# Research concerning a rocket-launching device system oscillations on a heavy vehicle 

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#### Abstract

Taking into consideration that during the firing, the rocket-launching device system oscillates and these oscillations may have a negative influence on the unguided rocket firing precision, it is necessary to evaluate the parameter oscillation in order to set up a precise rocket-launching device. This study intends to calculate the oscillations of a rocket-launching device system during firing, by means of numerical calculations on a nonlinear differential equation system [1]. In order to evaluate the rocket-launching device system oscillations a scheduling algorithm consisting in numerical integrating a differential equation system defining the movement of the system during launch is used.


Key-Words: - launching device, oscillation, disturbances, scheduling algorithm, vehicle chassis, tilting platform, unguided rocket, heavy vehicle, numerical results

## 1 Introduction

We use a rocket-launching device system that has the following components: the vehicle chassis (front side on the wheels, and back side on the blockage jacks) upon which is laid the launching device basis with the revolving support of the mechanism, the tilting platform (with the containers for the rockets) and the rockets (including the moving rocket) [1].

The system, during the rocket launch, is a complex oscillating system. The system is viewed as a set of rigid bodies bound together by elastic elements. The system has a high freedom degree number leading to a complex study of the oscillations. To simplify the calculus, but without limiting the generality of the study, we consider the movement of the rocket-launching device system during firing being completely described by 6 state variables [1]: the rocket linear translation in the container's guiding tube, s, two angles that define the tilting platform's position (pitch and gyration oscillation), $\varphi_{y}, \varphi_{z}$, other two angles that define the vehicle chassis pitch and rolling motions, $\gamma_{\mathrm{x}}$, $\gamma_{y}$, and the chassis center of masse oscillating vertical displacement, $\mathrm{z}_{\mathrm{S}}$. In this study all forces and moments acting on the rocket-launching device system during firing are taken into consideration.

The differential equation system [1] that defines the system's oscillating movement, being so complex, doesn't allow computing an analytical solution therefore we need to use a numerical
solving. So, it was necessary to create a scheduling algorithm consisting in numerical solving a rocketlaunching device system's movement equations by successive iterations. With each iteration the rocket translation is computed by means of the container guiding tube movement equation, of the tilting platform and of chassis angular oscillation equations, as well as of the chassis vertical translation equation.

## 2 Scheduling algorithm used to calculate the launching device's oscillations

The oscillations of the launching device are computed by numerical solving a differential equation system that describes the rocket-launching device system's movement. Starting from the vectorial form of the equation system [1], by projecting upon convenient systems of axes, by using matrices transformations from a system to another system and by expressing the forces and moments present, the equation system will be brought to a scalar 18 equation system.

Six differential second-order equations from the whole 18 obtained, allow calculating the 6 variables that define the system $\mathrm{z}_{\mathrm{s}}, \gamma_{\mathrm{x}}, \gamma_{\mathrm{y}}, \varphi_{\mathrm{y}}, \varphi_{\mathrm{z}}$ and s , while the other 12 equations are used to calculate the connection forces and moments $F_{R y}, F_{R z}, \mathbf{M}_{R x}$,
$\mathbf{M}_{\mathrm{Ry}}, \quad \mathbf{M}_{\mathrm{Rz}}, \quad \mathrm{R}_{\Pi \mathrm{x}}, \quad \mathrm{R}_{\Pi y}, \quad \mathrm{R}_{\Pi z}, \quad \mathbf{M}_{\eta \mathrm{x}}, \quad \mathrm{F}_{\mathrm{p} 1 \mathrm{x}}$, $\mathrm{F}_{\mathrm{p} 2 \mathrm{x}}$ and $\mathrm{F}_{\mathrm{py}}$.

In order to pass from the second order to the first order differential equations, a series of additional variables will be aided (first order derivative from the basis variables, which represents linear or angular physical velocity):

$$
\begin{align*}
& \mathrm{v}_{\mathrm{s}}=\dot{\mathrm{s}}  \tag{1}\\
& \omega_{\varphi_{\mathrm{y}}}=\dot{\varphi}_{\mathrm{y}}  \tag{2}\\
& \omega_{\varphi_{\mathrm{z}}}=\dot{\varphi}_{\mathrm{z}}  \tag{3}\\
& \mathrm{v}_{\mathrm{z}_{\mathrm{S}}}=\dot{\mathrm{z}}_{\mathrm{S}}  \tag{4}\\
& \omega_{\gamma_{\mathrm{x}}}=\dot{\gamma}_{\mathrm{x}}  \tag{5}\\
& \omega_{\gamma_{\mathrm{y}}}=\dot{\gamma}_{\mathrm{y}} \tag{6}
\end{align*}
$$

The connection forces and moments can be developed using some functions, as follows:

$$
\begin{align*}
& \mathrm{F}_{\mathrm{Ry}}=\mathrm{f}_{\mathrm{FRy}_{\mathrm{R}}}\left(\mathrm{v}_{\mathrm{s}}, \omega_{\varphi_{\mathrm{y}}}, \omega_{\varphi_{\mathrm{z}}}, \mathrm{v}_{\mathrm{zS}}, \omega_{\gamma_{\mathrm{x}}}, \omega_{\gamma_{\mathrm{y}}}\right) ;  \tag{7}\\
& \mathrm{F}_{\mathrm{Rz}}=\mathrm{f}_{\mathrm{F}_{\mathrm{Ry}}}\left(\mathrm{v}_{\mathrm{s}}, \omega_{\varphi_{\mathrm{y}}}, \omega_{\varphi_{\mathrm{z}}}, \mathrm{v}_{\mathrm{zS}}, \omega_{\gamma_{\mathrm{x}}}, \omega_{\gamma_{\mathrm{y}}}\right) \text {; }  \tag{8}\\
& \mathbf{M}_{R \mathrm{Rx}}=\mathrm{f}_{\mathbf{M}_{\mathrm{Rx}}}\left(\mathrm{v}_{\mathrm{s}}, \omega_{\varphi_{y}}, \omega_{\varphi_{\mathrm{z}}}, \mathrm{v}_{\mathrm{z}_{\mathrm{S}}}, \omega_{\gamma_{\mathrm{x}}}, \omega_{\gamma_{\mathrm{y}}}\right) ;  \tag{9}\\
& \mathbf{M}_{\text {Ry }}=\mathrm{f}_{\mathbf{M}_{\mathrm{Ry}}}\left(\mathrm{v}_{\mathrm{s}}, \omega_{\varphi_{\mathrm{y}}}, \omega_{\varphi_{\mathrm{z}}}, \mathrm{v}_{\mathrm{z}_{\mathrm{S}}}, \omega_{\gamma_{\mathrm{x}}}, \omega_{\gamma_{\mathrm{y}}}\right) ;  \tag{10}\\
& \mathbf{M}_{\mathrm{Rz}}=\mathrm{f}_{\mathbf{M}_{\mathrm{Rz}}}\left(\mathrm{v}_{\mathrm{s}}, \omega_{\varphi_{\mathrm{y}}}, \omega_{\varphi_{\mathrm{z}}}, \mathrm{v}_{\mathrm{zS}}, \omega_{\gamma_{\mathrm{x}}}, \omega_{\gamma_{\mathrm{y}}}\right) ;  \tag{11}\\
& \mathrm{R}_{\Pi \mathrm{x}}=\mathrm{f}_{\mathrm{R}_{\Pi \mathrm{x}}}\left(\mathrm{v}_{\mathrm{s}}, \omega_{\varphi_{\mathrm{y}}}, \omega_{\varphi_{\mathrm{z}}}, \mathrm{v}_{\mathrm{zS}}, \omega_{\gamma_{\mathrm{x}}}, \omega_{\gamma_{\mathrm{y}}}\right) ;  \tag{12}\\
& R_{\Pi y}=f_{R_{\Pi y}}\left(v_{s}, \omega_{\varphi_{y}}, \omega_{\varphi_{z}}, v_{z \mathrm{~S}}, \omega_{\gamma_{\mathrm{x}}}, \omega_{\gamma_{y}}\right) ;  \tag{13}\\
& \mathrm{R}_{\Pi \mathrm{z}}=\mathrm{f}_{\mathrm{R}_{\Pi \mathrm{z}}}\left(\mathrm{v}_{\mathrm{s}}, \omega_{\varphi_{\mathrm{y}}}, \omega_{\varphi_{\mathrm{z}}}, \mathrm{v}_{\mathrm{z}_{\mathrm{S}}}, \omega_{\gamma_{\mathrm{x}}}, \omega_{\gamma_{\mathrm{y}}}\right) ;  \tag{14}\\
& \mathbf{M}_{\eta \mathrm{x}}=\mathrm{f}_{\mathbf{M}_{\mathrm{\eta x}}}\left(\mathrm{v}_{\mathrm{s}}, \omega_{\varphi_{\mathrm{y}}}, \omega_{\varphi_{\mathrm{z}}}, \mathrm{v}_{\mathrm{zS}}, \omega_{\gamma_{\mathrm{x}}}, \omega_{\gamma_{\mathrm{y}}}\right) ;  \tag{15}\\
& \mathrm{F}_{\mathrm{plx}}=\mathrm{f}_{\mathrm{F}_{\mathrm{p} 1 \mathrm{x}}}\left(\mathrm{v}_{\mathrm{s}}, \omega_{\varphi_{\mathrm{y}}}, \omega_{\varphi_{\mathrm{z}}}, \mathrm{v}_{\mathrm{z}_{\mathrm{S}}}, \omega_{\gamma_{\mathrm{x}}}, \omega_{\gamma_{\mathrm{y}}}\right) ;  \tag{16}\\
& \mathrm{F}_{\mathrm{p} 2 \mathrm{x}}=\mathrm{f}_{\mathrm{F}_{\mathrm{p} 2 \mathrm{x}}}\left(\mathrm{v}_{\mathrm{s}}, \omega_{\varphi_{y}}, \omega_{\varphi_{\mathrm{z}}}, \mathrm{v}_{\mathrm{zS}_{\mathrm{S}}}, \omega_{\gamma_{\mathrm{x}}}, \omega_{\gamma_{\mathrm{y}}}\right) ;  \tag{17}\\
& \mathrm{F}_{\mathrm{py}}=\mathrm{f}_{\mathrm{F}_{\mathrm{py}}}\left(\mathrm{v}_{\mathrm{s}}, \omega_{\varphi_{\mathrm{y}}}, \omega_{\varphi_{\mathrm{z}}}, \mathrm{v}_{\mathrm{zs}}, \omega_{\gamma_{\mathrm{x}}}, \omega_{\gamma_{\mathrm{y}}}\right) \text {. } \tag{18}
\end{align*}
$$

The simultaneous equations, which allow calculating the oscillations of the launching device during firing, using the expressions of the forces and moments and of the additional variables, become:

$$
\begin{align*}
\dot{\mathrm{v}}_{\mathrm{s}} & =\mathrm{f}_{\mathrm{v}_{\mathrm{s}}}\left(\mathrm{v}_{\mathrm{s}}, \omega_{\varphi_{\mathrm{y}}}, \omega_{\varphi_{\mathrm{z}}}, \mathrm{v}_{\mathrm{zS}}, \omega_{\gamma_{\mathrm{x}}}, \omega_{\gamma_{\mathrm{y}}}\right)  \tag{19}\\
\dot{\omega}_{\varphi_{\mathrm{y}}} & =\mathrm{f}_{\omega_{\varphi_{\mathrm{y}}}}\left(\mathrm{v}_{\mathrm{s}}, \omega_{\varphi_{\mathrm{y}}}, \omega_{\varphi_{\mathrm{z}}}, \mathrm{v}_{\mathrm{z}_{\mathrm{S}}}, \omega_{\gamma_{\mathrm{x}}}, \omega_{\gamma_{\mathrm{y}}}\right)  \tag{20}\\
\dot{\omega}_{\varphi_{\mathrm{z}}} & =\mathrm{f}_{\omega_{\varphi_{\mathrm{z}}}}\left(\mathrm{v}_{\mathrm{s}}, \omega_{\varphi_{\mathrm{y}}}, \omega_{\varphi_{\mathrm{z}}}, \mathrm{v}_{\mathrm{zS}}, \omega_{\gamma_{\mathrm{x}}}, \omega_{\gamma_{\mathrm{y}}}\right) ;  \tag{21}\\
\dot{\mathrm{v}}_{\mathrm{z}_{\mathrm{S}}} & =\mathrm{f}_{\mathrm{v}_{\mathrm{z}_{\mathrm{S}}}}\left(\mathrm{v}_{\mathrm{s}}, \omega_{\varphi_{\mathrm{y}}}, \omega_{\varphi_{\mathrm{z}}}, \mathrm{v}_{\mathrm{z}_{\mathrm{S}}}, \omega_{\gamma_{\mathrm{x}}}, \omega_{\gamma_{\mathrm{y}}}\right)  \tag{22}\\
\dot{\omega}_{\gamma_{\mathrm{x}}} & =\mathrm{f}_{\omega_{\gamma_{\mathrm{x}}}}\left(\mathrm{v}_{\mathrm{s}}, \omega_{\varphi_{\mathrm{y}}}, \omega_{\varphi_{\mathrm{z}}}, \mathrm{v}_{\mathrm{z}_{\mathrm{S}}}, \omega_{\gamma_{\mathrm{x}}}, \omega_{\gamma_{\mathrm{y}}}\right)  \tag{23}\\
\dot{\omega}_{\gamma_{\mathrm{y}}} & =\mathrm{f}_{\omega_{\gamma_{\mathrm{y}}}}\left(\mathrm{v}_{\mathrm{s}}, \omega_{\varphi_{\mathrm{y}}}, \omega_{\varphi_{\mathrm{z}}}, \mathrm{v}_{\mathrm{z}_{\mathrm{S}}}, \omega_{\gamma_{\mathrm{x}}}, \omega_{\gamma_{\mathrm{y}}}\right) ;  \tag{24}\\
\dot{\mathrm{s}} & =\mathrm{v}_{\mathrm{s}}  \tag{25}\\
\dot{\varphi}_{\mathrm{y}} & =\omega_{\varphi_{\mathrm{y}}}  \tag{26}\\
\dot{\varphi}_{\mathrm{z}} & =\omega_{\varphi_{\mathrm{z}}} \tag{27}
\end{align*}
$$

$$
\begin{align*}
& \dot{\mathrm{z}}_{\mathrm{S}}=\mathrm{v}_{\mathrm{zS}_{\mathrm{S}}}  \tag{28}\\
& \dot{\gamma}_{\mathrm{x}}=\omega_{\gamma_{\mathrm{x}}}  \tag{29}\\
& \dot{\gamma}_{\mathrm{y}}=\omega_{\gamma_{\mathrm{y}}} . \tag{30}
\end{align*}
$$

A iterative approximation method is used to calculate the independent variables $\mathrm{z}_{\mathrm{S}}, \gamma_{\mathrm{x}}, \gamma_{\mathrm{y}}, \varphi_{\mathrm{y}}$, $\varphi_{z}$ and s . Next, each system component is identified and extracted from the system (see fig. 1).


Fig. 1 The calculus diagram of the launching device
The scheduling algorithm of the launching device, shown in the fig. 2, consists in successive iterations, which are described following.


Fig. 2 The scheduling algorithm of the launching device

## First iteration

The independent variables are $\mathrm{s}, \varphi_{\mathrm{y}}, \varphi_{\mathrm{z}}, \mathrm{z}_{\mathrm{S}}, \gamma_{\mathrm{x}}$ and $\gamma_{y}$ and the connection forces and moments $\mathrm{F}_{\mathrm{Rbx}}$, $\mathrm{F}_{\mathrm{Rby}}, \mathrm{F}_{\mathrm{Rbz}}, \mathbf{M}_{\mathrm{Rbx}}, \mathbf{M}_{\mathrm{Rby}}, \mathbf{M}_{\mathrm{Rbz}}, \mathrm{R}_{\Pi \mathrm{x}}, \mathrm{R}_{\Pi y}$,
$\mathrm{R}_{\Pi \mathrm{z}}, \mathbf{M}_{\eta \mathrm{x}}, \mathbf{M}_{\eta \mathrm{y}}$ and $\mathbf{M}_{\eta \mathrm{z}}$ have an initial value of zero.

The rocket displacement in the container guiding tube, s is calculated at the moving rocket level. The connection forces and moments that act at the tilting platform: $\mathrm{F}_{\mathrm{Rbx}}, \mathrm{F}_{\mathrm{Rby}}, \mathrm{F}_{\mathrm{Rbz}}, \mathbf{M}_{\mathrm{Rbx}}, \mathbf{M}_{\mathrm{Rby}}$ and $\mathbf{M}_{\mathrm{Rbz}}$ are calculated for all the rockets in the container.

The variables thus computed, are inserted in the tilting platform movement equations whose computation shows the evolution of the $\varphi_{y}$ and $\varphi_{z}$ variables, and of the connection forces and moments $\mathrm{R}_{\Pi \mathrm{x}}, \mathrm{R}_{\Pi \mathrm{y}}, \mathrm{R}_{\Pi \mathrm{z}}, \mathbf{M}_{\eta \mathrm{x}}, \mathbf{M}_{\eta \mathrm{y}}$ and $\mathbf{M}_{\eta \mathrm{z}}$.

The independent variables of the rocket, the tilting platform levels and the connection forces and moments between the tilting platform and the vehicle chassis are used to calculate the chassis variables $\mathrm{z}_{\mathrm{S}}, \gamma_{\mathrm{x}}$ and $\gamma_{\mathrm{y}}$. The Runge-Kutta integration method will be used.

## Second iteration

Knowing the independent variables $\mathrm{s}, \varphi_{\mathrm{y}}, \varphi_{\mathrm{z}}, \mathrm{z}_{\mathrm{S}}$, $\gamma_{\mathrm{x}}$ and $\gamma_{\mathrm{y}}$ from the first iteration, the same algorithm as previously is used in order to obtain the results of the second iteration.

To analyze the algorithm's convergence, a comparison between the successive iterations will be used. We notice that $5 \div 6$ iterations are enough to obtain the results with acceptable errors. In order to reduce the calculus time, a modified version of the algorithm can be used: an initial calculus at rocket and tilting platform levels lead to the achievement of a good convergence of the results followed by a chassis level computation. In this case only $2 \div 3$ iterations are required.

## 3 Numerical results

The numerical application named ILANPRN [2], developed by the authors using the general mathematical model [1], and the scheduling algorithm presented above, allows calculating the oscillations of the rocket-launching device system.

Next, numerical results obtained by numerical integration of the differential simultaneous equations describing the dynamic behavior of the rocketlaunching device system are presented.

The 122 mm unguided rocket launching device is used with a container of 40 rockets.

Many simulations are computed in a single rocket firing case with the rocket in central position. The time history of state variables that describe the
rocket-launching device $\operatorname{system}\left(\mathrm{z}_{\mathrm{S}}, \gamma_{\mathrm{x}}, \gamma_{\mathrm{y}}, \quad \varphi_{\mathrm{y}}\right.$, $\varphi_{\mathrm{z}}, \mathrm{s}$ and $\left.\dot{z}_{\mathrm{S}}, \dot{\gamma}_{\mathrm{x}}, \dot{\gamma}_{\mathrm{y}}, \dot{\varphi}_{\mathrm{y}}, \dot{\varphi}_{\mathrm{z}}, \dot{\mathrm{s}}\right)$ are presented in fig. 3-14.

We notice that the displacement $z_{S}$ of the chassis center of masse has an oscillatory evolution (fig. 3) having a 0.75 s oscillating period. The oscillating amplitude has the initial value of 1.6 mm , and after the first period decreases to $1 \mathrm{~mm}(62.5 \%$ from the initial amplitude).


Fig. 3 Time history of the $\mathrm{z}_{\mathrm{S}}$ displacement
The chassis angle rotations $\gamma_{x}$ and $\gamma_{y}$ have also a damped oscillatory time history (fig. 4 and fig. 5). Moreover, the oscillating period for $\gamma_{\mathrm{x}}(0.4 \mathrm{~s})$ is smaller than for $\gamma_{y}(0.8 \mathrm{~s})$.


Fig. 4 Time history of the rotation angle $\gamma_{\mathrm{x}}$


Fig. 5 Time history of the rotation angle $\gamma_{y}$
The $\gamma_{\mathrm{x}}$ oscillating amplitude is 0.18 degree whereas the $\gamma_{y}$ oscillating amplitude is only 0.028 degree leading to the conclusion that the rolling oscillation is more important than the pitch oscillation.

As regards the oscillations of the tilting platform, the rotation angle $\varphi_{y}$ (see fig. 6 and fig. 7) has larger period ( 0.5 s ) and larger amplitude (0.8
degree) than $\varphi_{\mathrm{z}}(0.2 \mathrm{~s}$ oscillating period and 0.04 degree oscillating amplitude). So, we can say that at the tilting platform level, the main oscillation is the pitch motion ( $\varphi_{\mathrm{y}}$ ).


Fig. 6 Time history of the rotation angle $\varphi_{\mathrm{y}}$


Fig. 7 Time history of the rotation angle $\varphi_{z}$
Fig. 8 presents the trajectory of the rocket center of masse during the launching. The evolution range starts from zero (initial position) to 3 m (rocket center of masse position when it leaves the launching device).


Fig. 8 Time history of displacement s
Fig. 9-14 presents the time history of the linear and angular velocity for the main components of the rocket-launching device system. These evolutions were noticed to be similar to the corresponding state variables ( $\mathrm{z}_{\mathrm{S}}, \gamma_{\mathrm{x}}, \gamma_{\mathrm{y}}, \varphi_{\mathrm{y}}, \varphi_{\mathrm{z}}, \mathrm{s}$ ).


Fig. 9 Time history of the velocity $\dot{z}_{\mathrm{S}}$


Fig. 10 Time history of the angular velocity $\dot{\gamma}_{\mathrm{x}}$


Fig. 11 Time history of the angular velocity $\dot{\gamma}_{\mathrm{y}}$


Fig. 12 Time history of the angular velocity $\dot{\varphi}_{\mathrm{y}}$


Fig. 13 Time history of the angular velocity $\dot{\varphi}_{\mathrm{z}}$


Fig. 14 Time history of the velocity $\dot{\mathrm{s}}$
Having the main movement elements of the rocket-launching device system identified during firing (rocket translations, tilting platform angle rotation $\varphi$, chassis translation $\mathrm{z}_{\mathrm{S}}$ and chassis angle
rotation $\gamma$ ), we are able to calculate the movement evolution for any point located on the launching device.

Also, we can consider other launching cases: single rocket launcher on any container position, or launcher of a two or more consecutive rocket range.

In order to estimate the oscillation influence on the stability of the flying rocket, and also the effect on the firing accuracy, we computed the evolution of the launching device (points from the container where the rocket has the last contact with the container). Fig. 15 presents the time evolution of the z coordinate of the central launcher point. Fig 16 presents the pitch movement evolution for the same point in the case of the 4 rockets range launching. The two parameters shown have a dumping oscillatory evolution.


Fig. 15 Time history of the launching point z coordinate


Fig. 16 Time history of the launching point pitch movement

## 4 Conclusion

The evolution calculus of the rocket-launching device system state variables during firing sequences allows the evaluation of dynamic forces present at all levels of the launching device system component, and therefore the analysis of the dynamic behavior of the whole assembly system.

Evaluating the oscillation parameters of a rocket-launching device system, of their influencing to the system stability during firing, such as the initial rocket flight condition, leads implicitly to evaluating the firing accuracy, a must in the design of a precise rocket-launching device system.

In conclusion, a rocket launching phase design needs to take into account the system oscillations. These oscillations can be computed by numerically solving a theoretical model [1], also confirmed by the experimental results which validate and lead to the improvement of the numerical scheduling algorithm.

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