# Site-Dependent Response Spectral Attenuation Modelling

HING-HO TSANG
Department of Civil Engineering
The University of Hong Kong
Pokfulam Road
HONG KONG

NELSON T. K. LAM
Department of Civil and Environmental Engineering
The University of Melbourne
Victoria 3010
AUSTRALIA

S. H. LO
Department of Civil Engineering
The University of Hong Kong
Pokfulam Road
HONG KONG

*Abstract:* A site-dependent response spectral attenuation model is an important element in a site-dependent seismic hazard assessment. Seismic attenuation behaviour is controlled by a large number of wave modification mechanisms, some of which have characteristics specific to a local area or a particular site, whilst others can be generalised to the entire seismic region. Factors representing these mechanisms are often not resolved. An attenuation modelling approach is demonstrated in this paper, to evaluate individual regional and local wave modification factors. The upper-crust amplification factor computed from the modelled rock shear wave velocity profile was then combined with predicted attenuation parameters to determine the upper-crust modification filter function associated with Singaporean geological formations. Stochastic simulations of the seismological model for the magnitude 9.3 Aceh earthquake (Indonesia) on the 26th of December in 2004, were performed and compared with the response spectra recorded on a rock site in Singapore.

*Key-Words:* seismological model, attenuation, distant earthquake, response, Aceh, Indonesia, Singapore

## 1. Introduction

A site-dependent ground motion or response spectral attenuation model forms one of the key components in a site-dependent seismic hazard assessment, either deterministic or probabilistic. Attenuation relationships provide predictions for the intensity of ground shaking for any given earthquake scenario, expressed principally in terms of a combination of earthquake magnitude (*M*) and source-site distance (*R*).

The attenuation behaviour of earthquake ground shaking is highly complex, but can be approximated by a series of “filters”, each of which represents a seismic wave generation (or modification) mechanism along its entire transmission path between the source (at depth) and the site of interest (on the surface). The properties of these filters can be generalised to a region, an area or a site depending on the considered mechanism and the method of modelling. Thus, attenuation factors can be classified into (i) regional factors, (ii) local factors and (iii) site factors.

Despite distinctions between the three tiers of mechanism, their effects have seldom been resolved in existing attenuation models. In regions of high seismicity such as California where strong motion records are abundant, attenuation relationships are developed typically by regression of recorded ground shaking parameters of the whole region. Consequently, local conditions within the region have not been parameterised.

In the absence of recorded strong motion data in regions of low to moderate seismicity, attenuation modelling may be based on ground motions simulated stochastically in accordance with the seismological model, which characterises earthquake properties by their frequency content. An important advantage is that each event can be assigned a set of filter functions. Importantly, the various regional and local mechanisms identified above are represented by separate source and path factors within such a model. For instance, the anelastic whole path attenuation can be characterised by an event-dependent quality (Q) factor of the rock crust, along the travel path of the seismic waves.

## 2. Overview of Seismological Model

In the seismological model, the Fourier amplitude spectrum of displacement $A(f)$ of seismic waves reaching the exposed surfaces of bedrock may be expressed as the product of a number of factors:

$$A(f) = C\, M_0\, S(f)\, G\, An(f)\, V(f)\, P(f) \qquad (1)$$

where $C$ is a scaling factor, $M_0$ the seismic moment, $S(f)$ the regional source spectrum, $G$ the regional geometric attenuation factor, $An(f)$ the regional anelastic whole path attenuation filter, $V(f)$ the local upper crust amplification filter, and $P(f)$ the local upper crust attenuation filter.

### 2.1 Regional Source Factor $S(f)$ and Mid-crust Factor $\gamma_{mc}$

Regional, and generic, source factors have been used to generalise the average behaviour of seismic waves generated at the source of the earthquake to the whole region. For example, the generic source factor of intra-plate earthquakes as developed by Atkinson [1] has been used to represent conditions in the whole of Central and Eastern North America (CENA). The generic source factor $S(f)$ (for displacement amplitude) defining the Fourier spectrum of the seismic shear waves generated at the source of the earthquake is given in Equations (2) to (8):

$$S(f) = CM_0\,[(1-\varepsilon)\, S_A + \varepsilon\, S_B] \qquad (2)$$

where

$$S_A = 1 / [\,1 + (f / f_A)^2] \qquad (3)$$

$$S_B = 1 / [\,1 + (f / f_B)^2] \qquad (4)$$

$$C = R_p F V / 4\pi\rho\beta^3 \qquad (5)$$

$M_0$ is the seismic moment, $R_p$ the wave radiation factor, $F$ the free surface amplification factor, $V$ the factor partitioning energy in the two orthogonal directions (the product $R_pFV$ has been taken as 0.78 [2] ) $\rho$ the density of the rock at the depth of rupture and $\beta$ the shear wave velocity (*SWV*) of the rock at the depth of rupture. The intra-plate source model was based on the generic *hard rock* conditions of CENA with $\rho$ = 2.8 t/m$^3$ and $\beta$ =3.8 km/s at a depth of approximately 12 km.

The magnitude-dependent corner frequencies $f_A$, $f_B$ and the proportioning factor $\varepsilon$ are listed as follows:

$$\log f_A = 2.41 - 0.533\, M \qquad (6)$$

$$\log f_B = 1.43 - 0.188\, M \qquad (7)$$

$$\log \varepsilon = 2.52 - 0.637\, M \qquad (8)$$

where $M$ is the moment magnitude which has also been denoted as $M_w$.

The amplitude of *S*-waves generated from the source of the earthquake is inversely proportional to the *SWV* of the surrounding crust raised to a power of 3, according to Equation (5). Adjustments to allow for other parameter values can be made through the mid-crust modification factor, defined as:

$$\gamma_{mc} = \left(\frac{3.8}{V_{s,8}}\right)^3 \left(\frac{2.8}{\rho_8}\right) \qquad (9)$$

where $V_{s,8}$ and $\rho_8$ is the crustal *SWV* and density respectively at 8 km depth.

### 2.2 Regional Geometrical Attenuation Factor *G*

The Geometrical (*G*) factor represents the attenuation of the amplitude of the radiated seismic waves resulting purely from the geometrical spread of energy (as opposed to dissipation of energy). The *G* factor in the near-field conforms to spherical attenuation and is independent of regional conditions [refer Equation (10)]. The *G* factor becomes regionally dependent in the far-field where the attenuation pattern is influenced significantly by seismic waves reflected from the Moho discontinuity, which defines the interface between the earth's crust and the underlying lithosphere. The significance of the Moho reflection increases with decreasing thickness of the earth's crust, according to Equations (11) and (12).

$$G(R,D) = \frac{30}{R} \quad (\text{for } R \le 1.5\, D) \qquad (10)$$

$$G(R,D) = \frac{30}{1.5D} \quad (\text{for } 1.5\text{ D} \le R \le 2.5\, D) \qquad (11)$$

$$G(R,D) = \frac{30}{1.5D}\sqrt{\frac{2.5D}{R}} \quad (\text{for } R \ge 2.5\, D) \qquad (12)$$

where $R$ is the source-site distance of the earthquake and $D$ is the crustal thickness.

### 2.3 Regional Whole Path Anelastic Attenuation Factor *An* ( *f* )

Whole path attenuation is particularly important in the modelling of ground shaking from distant earthquakes [3]. Large-magnitude earthquakes generated at source-site distance (*R*) exceeding 100 km are typified by low-frequency (long-period) seismic waves, since the high-frequency components have greatly diminished in amplitude as a result of energy absorption along the source-site wave travel path. The attenuation mechanism may be characterized by the value of the seismological quality factor *Q* (equivalent to $Q_0$, namely *Q* at frequency of 1 Hz) as obtained from seismological monitoring in the region. The value of *Q* may be substituted into Equation (13) to develop the filter function *An*( *f* ) representing the effects of whole path attenuation of seismic waves propagating within the earth's crust:

$$An(f) = e^{-\frac{\pi . f . R}{Q(f).V_s}} \qquad (13)$$

where *f* is the wave frequency, *R* is the length of the wave travel path and $V_s$ is the *SWV*.

The *Q*( *f* ) function is then defined by:

$$Q(f) = Q_0 f^{\eta} \qquad (14)$$

Substitution of Equation (14) into Equation (13) yields the estimated whole path attenuation factor.

An empirical correlation between $Q_0$ and $V_{uc}$ has been developed [4]:

$$Q_0 = 100 + 2.5 V_{uc}^{4.5} \quad [V_{uc} \geq 1.6 \text{ km/s}] \qquad (15)$$

Further, an empirical correlation between $\eta$ and $Q_0$ based on global database has been developed [5]:

$$\eta = 0.0000008 Q_0^{\,2} - 0.0014 Q_0 + 0.93 \qquad (16)$$

### 2.4 Local Upper Crustal Amplification Factor *V* ( *f* )

Upwardly propagating shear waves are amplified when the waves cross from one medium to a lower velocity medium and can be explained by the principle of conservation of energy. Upper-crust amplification is a function of the *SWV* profile (its value and gradient) in the earth's crust, particularly at shallow depths and is period (or frequency) dependent. The extent of upper-crust amplification may be predicted from Equation (17), using $\rho_B$ and $V_B$ to represent the rock density and *SWV* at the source depth, which is typically assumed as $z = D = 8$ km, and $\rho_A$ and $V_A$ at a depth corresponding to the period of interest.

$$V(V_s, \rho) = \sqrt{\frac{\rho_B V_B}{\rho_A V_A}} \qquad (17)$$

To relate the period of interest to the rock depth, the quarter-wavelength approximation method [6] is required. This method allows the values of $\rho_A$ and $V_A$ to be averaged to a depth equivalent to the quarter-wavelength of the upwardly propagating shear wave, for applying Equation (17).

### 2.5 Local Upper Crustal Attenuation Factor *P* ( *f* )

Wave transmission quality within bedrock is not uniform with depth. Attenuation in the upper crust is a local phenomenon and is represented by a local factor and the mechanism occurs over a short transmission distance, as for attenuation in soft soil sediments.

The upper crustal attenuation factor *P* ( *f* ) in the seismological model has been defined by Equation (18):

$$P(f) = e^{-\pi f \kappa} \qquad (18)$$

where the parameter $\kappa$ (in units of seconds and pronounced "*Kappa")* can be measured from analysis of the Fourier transform of seismic waves recorded from the very near-field [7].

The parameter $\kappa$ is generally difficult to measure in regions of low and moderate seismicity because of magnitude or epicentral distance requirements associated with the measurements. A method for estimating $\kappa$ is to make inferences from the *SWV* near to the rock surface. Empirical correlations of $\kappa$ with the average *SWV* of the upper crust $V_{uc}$ (taken as the upper 4 km depth) as well as at 30 m depth, have been developed [4]:

$$\kappa = 0.145 - 0.12 \ln(V_{uc}) \geq 0 \quad [V_{uc} \geq 1.6 \text{ km/s}] \qquad (19)$$

$$\kappa = \frac{0.057}{V_{s,0.03}^{\,0.8}} - 0.02 \qquad [0.5 \text{ km/s} \leq V_{s,0.03}] \qquad (20)$$

### 2.6 Soil Site Response Function *F* ( *f* )

Experience from previous earthquakes has repeatedly shown that the intensity of ground shaking, and the intensity of the damage produced, are strongly influenced by local site conditions, in particular the influence of relatively shallow geologic materials on (nearly) vertically propagating body waves. As soils behave nonlinearly when subjected to strong levels of ground shaking, it is more appropriate to account separately for site effects from bedrock and soil layers, and hence a site response transfer function $F\,(\,f\,)$ can be added to Equation (1).

It is emphasized that the soil site response function $F\,(\,f\,)$ is not a single value for a specific site, as it depends on the level of soil damping and is, in turn, related to the shaking level. Together with different resonant conditions, which are interactive effects arising between the earthquake scenario and site condition, the soil site response factor would vary for different scenario earthquake events. That represents a unique and distinctive feature of using the combination of the seismological model and site response transfer function as the attenuation model.

### 2.7 Uncertainties

As with empirical attenuation relationships, there are invariably very large uncertainties associated with the stochastic simulations of the seismological model. The uncertainties in ground motion are now generally classified into two different types, namely aleatory and epistemic. Aleatory uncertainty is related to the unpredictability of the earthquake generating process, and cannot be reduced by collecting additional information. It can be obtained by stochastic simulation of the seismological model, given the associated probability density function (*PDF*) for each input parameter. The aleatory uncertainty can then be represented by the standard deviation of the *PDF* for the ground motion prediction and it is expected to be a site-dependent quantity and is greatly affected by the availability of the level of information. Hence, by undertaking a site-by-site assessment, the aleatory uncertainty can be more accurately quantified. Also, this quantity can be varied with the level of ground shaking, different frequency components of seismic waves, and so forth.

Epistemic uncertainty is due to the lack of knowledge of the earthquake processes, which in theory can be reduced through the accumulation of greater knowledge. Hence, more detailed measurements of the earth's crust, as demonstrated in a companion paper [8], could reduce the epistemic uncertainties associated with the attenuation relationship. This type of uncertainty can also be resolved by using a logic-tree formulation, which can be used to represent alternative, scientifically valid, models and their probabilities. Input parameter alternatives are represented by a branch point in the logic tree. A logic tree can be sampled using a Monte Carlo technique [9] to generate a set of alternative models, and results in a *PDF* of possible ground motions at the end of all its branches.

## 3 Stochastic Simulations of Aceh Earthquake in 2004

### 3.1 Background Information

The city state of Singapore, which is located at the southern tip of the Malay Peninsula, is on a stable part of the Eurasian plate and is in a region of low seismicity. Singapore, has a long history of experiencing tremors generated from long distance earthquakes in the Sumatra fault about 450 km away as well as from the subduction zone off-shore of Sumatra. This major seismic source generated the magnitude 9.3 "Aceh" earthquake on the 26th of December, 2004, which was amongst the largest magnitude earthquakes recorded worldwide and was the cause of the South Asian tsunami which has a death toll of over 220,000. Singapore was not subject to the tsunami and was subject to only very light ground shaking in this event because it was some 1000 km away from the fault source (refer Fig. 1).

However, the thousands kilometer long subduction zone which generated this earthquake has a closest distance to Singapore of around 600 km. Seismic hazard modelling for Singapore in the future will need to consider earthquake magnitudes in this range. Before these major events, there was already growing concern in Singapore of the need to address seismic risks and develop effective mitigation measures, in view of a high population density, concentration of commercial activities and local engineering practices that have not embraced aseismic design principles.


Fig. 1 Location of Aceh earthquake in 2004.

### 3.2 Seismological Parameters

The mean focal depth of the interface subduction source was taken as h = 20 km in the seismological modelling. This assumption is close to h = 24 km as assumed in other simulation studies conducted in the region previously. With this depth range it was considered that $\rho = 2.8$ t/m$^3$ and $\beta = 3.7$ km/s, which are consistent with values estimated from CRUST2.0 [10]. The mid-crust modification factor is accordingly equal to 1.0 – 1.1. The value of *D* for the region has been identified using data from CRUST2.0 to be 30 km. The Quality Factor $Q_0$ = 150 and the upper crustal attenuation factor $\kappa$ = 0.02 have been obtained from Equations (15) and (19) and by making inferences from the regional shear wave velocity parameters.

To develop representative local factors representing these two mechanisms, the *SWV* profile of the crustal rock (refer Fig. 2) has been developed based on CRUST2.0 and the methodology of constructing rock SWV profiles [11].

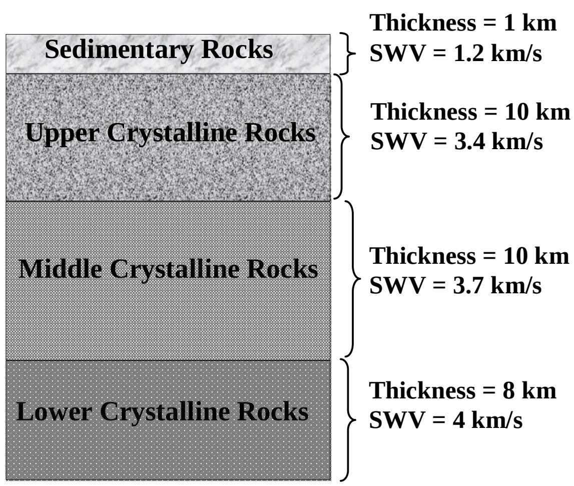
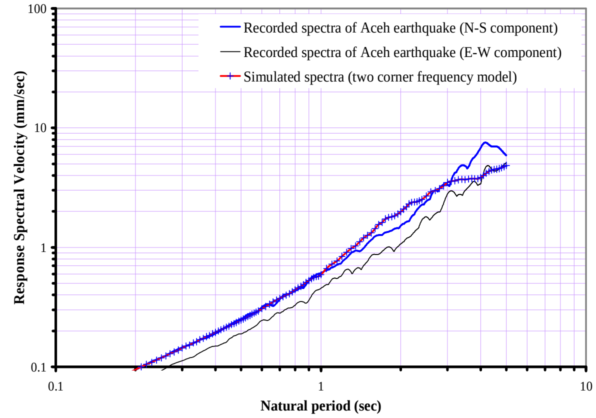



Fig. 2 Crustal model for region surrounding Singapore [10].

### 3.3 Comparison with Response Spectra Recorded in Singapore

Synthetic accelerograms have been simulated stochastically using the computer program GENQKE [12]. The response spectra calculated from 18 accelerograms with random phase angles were averaged for different earthquake scenarios. The average response spectra computed have been shown in Fig. 3. The response spectra recorded on a rock site in Singapore at both N-S and E-W directions for the Aceh earthquake have been plotted on the same graph for direct comparison.




ACEH earthquake M = 9.3 R = 1200 km
Based on extract from Lam *et al.* (submitted) [13]

Fig. 3. Comparison of recorded and simulated response spectra

The second earthquake generated from Nias (Mw = 8.7), which was comparable in magnitude to the Aceh earthquake, offered the opportunity to test the robustness of the stochastic model. The site-source distance of this second event from Singapore was only about 750 km (which was some 0.6 times the site-source distance of the Aceh earthquake). The record-model comparison for this second event (details not shown herein) displays a similar level of consistencies as with the first event at Aceh [13]. The very different site-source distance of the Aceh and Nias events means that significant record-model discrepancies would have surfaced with one of the events had the adopted attenuation parameters (Q in particular) been not representative of the real conditions of the wave travel path.

## 4 Conclusions

1. The state-of-the-art development of the seismological model has been reviewed. The attenuation behaviour of earthquake ground shaking can be classified into (i) regional factors, (ii) local factors and (iii) site factors.
2. The generic source factor of intra-plate earthquakes has been adopted, along with a mean focal depth of h = 20 km and accordingly a mid-crust amplification factor of 1.0 – 1.1. The geometrical attenuation relationship has been based on a regional crustal thickness of D = 30 km. The anelastic attenuation behaviour has been characterised by a $Q_0$ factor of 150.

3. The *SWV* profile has been obtained from CRUST2.0. The value of $\kappa = 0.02$ for the upper crust has been determined from an empirical expression obtained in a previous study. The upper crust amplification and attenuation filter functions have been modelled accordingly.
4. Artificial accelerograms were generated in accordance with the seismological model developed for Aceh earthquake. Response spectra were calculated accordingly for different earthquake scenarios.
5. The recorded and simulated response spectra have been shown to be very consistent with the interface subduction events even though a generic intraplate source model has been used in the simulations. This investigation into the major subduction events at Sumatra involving seismological modelling suggests a high degree of generality of the source model in spite of the fact that it was originally developed from the ECTN database for Eastern North America.

*Acknowledgments:*
The work described was supported by a grant from the Research Grants Council of Hong Kong Special Administrative Region (Project No. HKU7117/04E), whose support is gratefully acknowledged. Contributions from Prof T Balendra from the *National University Singapore* and *Meteorological Services Singapore* in the supply of the recorded data are also gratefully acknowledged.

*References:*
[1] Atkinson GM. Earthquake Source Spectra in Eastern North America. *Bull. Seism. Soc. Am.*, Vol. 83, 1993, pp. 1778-1798.
[2] Lam N, Wilson J, Chandler A, Hutchinson G. Response Spectral Relationships for Rock Sites derived from Component Attenuation Model. *Earthquake Eng. Structural Dyn.*, Vol. 29, 2000, pp. 1457-1489.
[3] Chandler A, Lam N. An Attenuation Model for Distant Earthquakes. *Earthquake Eng. Structural Dyn.*, Vol. 33, No. 2, 2004, pp. 183-210.
[4] Chandler AM, Lam NTK, Tsang HH. Near-Surface Attenuation Modelling based on Rock Shear Wave Velocity Profile. *Soil Dyn. Earthquake Eng.*, Vol. 26, No. 11, 2006, pp. 1004-1014.
[5] Mak S, Chan LS, Chandler AM, Koo RCH. Coda Q Estimates in the Hong Kong Region. *Journal Asian Earth Sci.*, Vol. 24, 2004, pp. 127-136.
[6] Boore DM, Joyner WB. Site Amplifications for Generic Rock Sites. *Bull. Seism. Soc. Am.*, Vol. 87, No. 2, 1997, pp. 327-341.
[7] Anderson JG, Hough SE. A Model for the Shape of the Fourier Amplitude Spectrum of Acceleration at High Frequencies. *Bull. Seism. Soc. Am.*, Vol. 74, No. 5, 1984, pp. 1969-1993.
[8] Tsang HH, Lam NTK, Lo SH. The Importance of Crustal Shear Wave Velocity Profile for Ground Motion Modelling. *Proc. 1st IASME/ WSEAS International Conference on Geology and Seismology*, Slovenia, 15-17 May, 2007.
[9] Cramer CH, Petersen MD, Reichle MS. A Monte Carlo Approach in Estimating Uncertainty for a Seismic Hazard Assessment of Los Angeles, Ventura, and Orange Counties, California, *Bull. Seism. Soc. Am.*, Vol. 86, 1996, pp. 1681-1691.
[10] CRUST2.0. 2001. Institute of Geophysics and Planetary Physics, Univ. of California, San Diego. Website: http://mahi.ucsd.edu/Gabi/rem.dir/crust/crust2.html
[11] Chandler AM, Lam NTK, Tsang HH. Shear Wave Velocity Modelling in Crustal Rock for Seismic Hazard Analysis. *Soil Dyn. Earthquake Eng.*, Vol.25, 2005, pp. 167 – 185.
[12] Lam NTK, Wilson JL, Hutchinson G. Generation of Synthetic Earthquake Accelerograms using Seismological Modelling: A Review. *J. Earthquake Eng*, Vol. 4, No. 3, 2000, pp. 321-354.
[13] Lam NTK, Balendra T, Wilson JL and Srikanth V (submitted). Response Spectrum Estimates for Long Distance Earthquakes Affecting Singapore, Submitted for publication in *Journal of Engineering Structures*.