T,\min}^{-b_T} - \Delta_{T,\max}^{-b_T}} \tag{12}$$

The number of events leading to spectral response exceeding certain amplitude is

$$N(\Delta_T) = N(\Delta_{T,\min}) \left(\frac{\Delta_T^{-b_T} - \Delta_{T,\max}^{-b_T}}{\Delta_{T,\min}^{-b_T} - \Delta_{T,\max}^{-b_T}} \right) \tag{13}$$

Further, the *PDF* can be obtained by differentiating the *CDF* with respect to Δ_T.

$$f(\Delta_T) = \frac{b_T \Delta_T^{-(b_T+1)}}{\Delta_{T,\min}^{-b_T} - \Delta_{T,\max}^{-b_T}} \qquad (14)$$

For the b_T–parameter, maximum likelihood estimation has been adopted. The b_T–parameter for each response-recurrence relationship may be obtained from

$$b_T = \frac{\overline{\Delta}_T}{\overline{\Delta}_T - \Delta_{T,\min}} \qquad (15)$$

where $\overline{\Delta}_T$ is the mean or the expected value of Δ_T.

Response-recurrence relationships have been obtained for the whole period range of engineering interest. Fig. 1 shows the doubly truncated spectral velocity response-recurrence relationship using Equation (13) for T = 0.2 s. Three independent studies have been employed in defining the maximum spectral amplitude [13-15]. Fig. 2 shows the variation of the b_T–parameter from each response-recurrence relationship. The b_T–parameter indicates the relative distribution between different levels of response amplitudes. It is observed that the value can greatly vary from around 1.5 to 3.2.
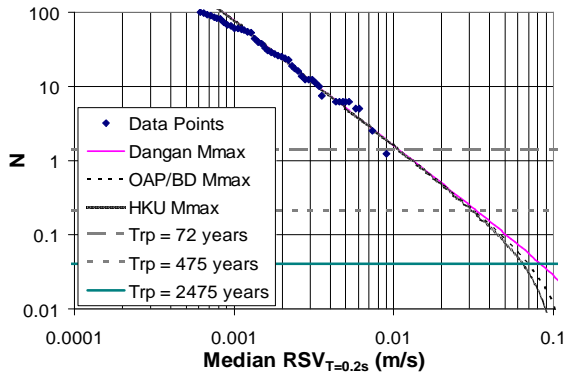

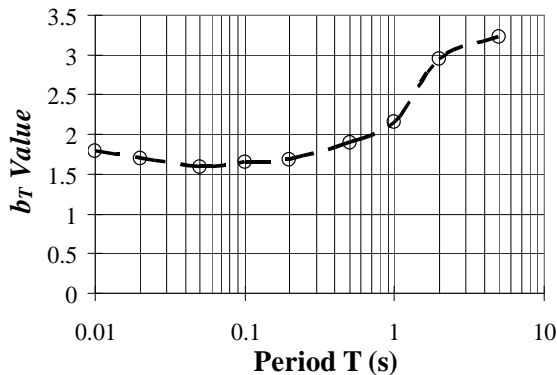Fig. 1: Doubly truncated spectral velocity response-recurrence relationship for T = 0.2 s.

Fig. 2: The b_T-parameter of each response-recurrence relationship obtained by Equation (15).

6   Results

A generic analytical solution has been derived for the proposed *DAB* approach [Equation (16)], in order to compute the probabilities of exceeding different levels of SV_T, and hence form a seismic hazard curve for each natural period. As there is no specification of seismic sources, and also, the spectral response amplitudes are computed before performing the integration, the closed-form solution would hence be generic in its nature. This forms a significant additional advantage of the proposed approach.

$$P[SV_T > z] = \frac{\eta_T}{z^{b_T}} \left\{ \exp\left(\frac{m_T^2}{2}\right) \left[D(u_T + m_T) - \frac{1}{2} \right] \right.$$
$$\left. - \exp(-m_T u_T) D(u_T) \right\}_{\Delta_{T,\min}}^{\Delta_{T,\max}} \qquad (16)$$

where $\quad u_T = \frac{1}{\sigma_T} \log\frac{\Delta_T}{z}$, $\quad m_T = \frac{b_T \sigma_T}{\log e}$

and $\quad \eta_T = \frac{N(\Delta_{T,\min})}{\Delta_{T,\min}^{-b_T} - \Delta_{T,\max}^{-b_T}}$

It is noted that, apart from the b_T–parameter, period-dependent behaviour also exists in other parameters, including the standard deviation of the response spectral attenuation relationship σ_T (which was held constant at 0.3 for the whole range of periods), as well as the minimum and maximum response amplitudes ($\Delta_{T,\min}$, $\Delta_{T,\max}$) (refer Table 2).

Table 2: Maximum and minimum spectral response amplitudes ($\Delta_{T,\max}$, $\Delta_{T,\min}$) of recurrence relationships.

T	0.1	0.2	0.5	1.0	2.0	5.0
$\Delta_{T,\max}$ [13]	70.8	105	118	106	94.6	55.0
$\Delta_{T,\max}$ [14]	84.8	127	151	130	106	62.3
$\Delta_{T,\max}$ [15]	172	297	437	379	227	128
$\Delta_{T,\min}$	0.45	0.91	2.27	3.65	5.46	4.97

A set of seismic hazard curves can be computed, and hence, site-specific UHS can be constructed. Two pairs of velocity UHS with different return periods (72 and 2475 years) have been shown in Fig. 3. Results obtained using the proposed DAB approach were plotted in solid lines, whilst the pair plotted in dashed lines was obtained using Cornell's approach [14]. It is observed that results obtained using the two approaches are in the same order.

The velocity UHS in Fig. 3 have been converted to displacement UHS as shown in Fig. 4. More significant difference can be observed especially for longer natural periods. Noted that displacement, as opposed to force (associated to the acceleration response), has been recognised as the preferred criterion on seismic design and performance assessment of buildings and infrastructures.

It is emphasized that the full potential of the proposed approach could be realized by applying it to soil sites, for which the site-to-site variability is more significant.
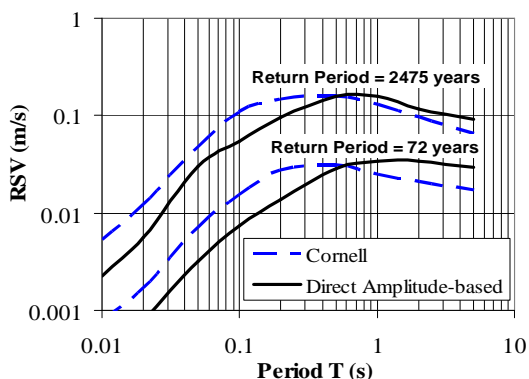

Fig. 3: Velocity UHS obtained using the proposed DAB approach, in comparison with that obtained using Cornell's approach.
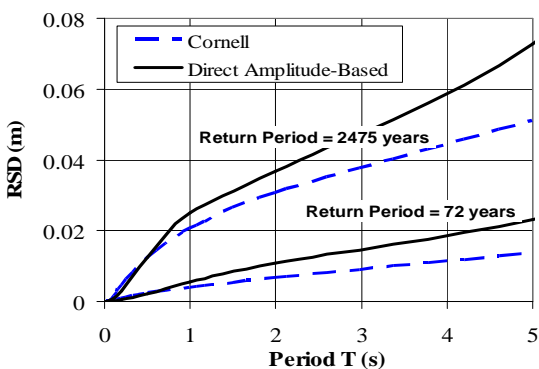

Fig. 4: Displacement UHS obtained using the proposed DAB approach, in comparison with that obtained using Cornell's approach.

7 Conclusions

1. A new procedure for constructing UHS has been presented, which can avoid those problems associated to the Cornell's PSHA methodology.
2. A generic site-specific and event-specific attenuation relationship has been introduced, and its suitability and advantages have been discussed.
3. The response-recurrence relationship and the corresponding *b*-parameter were presented.
4. An example was presented citing Hong Kong as a case study.

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