Port-Based Modelling for Open Channel Irrigation Systems

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Abstract: A port-based model for the water flow and level dynamics in open channel systems is derived. It is structured into dissipative and conservative sub-systems related through a symplectic geometric structure which expresses instantaneous power conservation. This model is equivalent to the classical Saint-Venant equations (nonlinear PDEs also called shallow water equations) but trivially exhibits some interesting properties (passivity, stability, stored energy, entropy production) which may be useful for analysis or control purposes. The use of a geometric reduction leads to a reduced port-controlled hamiltonian (PCH) system. A Comparison between simulation results and experimental ones , obtained on an experimental micro-channel , demonstrates the effectiveness of the reduced model.

Key–Words: Irrigation channel, Shallow water equations, Port Hamiltonian system, Stokes-Dirac structure, geometric reduction of distributed parameter system.

1 Introduction

Usually the dynamics of a single reach is modelled by a set of non linear hyperbolic partial differential equations : the Saint-Venant equations [1]. They are derived from mass and kinetic momentum balances in an infinitesimal length of the reach. Written in closed form these equations are rather intricate to analyze, control or even simulate. Although the model is supposed to express mass and momentum conservation, it is almost impossible to prove its dynamics stability, specially with the nonlinear boundary conditions introduced by the gates constitutive equations. In the same way passivity is intuitively obvious but very difficult to prove even though the model is built upon conservation of energy and dissipation assumptions.
In this paper, we propose an interesting modelling alternative to the classical set of partial differential equations and boundary conditions for reaches of open-air channels and hydraulic works. It is based on a modular description of kinetic and potential energy storage through the hydraulic network and on the phenomenological laws usually used for energy dissipation within the reaches or in the hydraulic works (gates). The various "energy-based" sub-models are connected through a symplectic differential interconnection structure (the same as the one used for the Maxwell equations, for instance). This model can be stated independently of the specific geometry of the reaches, boundary conditions may be chosen freely (it is in some sense a-causal) and some of its dynamic properties are trivial to establish (stability, passivity, stored energy, dissipation map or entropy production). Let us notice that contrarily to the work [2], where a similar approach is developed, our model applies for the general case of interconnected reaches with bed frictions and slopes.
We propose also a reduction scheme which preserves basic energetic properties of the Saint-Venant PDE model. This scheme is an extension of the mixed finite element method developed for Maxwell's equations in [3] and [4].
The paper is organized as follows. In section 2, we derive the port based model for the shallow water equations with a suitable choice of energy variables by using the expression of the total energy of the fluid flowing in the channel to define efforts variables (see [5] for the principles of port based modelling) . Constitutive relation of the dissipation is defined using the Manning-Strickler formula.
In section 3, a geometric reduction scheme based on the mixed finite element method is presented. This method leads to a reduced Port Hamiltonian system. It uses different approximation bases (one for each space of the differential forms) in such a way that the geometric structure of the model is preserved in the approximation space.
In section 4, Comparisons between simulations of the obtained reduced model and experimental results which are obtained using an experimental micro-channel are shown. Finally main features and properties of the method are recalled in the conclusion and

future works related to passivity based control of the reduced port hamiltonian model are presented.

2 The Port-based modelling

2.1 Shallow water equations

Consider a rectangular open channel with slope I, length L and width B and delimited by upstream and downstream gates. The flow and level dynamics of water in this channel is modelled by Shallow water equations which are derived from conservation laws of mass and momentum using some assumptions on the flow. First, we suppose that the slope is small and constant, to be able to approximate $sin(I) \simeq I$, The length is sufficiently long compared to the height of water level, the pressure repartition is supposed hydrostatic and the fluid is supposed to be incompressible ($\rho = cst$), we neglect also the internal viscosity effects. Equations (1) give the one- dimensional case.

$$\begin{cases} \dfrac{\partial h}{\partial t} = -\dfrac{1}{B}\dfrac{\partial Q}{\partial x} \\[2ex] \dfrac{\partial Q}{\partial t} = -\dfrac{1}{B}\dfrac{\partial (Q^2/h)}{\partial x} - gBh\dfrac{\partial h}{\partial x} - gBh(J-I) \end{cases} \tag{1}$$

with $h(x,t)$ the height water level, $Q(x,t)$ the water flow, g the acceleration of gravity and $J(Q,h)$ the friction slope which can be evaluated using the Manning-Strickler formula [1]:

$$J(Q,h) = \frac{Q|Q|}{K^2(Bh)^2(\frac{Bh}{2h+B})^{\frac{4}{3}}} \tag{2}$$

with K the Manning-Strickler parameter.

2.2 Port-based model of shallow water equations

Since the shallow water equations are derived from the mass and kinetic momentum balances, we consider an elementary volume (3) and a momentum density (4) as energy (state) variables. they are 1-differential forms on the spatial domain $Z = [0, L]$.

$$q(x,t) = Bh(x,t)dx \qquad \in \Omega^1(Z) \tag{3}$$

$$p(x,t) = \rho v(x,t)dx \qquad \in \Omega^1(Z) \tag{4}$$

$v(x,t)$ is the water velocity and ρ the mass density of water.

The energy density $\mathcal{H}$ of the fluid given in equation (5) is obtained by integrating its potential and kinetic energies within an elementary column of fluid. Then the total energy of the fluid H is obtained by integrating this energy density along the spatial domain Z.

$$\begin{aligned} H(t) &= \int_0^L \mathcal{H}(x,t) \\ &= \frac{1}{2}\int_0^L (\rho g B h^2 - 2\rho B I h g x + \rho B h v^2)dx \end{aligned} \tag{5}$$

From the expression of the energy of the fluid, we define distributed efforts variables (co-energy variables), calculated as a first order derivatives of energy density relatively to energy variables (called also variational derivatives):

$$\begin{aligned} e_q(x,t) &:= \delta_q H = \frac{1}{2}\rho v^2(x,t) + \rho g(h(x,t) - Ix) \\ e_p(x,t) &:= \delta_p H = Bh(x,t)v(x,t) \end{aligned} \tag{6}$$

These efforts are functions on the spatial domains (called 0-differential forms $\Omega^0(Z)$). The first effort ($e_q(x,t)$) is generally called hydrodynamic pressure and the second ($e_p(x,t)$) represents the flow of water in the channel.

Using the port variables defined above, we can write a canonical coupling of the two energy domains (kinetic and potential) by the mean of the canonical symplectic structure [5].

$$\begin{bmatrix} -\frac{\partial q}{\partial t} \\ -\frac{\partial p}{\partial t} \end{bmatrix} = \begin{bmatrix} 0 & d \\ d & 0 \end{bmatrix} \begin{bmatrix} \delta_q H \\ \delta_p H \end{bmatrix} \tag{7}$$

$$\begin{bmatrix} e_\partial^0(t) \\ e_\partial^L(t) \\ f_\partial^0(t) \\ f_\partial^L(t) \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \delta_q H_{|x=0} \\ \delta_q H_{|x=L} \\ \delta_p H_{|x=0} \\ \delta_p H_{|x=L} \end{bmatrix} \tag{8}$$

The energy balance in absence of dissipation is:

$$\frac{dH}{dt} = \int_{\partial Z} e_\partial \wedge f_\partial \tag{9}$$

And the instantaneous power conservation may be written as the condition:

$$\int_Z [e_q \wedge f_q + e_p \wedge f_p] + \int_{\partial Z} e_\partial \wedge f_\partial = 0 \tag{10}$$

In the Saint-venant model, the fluid viscosity is neglected and the frictions with the banks and the water bed are modelled by the empirical Manning-Strickler formula [1]. These friction forces may be viewed as losses in the momentum balance equation and thus as a dissipative flow:

$$f_d = \rho g J(Q,h)dx \qquad \in \Omega^1(Z) \tag{11}$$

The empirical expression of the Manning-Strickler formula allows to define this flow as:

$$f_d = G_d \wedge e_p \tag{12}$$

where $G_d \in \Omega^1(Z)$ is the lineic distributed conductance along the spatial domain Z:

$$G_d = \frac{\rho g|Q|}{K^2(Bh)^2(\frac{Bh}{2h+B})^{\frac{4}{3}}}dx \tag{13}$$

The dissipated flow may be easily withdrawn from the momentum balance equation and incorporated in the whole model. One obtains:

$$\begin{bmatrix} -\frac{\partial q}{\partial t} \\ -\frac{\partial p}{\partial t} \end{bmatrix} = \begin{bmatrix} 0 & d \\ d & 0 \end{bmatrix} \begin{bmatrix} \delta_q H \\ \delta_p H \end{bmatrix} + \begin{bmatrix} 0 \\ f_d \end{bmatrix} \tag{14}$$

Finally the canonical structure defined by (14) and (8) with constitutive relations (3,4,6,12,13) is the port hamiltonian formulation of the so called shallow water equations. This formulation of the the shallow water equations is the same used for many distributed parameter system (e.g. Maxwell's equations, wave equations ...). The energy-balance equation becomes:

$$\frac{dH}{dt} = \int_{\partial Z} e_\partial \wedge f_\partial - \int_{\partial Z} e_p \wedge f_d \tag{15}$$

Since the kinetic energy of the fluid is *non-negative* and the potential energy is bounded from below, the total energy of the fluid is bounded from below. Thus energy-balance equation (15) shows that the port hamiltonian system (14) (8) is passive from the input e_∂ to the output f_∂ and inversely .

3 Geometric Reduction Scheme

Among the reduction schemes for the Saint-Venant model, we may distinguish between total discretization schemes which lead to discrete time models, we cite the Preissmann implicit finite difference scheme and partial (spatial) discretization schemes (usually using pseudo-spectral methods) leading to continuous time but finite-dimensional approximations like orthogonal collocation. the orthogonal collocation method has been frequently used for simulation and to design simplified "control" models (see [9] for input-output linearization, [10] for backstepping or [11] for robust optimal control). Unfortunately, this approximation is also difficult to implement : the number and location of collocation points greatly affect numerical and dynamical stability. Moreover, the collocation scheme does not preserve dynamical stability of the actual PDE model, nor its energetic properties (energy conservation, dissipativeness).

These are the reasons why we defined a reduction scheme which preserves basic energetic properties of the Saint-Venant PDE model, that is the potential and kinetic energy stored in the channel, the dissipation inequality and the canonical coupling between kinetic and potential energies. This scheme is an extension of mixed finite element method developped for Maxwell's equations [3]. It is here applied to the port-based model and leads to a finite dimensional approximation with exactly the same sub-models structure : each finite element is modelled as the interconnection of a conservative subsystem with a dissipative element through an interconnection structure which satisfies instantaneous power conservation and allows power exchanges through the boundaries of the element. The mixed finite element method takes into account the geometrical variables nature: in our case 0- and 1-differential forms. We apply it in such a way that the reduced Hamiltonian and the power-conserving interconnection structure remain preserved. This allows us to derive all the needed constitutive equations of the reduced system.

3.1 Basic principle of the reduction method

We present the principal stages of the used reduction method.First, spatial subdivision into elements (cells) of the total spatial domain Z of the channel is defined. On a generic element $Z_{ab} = [a, b]$. The flow and energy variables, which are 1-differential forms on Z_{ab}, are approximated as follows:

$$\begin{array}{c} q(x,t) = q^{ab}(t)\ w_{ab}(x) \\ p(x,t) = p^{ab}(t)\ w_{ab}(x) \\ f_q(x,t) = -\frac{\partial q}{\partial t} = f_q^{ab}(t)\ w_{ab}(x) \\ f_p(x,t) = -\frac{\partial p}{\partial t} = f_p^{ab}(t)\ w_{ab}(x) \end{array} \tag{16}$$

where $w_{ab}(x)$ is a 1-differential form on Z_{ab} satisfying:

$$\int_{\mathcal{Z}_{ab}} w_{ab}(x) = 1 \tag{17}$$

Under this condition, the total volume and momentum on Z_{ab} are equal,respectively, to the reduced variables $q^{ab}(t)$ and $p^{ab}(t)$.

For intensive quantitities, or 0-differential forms (functions) on Z_{ab}, we use a two-dimensional spatial approximation base. This means that we are using linear elements and approximations defined by:

$$\begin{array}{c} e_q(x,t) = e_q^a(t)\ \ w^a(x) + e_q^b(t)\ \ w^b(x) \\ e_p(x,t) = e_p^a(t)\ \ w^a(x) + e_p^b(t)\ \ w^b(x) \end{array} \tag{18}$$

where $w^a(x)$ and $w^b(x)$ are 0-differential forms on Z_{ab}. In order to equal the boundary values (at $x = a$ and $x = b$) and the reduced variables $e_q^a, e_q^b, e_p^a, e_p^b$, the 0-forms $w^a(x)$ and $w^b(x)$ should satisfy the following conditions:

$$w^a(a) = 1 \quad w^b(a) = 0 \quad w^a(b) = 0 \quad w^b(b) = 1 \tag{19}$$

approximated efforts and flows connected, at any time t and for all spatial coordinate x, one gets compatibility conditions between the two approximation bases:

$$w_{ab}(x) = \mathrm{d}w^b(x) \tag{20}$$

$$w_{ab}(x) = -\mathrm{d}w^a(x) \tag{21}$$

and the constitutive relations between reduced variables:

$$\begin{array}{c} f_q^{ab}(t) = -e_q^a(t) + e_q^b(t) \\ f_p^{ab}(t) = -e_q^a(t) + e_q^b(t) \end{array} \tag{22}$$

The relations between reduced and a boundary port variables in (8) may be summarized:

$$\begin{bmatrix} e_\partial^a(t) \\ e_\partial^b(t) \\ f_\partial^a(t) \\ f_\partial^b(t) \\ f_q^{ab}(t) \\ f_p^{ab}(t) \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 1 \\ -1 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} e_q^a(t) \\ e_q^b(t) \\ e_p^a(t) \\ e_p^a(t) \end{bmatrix} \tag{23}$$

As it can be noticed, this constitutive equation (like the infinite dimensional Dirac structure) is not injective. In order to define an acausal interconnection structure between port variables, we must define internal effort variables. These will be conjugate variables of the flow variables $f_q^{ab}(t)$ and $f_p^{ab}(t)$, using the approximation schemes (16,18) and the power product given in (10) on the spatial domain Z_{ab}. Doing so, one obtains as internal effort variables :

$$\begin{array}{c} e_q^{ab}(t) = [\alpha_{ab} e_q^a(t) + (1-\alpha_{ab}) e_q^b(t)] \\ e_p^{ab}(t) = [(1-\alpha_{ab}) e_p^a(t) + \alpha_{ab} e_p^b(t)] \end{array} \tag{24}$$

With

$$\alpha_{ab} = \int_{\mathcal{Z}_{ab}} w^a(x) w_{ab}(x) \tag{25}$$

Instantaneous power conservation is then written with the help of the following new *non degenerated* power product:

$$e_q^{ab} f_q^{ab} + e_p^{ab} f_p^{ab} - e_\partial^a f_\partial^a + e_\partial^b f_\partial^b = 0 \tag{26}$$

We obtain the finite-dimensional *Dirac* structure defined by the following constitutive relation between reduced variables:

$$E_{ab} e^{ab}(t) + F_{ab} f^{ab}(t) = 0 \tag{27}$$

where $f^{ab} := [f_q^{ab} f_p^{ab} f_\partial^a f_\partial^b]^T$ and $e^{ab} := [e_q^{ab} e_p^{ab} e_\partial^a e_\partial^b]^T$ are the vectors of conjugated flow and effort variables, and:

$$E_{ab} := \begin{bmatrix} -1 & 0 & -\alpha_{ab} & -(1-\alpha_{ab}) \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 \end{bmatrix}$$

$$F_{ab} := \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & (1-\alpha_{ab}) & \alpha_{ab} \\ 1 & 0 & 1 & -1 \\ 0 & 1 & 0 & 0 \end{bmatrix} \tag{28}$$

Using the base for the 1-forms, we write a reduced hamiltonian of (5) depending on the reduced variables $q^{ab}(t)$ and $p^{ab}(t)$:

$$\bar{H}(t) = \frac{1}{2}\frac{q^{ab}(t)^2}{C_{ab}} - \rho g I K_{ab} q^{ab}(t) + \frac{1}{2}\frac{q^{ab}(t)p^{ab}(t)^2}{L_{ab}} \tag{29}$$

where

$$\begin{array}{c} C_{ab}^{-1} = \int_{\mathcal{Z}_{ab}} *(\frac{w_{ab}(x)}{(\frac{B}{\rho g})}) w_{ab}(x) \\ L_{ab}^{-1} = \int_{\mathcal{Z}_{ab}} (\frac{*(w_{ab}(x))^2}{\rho}) w_{ab}(x) \\ K_{ab} = \int_{\mathcal{Z}_{ab}} x w_{ab}(x) \end{array} \tag{30}$$

C_{ab} is related to the accumulation of potential energy due to the water level in the element Z_{ab} (hydraulic capacitance), L_{ab} is related to the accumulation of kinetic energy (hydraulic inductance) in the same element and K_{ab} is related to the accumulation of potential energy resulting from the channel slope[1]. From the expression of the reduced hamiltonian, we may define effort variables as in any finite-dimensional port hamiltonian system (see [13]). We obtain:

$$e_q^{ab}(t) = \frac{\partial \bar{H}}{\partial q^{ab}} = \frac{q^{ab}}{C_{ab}} - \rho g I K_{ab} + \frac{p^{ab}(t)^2}{2L_{ab}} \tag{31}$$

$$e_p^{ab}(t) = \frac{\partial \bar{H}}{\partial p^{ab}} = \frac{q^{ab}(t)p^{ab}(t)}{L_{ab}} \tag{32}$$

In the same way,we define a reduced dissipative characteristic for the power dissipated by friction:

$$\begin{aligned} P_d &= \int_{\mathcal{Z}_{ab}} e_p(x,t) \wedge f_d(x,t) \\ &= \int_{\mathcal{Z}_{ab}} e_p(x,t) \wedge (G_d(q,p) \wedge e_p(x,t)) \\ &\simeq G_d^{ab}(q^{ab}, p^{ab}) \quad e_p^{ab}(t)^2 \end{aligned} \tag{33}$$

[1] In these definitions we use the *Hodge Star* operator which allows us to define functions from a 1-forms and reciprocally by a duality relation (see [3]). In our case we use the relations $*(q(x,t) = Bh(x,t)$ and $*(p(x,t)) = \rho v(x,t)$ for the state variables and $*(q^{ab}(t) \;\; w_{ab}(x)) = q^{ab}(t) \;\; * w_{ab}(x)$ and $*(p^{ab}(t) \;\; w_{ab}(x)) = p^{ab}(t) \;\; * w_{ab}(x)$ for their approximations.

where G_d^{ab} is the finite dimensional reduced conductance:

$$G_d^{ab} = *\bar{G}(q^{ab}, p^{ab}) \frac{\int_{\mathcal{Z}_{ab}} (*w_{ab}(x))^4 dx}{[\int_{\mathcal{Z}_{ab}} (*w_{ab}(x))^2 w_{ab}(x)]^2} \quad (34)$$

where $\bar{G}(q^{ab}, p^{ab})$ is derived from the approximation of the distributed conductance (using the chosen approximation base for 1-forms):

$$\bar{G} = g \frac{q^{ab}(t) p^{ab}(t)}{K^2 q^{ab}(t)^2} \left(\frac{B + \frac{2}{B} q^{ab}(t) \quad * w_{ab}(x)}{q^{ab}(t) \quad * w_{ab}(x)} \right)^{\frac{4}{3}} dx$$

We obtain then the reduced dissipative flow

$$f_d^{ab} = G_d^{ab}(q^{ab}, p^{ab}) e_p^{ab}(t) \quad (35)$$

This dissipative flow is incorporated into the reduced model by mean of a junction expressing the conservation of the total momentum flux (in the sense of parallel electric junctions):

$$\dot{p}^{ab}(t) + f_d^{ab}(t) + f_p^{ab}(t) = 0 \quad (36)$$

We have obtained a reduced PCH model which apply for any approximation bases choice. Making a particular choice of $w_{ab}(x)$ will allow us to derive the values of all physical parameters in the model. For instance, the choice:

$$w_{ab}(x) = \lambda dx \quad (37)$$

where λ is constant along Z_{ab}, this leads to functions $w^a(x) = \lambda(b - x)$ and $w^b = \lambda(x - a)$. Hence with conditions (19), we obtain $w_{ab}(x) = \frac{1}{(b-a)} dx$, $w^a(x) = \frac{(b-x)}{(b-a)}$ and $w^b(x) = \frac{(x-a)}{(b-a)}$. The numerical values of the reduced elements are then:

$$\begin{aligned} &\alpha_{ab} = \tfrac{1}{2},\ C_{ab} = \tfrac{B}{\rho g}(b-a),\ L_{ab} = \rho(b-a)^2 \\ &K_{ab} = \tfrac{b+a}{2} \\ &G_d^{ab} = \frac{g p^{ab}(t)}{K^2 q^{ab}(t)} \left(\frac{B^2(b-a) + 2q^{ab}(t)}{B q^{ab}(t)} \right)^{\frac{4}{3}} (b-a) \end{aligned} \quad (38)$$

Finally the finite *Dirac* structure (27,28) with the constitutive relations (29,31,32,36,38) defines a reduced implicit port controlled hamiltonian system equivalent to the Saint-Venant (shallow water).

4 Simulation and Experimental Results

We first consider a scenario in which the downstream gate remains with a constant opening $\theta_{down}(t) = 0.06m$ and in which the upstream gate is progressively opened with the help of a second order response (which is used in practice to model the dynamic of the motor driving the gate) from an initial position $\theta_{up}(t) = 0.03m$ to a final one $\theta_{up}(t) = 0.08m$. Simulation and experimental results of the cited scenario are presented in (Fig.1)

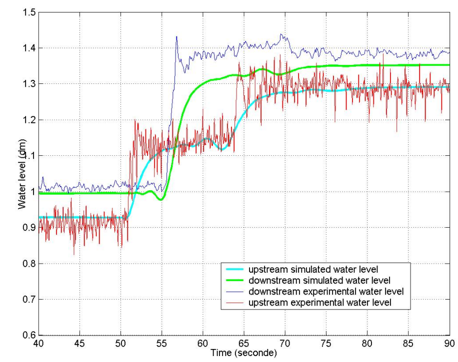

Figure 1: Upstream and Downstream heights of the water level response to progressive opening of upstream gate

We consider a second scenario where the upstream gate is closed progressively as in the first one from an initial position $\theta_{up}(t) = 0.08m$ to a final one $\theta_{up}(t) = 0.06m$. The downstream gate remains with a constant opening $\theta_{down}(t) = 0.06m$. The results are presented in (Fig.2).

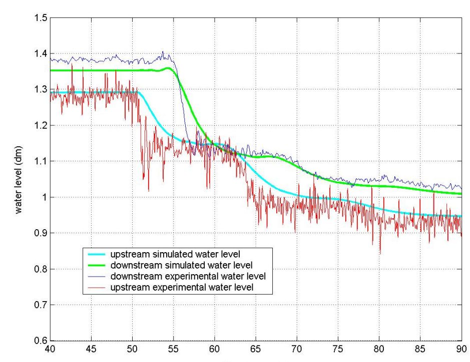

Figure 2: Upstream and Downstream heights of the water level response to progressive closing of upstream gate

From the results of this smooth scenarios we can extract some intrinsic properties of the flow dynamics like the flowing time from upstream to downstream, and conversely from downstream to upstream. These two different flowing times are dependent on the wave propagation velocity and the mean fluid velocity across the channel. They have been proven to be very accurate approximations of the measured values.

5 Conclusions and Future Works

5.1 Conclusions

In this paper, a port-based model of shallow water equations is derived which exhibits some interesting features. It is based on a description of the power transfers within the system and through its boundary which are explicitly expressed with the help of power-conserving geometric structure (called *Stokes-Dirac* structure). A geometric reduction scheme for shallow water equations is developed. This reduction scheme leads to a reduced port-controlled hamiltonian (PCH) system by defining a reduced elements with their reduced constitutive equations. Comparisons between simulations of the reduced model and experimental results, obtained using the experimental micro-channel, are presented and show that the reduced model recovers satisfactorily the dynamics of the micro-channel.

5.2 Future Works

We will develop the work presented here into two directions. The first one concerns generalizations and improvements of the geometric reduction scheme presented here. The second one concerns the use of the PCH reduced model as a control model for regulation using passivity based approaches.

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