Endochronic model applied to earthfill dams with impervious core: design recommendation at seismic sites

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Abstract: - Earthfill dams are geostructures which may be especially affected by seismic loadings, because soil skeleton that form them suffers remarkable modifications in its mechanical properties, as well as flow of water through their pores, when subjected to vibrations. The most extreme situation is the dam failure due to soil liquefaction. The application of a fully coupled endochronic based numerical model, developed and validated by the authors, to a set of theoretical cross sections of earthfill dams with impervious core, is presented. All these dams are same height and have the same volume of impervious material at the core. The influence of the core location inside the dam on its response against seismic loading is numerically explored. The analyzed theoretical dams are previously designed as strictly stable under static loads. As a result of this research, a design recommendation on the location of the impervious core is obtained for this type of earth dams, on the basis of the criteria of minor liquefaction risk, minor soil degradation during the earthquake and minor crest settlement.

Key-Words: - Soil dynamics, Earthquakes, Earthfill, Dam design, Constitutive models

1 Introduction
Earth dams are geostructures usually analyzed very carefully from the seismic point of view, because of the risk for the human lives that their failure may imply. In the last century, geotechnical earth dam engineers mainly focused their attention in the knowledge of empirical standards of design of these structures against earthquakes, guided by past experiences [12]. Characteristics of the materials to be used in each zone of heterogeneous dams were widely investigated. A significant event in the design of earthfill dams at seismic sites was the failure of the Lower San Fernando dam (Los Angeles area), in the earthquake of the same name, in 1971. Some laboratory and field researches after this event demonstrated that the flow failure of the liquefied upstream sand fill happened few minutes after the end of the earthquake (Fig.1)[10,11,15]. Due of the great amount of reported data available about this dam, it has become the main case study used for validating every constitutive law for saturated granular soils under vibrations developed after that year. Although it is the most famous case of liquefaction failure in earth dams, it is not the only one [9].

Since 1971, several analyses of possible liquefaction in already constructed earth dams at seismic sites were conducted, in order to upgrade the most dangerous ones against human life [7].

Not too much research about design of new heterogeneous earth dams at seismic sites is available in the literature, especially concerning geometry and location of the impervious core within the dam. Nevertheless, it is well known that, if the seismic risk is high, the location of the impervious core close to the upstream slope is better than a central core, and these cross sections are frequently used at sites where the seismic risk is medium or high [13]. But this issue is again more a knowledge derived from successful past experiences than a scientifically analyzed phenomenon.
Several constitutive laws have been especially developed for modelling soils under dynamic loadings [6,1,8,5]. Between all these models, those based on classical plasticity theories usually involve a lot of parameters without a clear physical meaning, and difficulties arise when the calibration process is to be carried out. Some approaches have been made in order to model the dynamic sand behaviour by means of densification based models, in which the rate of development of excess pore water pressure is related to changes in volumetric strain of the same sand when it is dry and subjected to vibrations. This type of models realistically represents the physical process of the loss of effective stress in loose sand if its behaviour is mainly contractive, but the dilative trend of dense sand can not be taken into account by most of these approaches. In this paper, a newly developed model based on densification, which also includes a flow rule for predicting the dilative behaviour and collapse of the soil, has been used.

This constitutive model has been implemented in a coupled 2D finite element code developed by the authors, which has been deterministically applied to several ideal cross sections of earth dams with impervious core. All these dams have been designed as 100 meters height, and they have the same volume of impervious material at the core, the main difference between them being the location of the core inside each dam, which ranges from centered to totally leaned to the upstream slope. Both upstream and downstream slope angles have been previously calculated in order to make the dams strictly stable under static loadings (i.e. security factor of one for both downstream slope – reservoir at its maximum level – and upstream slope – rapid drawdown). For the dynamic analysis, the same earthquake has been applied at the base of all these dams as input loading, in order to explore their risk against liquefaction failure and crest settlements. On the basis of this analysis, a design recommendation of the impervious core location in this case is numerically justified at seismic sites.

In this paper the governing equations used in the numerical model are firstly shown. After that, the applied constitutive law is referred. Input loading, geometry, material characteristics and boundary conditions of the analyzed dams are presented, and numerical results are evaluated.

2 Governing equations

The mechanical behaviours of both solid and fluid phases, as well as the coupling between them, are given by Biot's equations, in which transmission of waves in saturated porous media is established [15]. The "u-w" formulation is employed herein (u and w denote, respectively, absolute displacement of the solid phase and relative displacement of the fluid phase to the solid phase). This numerical scheme gives stable solutions even for extremely low values of the soil permeability although linear approximation functions are used, which represents and advantage if compared with the more conventional u-pw formulation (pw denotes excess pore water pressure) [2]. The incremental governing equations, in each time step, are finally rearranged as follows, using matrix notation:

\[
\begin{align*}
(S^T \cdot D^s \cdot S - S^T \cdot D^f \cdot S) \cdot du + M_d \cdot V(\nabla^2 du) + & M_s \cdot V(\nabla^2 dw) - \rho d\ddot{u} - \rho f d\ddot{w} + \rho db \\
M_d \cdot V(\nabla du) + M_s \cdot V(\nabla dw) - & k^{-1} \cdot d\dot{w} - \rho f d\ddot{u} - \frac{\rho_f}{n} d\ddot{w} + \rho f db = 0
\end{align*}
\]  

In these equations, \(M_d\) is the tangent constrained modulus of the drained sand; \(\rho\) and \(\rho_f\) are the...
densities of the two-phase medium and the fluid, respectively; \( n \) is the porosity of the soil; \( k \) is the soil permeability tensor (expressed in units of length\(^3\)·time/mass), \( \text{db} \) is the vector of incremental external acceleration due to the earthquake; \( S \) is a matrix operator; \( D^p \) is the plastic tensor, defined by the constitutive law of the material; and \( D^e \) is the elastic tensor. In eq. (2), the term \( k^{-1} \text{d} \dot{v} \) stands for the viscous coupling between solid and fluid phases, on the basis of the assumption of the porous media flow Darcy’s law. An implicit Newmark scheme is used for integrating in time the above equations.

3 Saturated sand behaviour under dynamic loading

Fig.2 shows a typical experimental result obtained on saturated loose undrained sand subjected to cyclic shear stress test [14]. In this figure the stress path is sketched. The initial effective stress, \( \sigma'_{v0} \), is depicted as point 1. Cycle by cycle, the effective stress, \( \sigma'_{v} \), decreases until the stress path reaches the phase transformation line (point 2): until this point, the sand behaves in a contractive manner. From this point until unloading (point 3), the sand dilates, recovering some of the initial effective stress. From this situation until failure due to liquefaction (null effective stress) only few cycles of loading are required.

Fig.2. Cyclic shear stress test on saturated loose undrained sand. Stress path, failure and phase transformation lines. (after [14])

In Fig.2, failure lines in both positive and negative shear stress planes (with slope \( M_p \)) and phase transformation lines (with slope \( M_p \)) are sketched. Failure lines are never crossed by the stress path. The looser the sand, the closer the failure and phase transformation lines (i.e. the sand behaviour is mainly contractive).

Dense sand behaves mainly in dilative manner. An example of a stress path of saturated dense sand under cyclic shear stress is given in Fig.3 [14].

Fig.3. Cyclic shear stress test in dense saturated undrained sand. Stress path (after [14])

The constitutive law implemented in the numerical model is described in [2]. It is based on the calculation of the plastic volumetric strains of dry sandy soil under vibrations, which is related to the changes in the effective stress of the saturated soil by means of the constrained modulus of the sand, \( M_d \). This model accounts for all the above mentioned patterns of behaviour, and allows us to model both contractive and dilative trends of the sand. It requires seven parameters to be calibrated by means of undrained cyclic tests. Other nine parameters have to be determined from static and dynamic tests, although not by calibration but by direct determination.

Both numerical behaviour and modelling of real situations, in free field and laboratory (centrifuge tests) have been assessed [3,4]. It has been demonstrated that this constitutive model is accurate enough for determining pore water pressure changes, and computed strains are also in good agreement with practical observations.

4 Application of the model

4.1 Geometry and materials of the dams

Fig.4 represents the analyzed earth dam cross sections. All of them are 100 meters height. The angle between the axis of the impervious core and the \( y \) axis (vertical), \( \delta \), ranges from zero to the same angle of the upstream slope (core totally leaned to this slope). The volume of the impervious core is the same in all the dams. Five earth dam cross sections have been analyzed. Both upstream and downstream slopes have been previously determined in order to make the dams strictly stable under static loadings (without seismic solicitation), by applying the Bishop’s method, and considering the materials of the dam as isotropic and Mohr - Coulomb type. The used
characteristics of the soils for the static stability analysis (specific weight of the particles, $\gamma_s$, cohesion, $c'$, and internal friction angle, $\phi'$) as well as the values of the parameters of the constitutive model mentioned in section 3, are given in Table 1. These values are typical for both granular and impervious soils, respectively.

Table 1. Parameters used for the materials in the numerical model [2].

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Sand</th>
<th>Impervious soil</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C$</td>
<td>10</td>
<td>0</td>
</tr>
<tr>
<td>$K_d$</td>
<td>12000</td>
<td>1</td>
</tr>
<tr>
<td>$m^*$</td>
<td>0.372</td>
<td>0.372</td>
</tr>
<tr>
<td>$\alpha_p$</td>
<td>-0.99</td>
<td>-0.99</td>
</tr>
<tr>
<td>$I_{1}$ (before phase tr. or collapse)</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$I_{2}$ (after phase tr. or collapse)</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>$B_g$</td>
<td>220</td>
<td>170</td>
</tr>
<tr>
<td>$C_i$</td>
<td>1.6</td>
<td>1.0</td>
</tr>
<tr>
<td>$e_{\text{min}}$</td>
<td>0.68</td>
<td>0.50</td>
</tr>
<tr>
<td>$e_{\text{max}}$</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>$D\theta_0$</td>
<td>0.60</td>
<td>0.64</td>
</tr>
<tr>
<td>$M_p$</td>
<td>0.65</td>
<td>1.20</td>
</tr>
<tr>
<td>$M_f$</td>
<td>0.17</td>
<td>0.28</td>
</tr>
<tr>
<td>$k$ (m/s)</td>
<td>$4.3 \times 10^{-4}$</td>
<td>$5.10^{-8}$</td>
</tr>
<tr>
<td>$\gamma_s$ (kN/m$^3$)</td>
<td>26.56</td>
<td>26.17</td>
</tr>
<tr>
<td>$c'$ (kN/m$^2$)</td>
<td>5</td>
<td>30</td>
</tr>
<tr>
<td>$\phi'$ (º)</td>
<td>35</td>
<td>20</td>
</tr>
</tbody>
</table>

The calculation of the downstream slope has been made assuming maximum water elevation in the reservoir. For determining the upstream slope, rapid drawdown has been considered. Table 2 gives the results of this static analysis (slopes expressed as horizontal distance per one unit of vertical distance).

Table 2. Values of the upstream (ml) and downstream (mr) slopes for equilibrium under static loads, as a function of the angle of the impervious core, $\delta$.

<table>
<thead>
<tr>
<th>$\delta$ (º)</th>
<th>ml</th>
<th>mr</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>3.5</td>
<td>1.8</td>
</tr>
<tr>
<td>20.0</td>
<td>3.6</td>
<td>1.7</td>
</tr>
<tr>
<td>40.0</td>
<td>3.9</td>
<td>1.6</td>
</tr>
<tr>
<td>60.0</td>
<td>4.2</td>
<td>1.6</td>
</tr>
<tr>
<td>76.2</td>
<td>4.2</td>
<td>1.5</td>
</tr>
</tbody>
</table>

Fig. 5 gives liquefaction degrees ($r_u = \Delta p_{o}/\sigma'_{vo}$) at the end of the earthquake in the five geometries. Values of $r_u$ equal to 1 mean liquefaction. Therefore, on the basis of this figure, the five dams reach liquefaction. In the first three dams the critical points (where the maximum values of $r_u$ are reached) are located at the highest elevation of the boundary between impervious and granular materials. In the fifth dam liquefaction takes place mainly in a less concentrated zone of the dam, at the top of the contact between downstream earthfill and impervious material. In this dam the core works as an impervious barrier against the water flow to the reservoir, which means that consolidation should be a slower process which takes place after the end of the earthquake, and the water does not flow to the reservoir unless the core is broken. It is well known that in the mentioned Lower San Fernando dam the upstream slope failed few minutes after the earthquake, during the consolidation stage, because of the instability in the upstream earthfill probably due, amongst other reasons, to the flow of the water under pressure within the dam towards the reservoir [10, 15].

4.2 Results of the numerical model

The numerical model mentioned in section 2 has been applied to the five earth dam cross sections. Above the saturation line the soil is assumed completely dry, and below it, totally saturated. As input motion the horizontal accelerogram recorded at Pacoima dam in the San Fernando earthquake in 1971 (Los Angeles area, 254, CDMG Station 279), scaled to the maximum horizontal acceleration of 0.6 g, has been applied to the base rock of all the dams. This accelerogram is usually employed in studies of the failure of the lower San Fernando dam [9].

4.3 Optimal cross section

In order to obtain the optimal cross section of earth dams with impervious core against liquefaction at
seismic sites between all the analyzed geometries, the next criteria should be taken into consideration:

- Maximum liquefaction degree.
- Highest percentage of closely liquefied soil in the dam.
- Crest settlement.

From the numerical results, and for the considered input accelerogram, maximum liquefaction degree is not a valid criterion in this case, because all the analyzed dams suffer liquefaction. In Fig. 6, the degradation of the materials of the dams at the end of the earthquake, in terms of percentage over a certain liquefaction degree, \( r_u \), against the angle of the impervious core, \( \delta \), is summarized. Results for liquefaction degrees \( r_u = 0.2, 0.6, \) and 0.9 are given. It is noticeable that the cross section with the minimum percentage of soil over \( r_u = 0.9 \) is the one corresponding to the most inclined core, (i.e. the core totally leaned to the upstream slope), which is 13\%. For the other two graphs of this figure, corresponding to the other values of \( r_u \), it is clear that the minimum values of percentages are always reached for the highest \( \delta \) which means that for this dam the degradation state of the soil is minimum.

Fig. 7 shows the vertical displacement computed at the crest dams at the end of the earthquake (negative sign means settlement). According to this criterion, the best cross section is again the corresponding to the maximum value of \( \delta \), with a total amount of settlement of 43 cm.

5 Conclusions

In this paper the application of a newly developed coupled numerical model to the dynamic analysis of earth dams under earthquakes has been shown, in order to explore the influence of the impervious core location inside the dams on the risk of failure due to liquefaction. After a brief description of the model, the geometry of the five analyzed dams has been defined. All dams are 100 m height, and the core
location ranges from totally centered to totally leaned to the upstream slope. The five analyzed dams are strictly stable under static loadings, and are subjected to the same earthquake at the base. The main findings of this numerical simulations show that:

- Liquefaction is reached in all these dams under the selected accelerogram.
- The optimal cross section against soil degradation is the one with the core totally leaned to the upstream slope.
- The dam which suffers the minor crest settlement is again the one with the most inclined core.

Therefore, the location of the impervious core by the upstream slope may be a design criteria in seismic sites, although in every particular situation, an analysis similar to the one developed in this paper should be done, following the same addressed guidelines by considering the exact dam height and geometry, material properties and seismic loading.

In addition, it is known that in this type of dams, the excess pore water pressure is dissipated through the downstream earthfill, which improves the safety during the consolidation stage.

The optimal cross section obtained herein is equivalent to earth dams with upstream impervious membranes, like concrete or asphalt, which work as barriers preventing the flow of liquefied material through the dam towards the reservoir. In order to avoid the dam collapse during of after an earthquake, the resistance of the membrane itself against seismic loading has to be assessed, to avoid as much as possible the development of cracks and leaks which the liquefied soil may pass through. On this sense, the use of asphalt may be more adequate than concrete, which is a more rigid material.

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References: