Dynamic model and estimation of the future eutrophication for the Lake Prespa

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Abstract: - This paper presents a new computing analysis, based on Matlab Simulink, how to build a dynamic model for aquatic surface water bodies in our case the Lake Prespa. The dynamic model, which is more complex than the other models, it involves equation that describes the inside biological and chemical process of the lake and connections between living organisms and phosphorous component, and can be used to long-term prediction of lake behaviour and it's eutrophication.

The model that we are presenting has some limitations, because the nature of the problem is more chaotic than deterministic. We are limited by the changeable outside conditions and the other inside process that are existing into the lake, than we are not have considered. Our model takes only the differential equation that describes the phosphorus cycle inside of the lake.

The main contribution of building such model is providing us with future estimating of the eutrophication inside of the lake. In aid of this study is software application called Matlab Simulink which it has unique possibility to simulate a dynamic development (evolution) of the ecosystem for given parameters and input value of the measured components. Many scenarios that have been developed can show the future development of the ecosystem in long-term period, for over 10 years. The experimental results that we have gain are compared with measured values and it has error not more than 10%, which classify our model as good.

Key-words: - Lake Prespa, Dynamic Model, Eutrophication, Phosphorus concentration, Scenario ca, Scenario cc.

1 Introduction

Aquatic ecological models can be classified into three groups: empirical, dynamic and combine. These ecological models have proven to be very useful in solving ecological problems using computing technicians. Due to their ability to used little measured parameters to achieve their goal, the most successful ecological model for long-term prediction is the dynamic model. The dynamic model used with different scenarios and build in various computer programs has been very successful in achieving optimal solution for variety of problems based on prediction of the future state of the ecosystems.

This paper introduces the dynamic model for Lake Prespa, a rare aquatic ecosystem with his own flora and fauna. Lake Prespa has her own geographical specification we have embedded into the model and we will show that state of the lake is corresponding with our results gain from the model.

The dynamic model has been evaluated with Matlab Simulink, which provide us with simulation with different input parameters and possibility to change many constants that appears in the simulated model [3]. We have achieved, only the simulation of the first of the three differential equations that describes the behaviour of the biological and chemical processes in the lake and that's the differential equation for available phosphorous cycle [5].

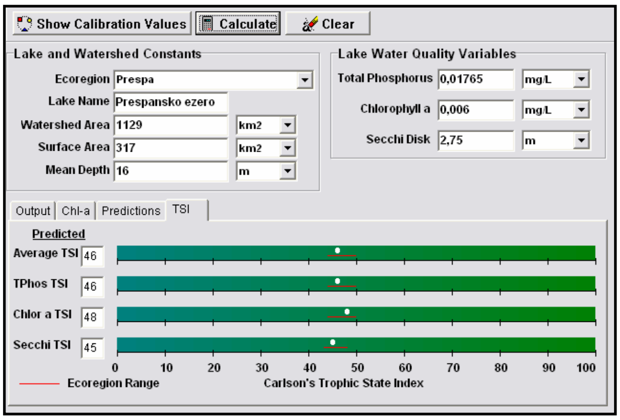
The eutrophication of the lake is evaluated by using the Trophic State Index (TSI) with different approaching methods, which one of them includes empirical model presented in section 2 and the other, the Matlab Fuzzy Logic Toolbox which is presented in section 3.2. In section 2 we present the formal definition and inside process of the dynamic model and in section 3 the actual inside look of the dynamic model for Lake Prespa in Matlab Simulink. The experimental results gain from the model and the future development scenarios are given in section 3.1, while section 4 concludes the paper and our areas of further research.

2 Eutrophication of the Lake Prespa

The eutrophication of the lakes has been established as a scientific fact since 1977 when, Carlson has provided methods to classified aquatic surface water bodies like lake, into three groups according to the level of the phosphorus component, chlorophyll-a(chl-a) and the secchi disk (SD) [6]. These three parameters are putting the lake into one of three groups in a scale of 0 to 100:

- oligothrophic lake (low productive lake);
- mesotrophic lake (medium productive lake);
- eutrotrophic lake (high productive lake);

Each of this groups has defined index as TSI, which is showing in which group is the lake. There are many methods to obtain TSI, but in this case we will use the empirical model [7]. Many software programs have been developed to estimate the TSI, but we estimated using special module call, Lake Eutrophication Analysis Procedure (LEAP), which is a part of software package, Wisconsin Lake Modelling Suite (WiLMS) [10]. This module has ability of automatic calculating TSI based on a Carlson's TSI formula. LEAP takes the results of the water quality that we have observed and calculates the future values of the three main variables that we have measured. On a second hand we have future results of the chlorophyll-a changing quantity depending of the model error variations. In the end comes the summary of the TSI, which is represented in Fig. 1.



Lake and Watershed Constants

Ecoregion: Prespa
Lake Name: Prespansko ezero
Watershed Area: 1129 km2
Surface Area: 317 km2
Mean Depth: 16 m

Lake Water Quality Variables

Total Phosphorus: 0,01765 mg/L
Chlorophyll a: 0,006 mg/L
Secchi Disk: 2,75 m

Average TSI: 46
TPhos TSI: 46
Chlor a TSI: 48
Secchi TSI: 45

Figure 1.*Carlson trophic state index for Lake Prespa.*

As we can see from the Fig.1 Lake Prespa has TSI of 46.3 indexes, which puts this lake into mesotrophic group of lakes. According to some recent papers Lake Ohrid, has TSI of 22 indexes which is oligothrophic [5]. According Carlson, the oligothrophic lakes are less eutrophicated then mesotrophic lakes, because they have lower values for TSI. This lead to a conclusion that this kind of lakes, in this case study Lake Prespa must have some system that will decrease this TSI index. Decreasing can be only done by eliminating the phosphorus component from the lake and also this will have impact on the food chain of the living organisms, also on the biological and chemical process inside of the lake. To prevent further eutrophication of the Lake Prespa we must take inside look of the process that have occur in the past and now, and this only can be done by examine the dynamic model of the Lake Prespa.

2.1 Process inside of the Lake Prespa

To understand what casing and how to prevent the further eutrophication of this ecosystem we have to understand the way how the phosphorus component is having effect on the Lake Prespa. Our search for the cause starts with the conceptual diagram which illustrates the flow of phosphorous through the food web. The model takes into account only the water column (sediment layer has not been consider in this research). This conceptual diagram is showing how the phosphorus component is effecting on the living organisms and how they are using it. But, the phosphorus that comes from the outside is the bigger problem that we are facing, so the lake faces accelerated deterioration of its waters and a change of the TSI of this aquatic ecosystem, mainly caused by nutrient load. 'Man-made' eutrophication, in absence of control measures, proceeds much faster than a natural phenomenon and is the major reason for raising eutrophication of this lake. A conceptual/functional diagram which is a basis for the dynamic model is presented in Fig.2.

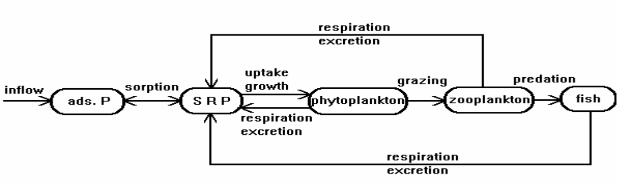


Figure 2.*Conceptual diagram showing the components of the model of the phosphorous cycle.*

Such dynamic model, which tends to predict future eutrophication and the trophic state lake-wide, is the first attempt undertaken in describing the ecological state of Lake Prespa. On the basis of the biological and chemical processes in the lake, the dynamical model is composed from the differential equation that is describing the behaviour of the available phosphorous (AvlP).

AvlP which is the only differential equation taken in consideration here (1), is described mathematically as the sum of the decomposition and the algae respiration processes, minus primary production, which are highly phosphorous dependable.

$$\frac{dAvlP}{dt} = \frac{P}{C}\left(D - \Pr + R_a\right) \qquad (1)$$

The AvlP is only a component that has enormous impact on the live inside of the Lake, but we must not forget that this AvlP comes as part of the Total Phosphorus (TP) inside of the lake which is sum of the existed phosphorus inside of the Lake and phosphorus concentration that comes from “man-made” eutrophication.

So, the task that we have put in front of us is to calculate AvlP and TP for the next years, so we could be able to predict the further eutrophication of the lake.

3 The dynamic model of Lake Prespa

In the previous section we have look the cause of the eutrophication and the basis of the process that we have to model them now, to get future development of the eutrophication. The dynamic model of the Lake Prespa has been built with Matlab Simulink, program that allows us to obtain results for long-term period with satisfied accuracy. The model that we have built has three main parts, which represents the dynamical process of the phosphorus component inside of the lake. Differential equation (1) contains from three main parts.

The Pr represents the primary productions of algae; Ra represents respiration of the algae and D the decomposition. P/C is constant that we have to change to gain minimal difference between measured values and predicted values for the given period of time. The three main parts are consisted with more mathematical equation that more closely are describing the process of phosphorus component and the direct impact on living organisms. The equations that are describing these processes are

$$P_r = P_m * UP * T_r * B \qquad (2)$$

$$Ra = \left(K_r + K_r^1 * UP\right) * T_r * B \qquad (3)$$

$$D = \beta \times T \times X \qquad (4)$$

, where P_m stands for maximal rate of growing, UP is redacting the phosphorus component, K_r and K_r^1 are constants that represents the rate of respiration and rate of proportional, respectively, T_r represents temperature factor, B is biomass. The T component represents input temperature, β is constant that describes the rate of transformation and X stands for input of the phosphorus concentration. As we can see from the equation (2), (3) and (4) the model is complex, considering more inside look between important components [8].

All this is considered when we have built the dynamic model. The illustrated diagram is shown in Fig.3. The model allows us for given input of two variables that are responsible for the eutrophication of the lake, the temperature and the phosphorus component to see how they contributed in the future development of the ecological sate of the Lake Prespa. Measured data is taken from different measurement stations around the Lake, for one year period, 03/2005-04/2006 [11].

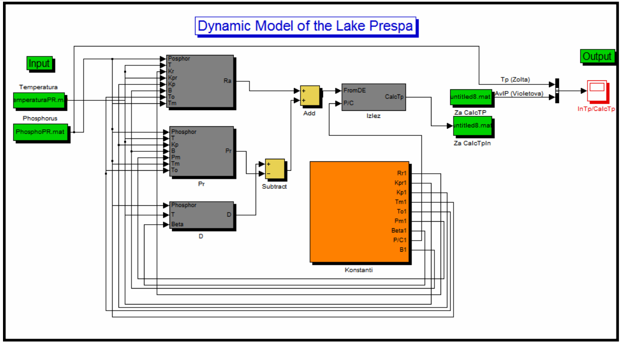


Figure 3.*The dynamic model of the Lake Prespa in Matlab Simulink*

The phosphorus component is very important, because it has direct impact on the state of the lake and the eutrophication. The AvlP represents the left component of the differential equation (1), so the will be represented as output of the dynamic model and then using discrete mathematics we will calculate TP which is present inside of the lake.

To develop fast response of the dynamic model between the measured and the results gain from the model for long-term period we have to modify the model to consider different changes between the two input parameters. This model is represented on Fig.4.

As we pointed earlier, now we can examine the long-term period of the eutrophication of the lake by taking input measured temperature values and values of the phosphorus component.

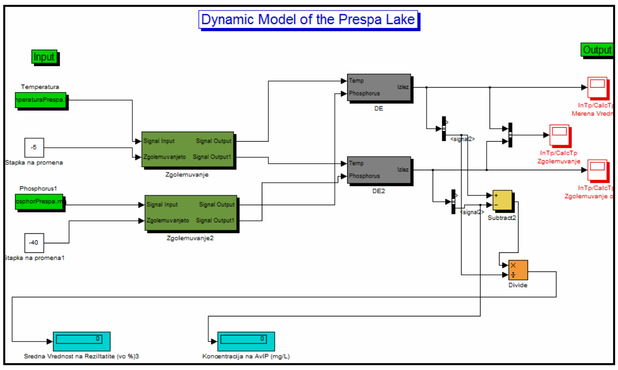


Figure 4.*The dynamic model of the Lake Prespa modified for different input parameters.*

3.1 Experimental results

Now, let’s simulate. First we do with the model is calibration. We have been changing the parameters constants to gain relevant output with the measured values. After we have done this, with acceptable error, we have extended the prediction period from 12 months to 120 months, or 10 years. Of course, there will be an error between the values that we have gain from the model, but we expect that these errors not extent the value of 10 % between these results and the values from the measured parameters.

After the simulation is run for the period of 12 months the output graphic is given on Fig.5.

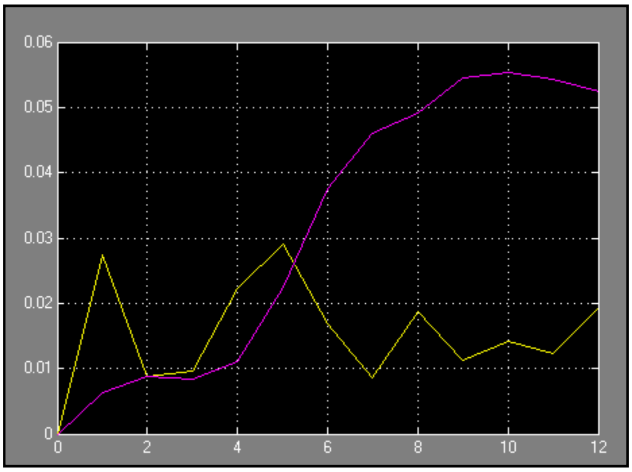


Figure 5.*Graphic output of the dynamic model for the Lake Prespa for 12 months period.*

Yellow line represents the AvlP concentration inside of the lake and the magenta colour line represents TP, which is important for the living environment. The next task is to gain results for the future development of the TP concentration. For this purpose we give periodical concentrations that are fiction, but more closely represents the past dynamic of the measured temperature and phosphorus values.

After simulation has ended we have gain data for different scenarios given in the Table 1.These scenarios represents different change of the measured values that are more likely to happen. For instance, the most used change for temperature values is -5 to +10 %, because we don’t expect dramatic change of this measured value in the next period, but the phosphorus component has to be treated with more precision, because it has direct impact on the living environment.

Name of the scenario	Change of temperature values (in %) – input value for the model	Change of phosphorus concentration (in %) – input value for the model	Change of TP concentration (Predicted/Input) – output value for the model
Scenario AA	-5	- 40	- 1.10
Scenario AB	-5	10	-0.03
Scenario BA	0	-50	-1.13
Scenario BB	0	-20	-0.42
Scenario BC	0	20	0.52
Scenario CA	5	-80	-1.57
Scenario CB	5	-40	-0.76
Scenario CC	5	20	0.75
Scenario DA	10	-100	-0.92
Scenario DB	10	-20	-0.08

Table 1 *Table of output values of the model for period of 5 years.*

This is according to Intergovernmental Panel for Climate Change (IPCC) the extreme events that probably could happen in next years in the Southern Europe, here are considered [9]. The cold winters with very low temperatures and dray summers with high temperatures with little rain will have major impact on the surface water resources and its pollution. Many watershed areas will drain-out and the concentration of the phosphorus in the lake will remain high, depending from the residence time of the water in the lake.

So the change of her concentration is in the range of -90% which represent’s system that has implemented water waste management, till +30 %

which will represent a system that is left with our care from the authorities.

Scenario CC is most possible to happen, because in next few years we expect to rise the temperature coefficient from +5 till +10% and change of phosphorus component to rise from +10 to +20%.

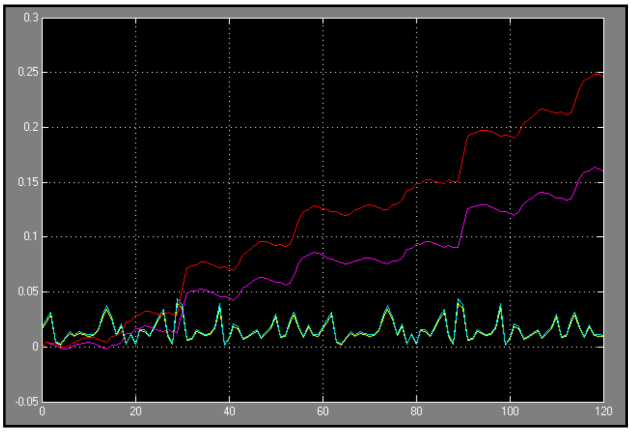


Figure 6.*Evolution diagram of Lake Prespa dynamics for scenario CC in next 10 years*

The evolution of the ecological system is given on Fig.6. The red line represents TP changed with parameters according scenario CC, and the magenta line represent evolution without influence to the main parameters. Even without any man-kind change of the inputted phosphorus, the dynamical model has pointed, that it need immediately building a water waste management system to eliminate phosphorus component from the surface water.

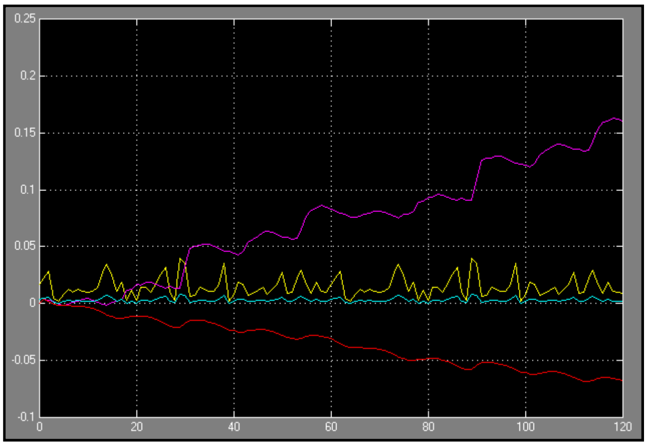


Figure 7.*Evolution diagram of Lake Prespa dynamics for scenario CA in next 10 years*

If we estimate that the water waste system can eliminate -40% and even -80% of phosphorus concentration from the surface water, we have scenario CB and scenario CA and we assume that temperature change will be not more than + 5%. The dynamic evolution is represented on Fig.7.

The scenario CA clearly indicates that if we want to prevent further pollution of the lake we must build a water waste management system to remove phosphorus component from the lake to reduce the eutrophication of the ecosystem. Next step is to calculate the TSI in order to show the increase of the eutrophication inside of the lake.

3.2 Estimating the future TSI of the Lake Prespa

A model for lake trophic state forecast has been developed based on a fuzzy expert system using a rule-based scheme in Matlab Fuzzy Logic Toolbox. Fuzzy logic is applies in order to obtain more realistic picture about the status of the lake and its eutrophication. A fuzzy inference procedure is used to evaluate the rules leading the lake system and to produce a trophic state index.

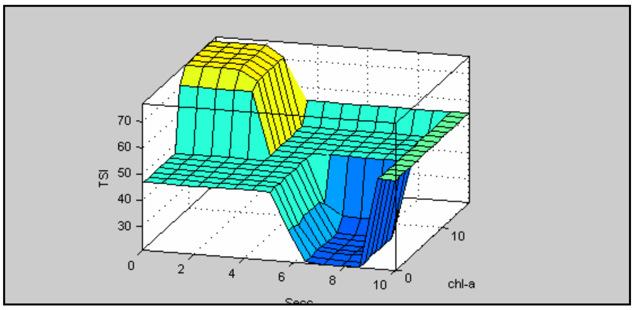


Figure 8.*Fuzzy rule system for the future developed TSI of Lake Prespa*

Because fuzzy theory deals with membership functions, we have to define membership function for the input and output variables. After defining membership functions for the input and output variables, a fuzzy expert system can be consulted.

Name of the scenario	Concentration of phosphorus (mg/L)	Value of TSI for given scenario	TSI state
Scenario AB	19,8	47	mesotrophic
Scenario CA	4	23	oligothrophic
Scenario CC	23	53	eutrotrophic

Table 2 *Summary of dynamic model for Lake Prespa with calculated TSI state*

This is an expert system that uses a collection of fuzzy membership functions and rules, instead of Boolean logic, to reason about data. The rules in a fuzzy system are usually of a term IF-THEN rules and it has 27 rules, 3 for each of the three parameters. This results in one fuzzy subset to be assigned to each output variable for each rule. Under composition, all of the fuzzy subset assigned to each output variable are combined together to form a single fuzzy subset for each output variable. Finally the defuzzyfication process is performed, which is used when it is useful to convert the fuzzy output set to a crisp number. On Fig.8 we can see the fuzzy rule system that has been built for Lake Prespa. For our purpose we will only use the procedure to obtain the results and only give the experimental results that we have gain from the model. After entering the new values for the future scenarios we can represent the eutrophication process in a certain moment of time. The results are represented in Table 2 which summarizes the results of our research.

4 Conclusion

This paper describes the dynamic model for Lake Prespa built and simulated in Matlab Simulink with tendency to predict long-term period eutrophication process of the ecosystem. We propose a new decision making mechanism with a new way of presenting the mathematical methods integrate within ecology and methods to present the future state trough trophic state and estimating eutrophication. The result is faster and more reliably model, able to integrate more changes and flexible prediction procedure that can be modified for different time periods. Using parameter estimated history; this model can be recalibrated for more precise prediction of output parameters and also can be updated to provide useful feedback for the eutrophication of the ecosystem.

From this point of view we can say that eutrophication of the Lake Prespa is climbing and it have a moderate supply of nutrients, are prone to moderate algal blooms, and have occasional oxygen depletions at depth.

We have shown that ecosystems, in our case Lake Prespa can be easily detached from the balance point and put into a state where environment is unable to be a place for the living organisms.

Our results have clearly pointed of alarming condition of the surface water and it’s pollution with phosphorous component. Building a water waste management can prevent this further growth of available phosphorous inside of the lake, so with this we are preventing this rare aquatic system from extinction of many fish spices and living organisms.

Based on these encouraging results, further research of building such model will be performed. Its implementation is more complex, but the other two differential equations will give us more reliable data and this will provide more details of the model quality and advantages.

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