Study of the dynamics of a rocket-launching device system on a heavy vehicle

PAMFIL SOMOIAG, FLORENTIN MORARU, DORU SAFTA, CRISTIAN MOLDOVEANU
Department of Mechanical Engineering
Military Technical Academy
No.81-83, George Cosbuc, Sector 5, Bucharest, cod 75275
ROMANIA

Abstract: - An important problem in studying the sloped rocket launch is to determine the oscillations and their effects on the initial conditions of the rocket path. This phenomenon influences the stability of the launching device and the firing precision. We suppose that the launching device and the moving rocket form a complex oscillating system that join together a sum of rigid bodies bound by elastic elements (the vehicle chassis, the tilting platform and the rockets in the containers). The article presents a general mathematical model that consists of a nonlinear differential equations system with variable coefficients. The system allows calculating the oscillations of the launching device during firing, evaluating the dynamic stability and also computing more accurately the rocket movement elements during launch.

Key-Words: - launching device, oscillation, disturbance, mathematical model, vehicle chassis, tilting platform, unguided rocket, heavy vehicle

1 Introduction

The study of launching device oscillations during firing is necessary for the design of precise and efficient rocket-launching device systems, especially in the case of unguided rockets.

In order to meet the following requirements: computation of the oscillation parameters of a launching device during firing and estimation of the influence of these parameters on the rocket's flight, this study presents a general mathematical model for the calculus of oscillations during rocket launching.

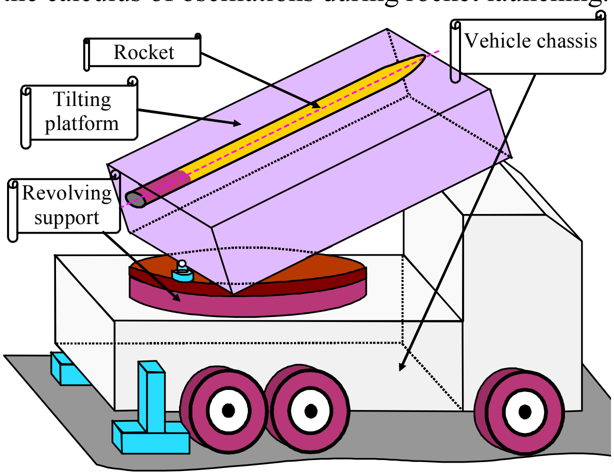

Fig.1 The components of the rocket-launching device system

We consider that the launching device with the moving rocket form an oscillating system, described by an assemble of the rigid bodies (fig.1) bound together by elastic elements, having as main components: the vehicle chassis (upon which is laid the launching device's basis with the revolving support of the mechanisms), the tilting platform (with the containers for the rockets) and the rockets (including the moving rocket).

To be able to approach the real analyzed phenomenon, we took into consideration all forces and moments that act upon the rocket-launching device system during firing.

The mathematical model consists of a nonlinear differential equations system with variable coefficients. This model can be used for the study of every launching device that resembles the one presented in fig. 1. This model allows the calculus of oscillations during firing, the evaluation of dynamical stability and a more accurate calculus of movement parameters for the launched rocket.

2 System of axes and position variables

The system of axes and the variables that define the position and movement of the launching device are chosen so that they do not limit the generality of the studied problem and as to lead to the equations that, with the appropriate transformations and simplifications, can be integrated by well known methods.

The movement of the whole system is related to a fixed system of axes "bound up with the ground", $O_T x_T y_T z_T$, O_T is considered to be on the same vertical line with the chassis' centre of masse, and contained in the longitudinal axial plane of the chassis.

In the chassis' centre of masse, O_S, we consider the "bound up with the chassis" system of axes $O_S x_S y_S z_S$ (fig.2) which describes the whole vehicle chassis' movements (the translation z_S on the vertical direction and the rotations γ_x and γ_y). The translations x_S and y_S as the rotation γ_z are considered small and can be neglected.

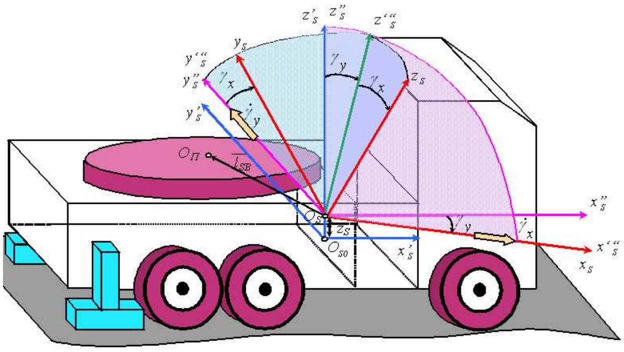

Fig. 2 The systems of axes describing the chassis movement

The bound between the chassis and the tilting platform is accomplished by a revolving support, considered integrated part of the chassis (it has the same movement as the chassis). In the centre of the revolving support, O_Π, we considered a "bound up with the revolving support" system of axes, $O_\Pi x_\Pi y_\Pi z_\Pi$. This system is obtained by rotation in horizontal plan with the direction angle φ_{H0} around the axis $O_S z_S$ of the chassis system of axes.

On the revolving support there is an important point for the system's kinematics, the point O_η, which represents the point where the axle collar of the tilting platform passes through the longitudinal plan of the launching device. The point O_η represents the origin of the "bound up with the axle collar of the tilting platform" system of axes, $O_\eta x_\eta y_\eta z_\eta$ (fig. 3). The initial position of the system is obtained by rotating the tilting platform in the vertical plan with the angle φ_{V0}. During firing, the moving rocket forces and moments impose on the tilting platform two rotations: φ_z and φ_y (the rotation φ_x around the longitudinal axes is neglected having smaller values than the others two rotations).

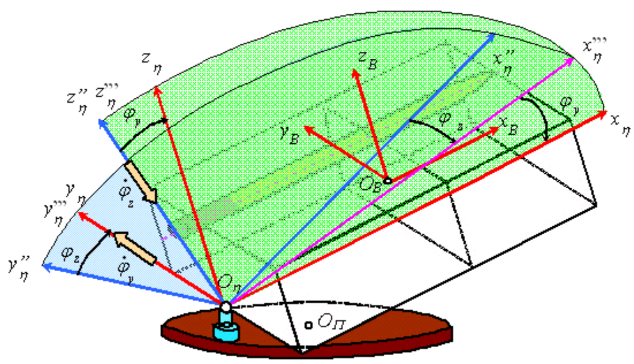

Fig. 3 The systems of axes defining the tilting platform movement

In the tilting platform's centre of masse, O_B, is considered the "bound up with the tilting platform" system of axes, $O_B x_B y_B z_B$ (fig. 3), with the axis parallel with $O_\eta x_\eta y_\eta z_\eta$.

During launching the rocket is submitted to a slow rotation movement, $\bar{\beta}$ ($\beta_x \equiv \beta$), aiming to accomplish a corresponding firing accuracy. Without limiting the generality of the study, we considered the initial angle $\beta_0 = 0$. Until leaving the launching device, the rocket accomplishes a translation movement s in relation to the tilting platform. The rocket's centre of masse is considered the origin of the "bound up with the rocket" system of axes $O_R x_R y_R z_R$ (see fig. 4).

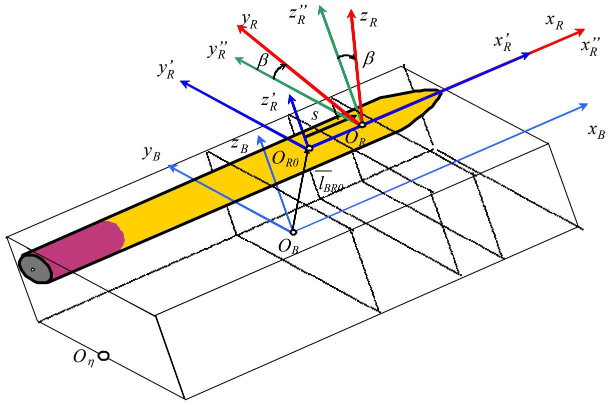

Fig. 4 The rocket systems of axes

In the study of the system composed of the vehicle chassis, the tilting platform and the moving rocket, we consider 6 position variables (they describe entirely the system movement): the vehicle chassis translation „z_S", the vehicle chassis rotation „γ_y" (the chassis pitch movement), the vehicle chassis rotation „γ_x" (the chassis rolling movement), the tilting platform rotation „φ_z" (the gyration movement around the vertical axes), the tilting platform rotation „φ_y" (the pitch movement) and the rocket translation „s".

3 Forces and moments

We will consider separately each component of the system: the rocket, the tilting platform and the vehicle chassis in order to highlight the exterior and interior forces that act on each one of them.

3.1 Forces and moments acting on the rocket

Upon the rocket act the following forces and moments: the rocket thrust $\overline{T}$ oriented along the $O_R x_R$ axis, the rocket weight, $\overline{G}_R$ and the connections forces and moments (see fig. 5).

The connection forces (including the friction forces) act on the entire contact surface between the rocket and the guiding tube. In order to reduce the complexity of the calculus, but without limiting the studied problem generality, these forces will be reduced to resulting forces and moments in the rocket centre of masse.

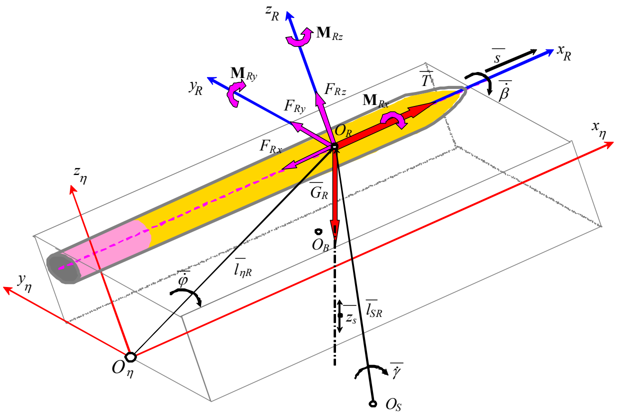

Fig. 5 The rocket forces and moments

The connection force between the rocket and the tilting platform, $\overline{F}_R$ (force of reaction) has, in the rocket transversal plan, the components F_{Ry} and F_{Rz} (forces of reactions between the rocket and the tilting platform), and the component F_{Rx} oriented along the $O_R x_R$ axis that consists in the withholding force $\overline{F}_{ret}$. This acts upon the rocket until the thrust overcomes the withholding force (the withholding force keeps the rocket from moving until the rocket thrust attains a certain imposed value). Additionally, the friction force $\overline{F}_f$ acts between the rocket and the tilting platform.

The connection moment between the rocket and the tilting platform, $\overline{\mathbf{M}}_R$ (the moment of reaction of the tilting platform on the moving rocket) is given in relation with the rocket centre of masse, having the components $\mathbf{M}_{Rx}$, $\mathbf{M}_{Ry}$ and $\mathbf{M}_{Rz}$ in the rocket system of axes $O_R x_R y_R z_R$.

3.2 Forces and moments acting on the tilting platform

In this paragraph we will describe the forces and the moments acting on the tilting platform: the weight, $\overline{G}_B$, the connection forces and the moments, between the tilting platform and the rocket and the same forces and moments operating between the tilting platform and the chassis through the axle collar of the tilting platform (fig. 6).

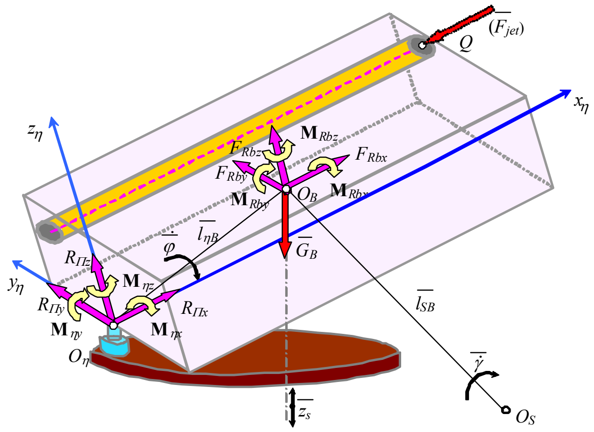

Fig. 6 Tilting platform forces and moments

The connection force between the tilting platform and the rocket is given by the relation:

$$\overline{F}_{Rb} = -\overline{F}_R - \sum_i \overline{F}_{Ri} - \overline{F}_{jet}\,, \qquad (1)$$

where $\overline{F}_R$ is the force of reaction of the lunching rocket on the tilting platform, $\overline{F}_{Ri}$ are the additional forces that act on the tilting platform, forces of the rockets that are in the container and lead to an increase of the tilting platform weight and $\overline{F}_{jet}$, the force of the rocket jet.

Similarly, the moment of reaction between the tilting platform and the rocket, $\overline{\mathbf{M}}_{Rb}$, is written:

$$\overline{\mathbf{M}}_{Rb} = -\overline{\mathbf{M}}_R - \sum_i \overline{\mathbf{M}}_{Ri} - \overline{l}_{BR} \times \overline{F}_R - \sum_i \overline{l}_{BR0_i} \times \overline{F}_{Ri} - \overline{l}_{BQ} \times \overline{F}_{jet}\,. \qquad (2)$$

The connection forces and moments between the tilting platform and the vehicle chassis are: $\overline{R}_\Pi$, the force of reaction on the tilting platform, with the application point in the O_η, $\overline{\mathbf{M}}_\eta$, the moment of reaction on the tilting platform of the vehicle chassis with the same application point having the components $\mathbf{M}_{\eta x}$, $\mathbf{M}_{\eta y}$ and $\mathbf{M}_{\eta z}$ in the tilting platform system of axes (the last two components are the counter-balance moments).

3.3 Forces and moments acting on the vehicle chassis

Furthermore, the forces and moments acting on vehicle chassis are: the weight, $\overline{G}_S$, the connections forces and moments between the chassis and the tilting platform, $\overline{R}_\Pi$ and $\overline{\mathbf{M}}_\eta$ (fig. 7 and fig.8), as well as the connection forces between the chassis and the ground: $\overline{F}_{a1}, \overline{F}_{a2}$ (at the anterior chassis axle), $\overline{F}_{p1}, \overline{F}_{p2}$ (at the posterior chassis axle).

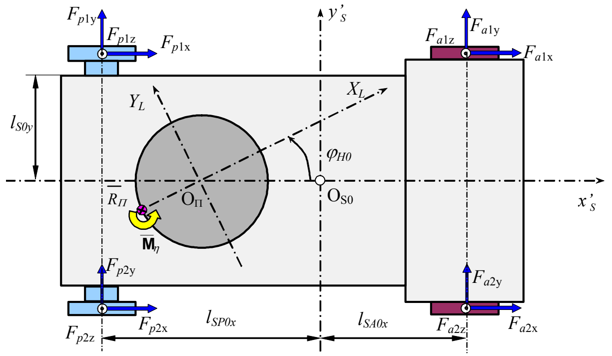

Fig. 7 Vehicle chassis forces and moments

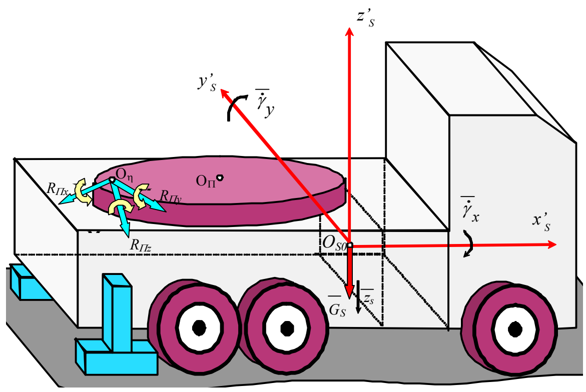

Fig. 8 Vehicle chassis forces and moments

In this paragraph we have presented two categories of forces and moments according to the degree of knowledge: known forces and moments and unknown forces and moments.

The known forces and moments are the forces and the moments that can be determined on the basis of known parameters like rocket thrust, weight of the components, vertical components of the connection forces (between the vehicle chassis and the ground) and the components of the tilting platform-chassis connection moment.

The unknown forces and moments are to be determined on the basis of the motion equations components: the rocket-tilting platform connection moments and tilting platform-vehicle chassis, horizontal components of the chassis-ground connection forces.

Knowing the forces and moments that act on the rocket-launching device system we will be able to determine the general dynamic equations that describe the movement of this system during launching.

4 General dynamic equations of the rocket-launching device system

In the rocket-launching device system there are 6 characteristic points (fig. 5): O_T (the fixed point bound up with the ground), O_S (the vehicle chassis centre of masse), O_Π (the centre of masse of the revolving support), O_η (the meeting point between the axle collar of the tilting platform and the longitudinal plan of the launching device), O_B (the tilting platform centre of masse) and O_R (the moving rocket centre of masse).

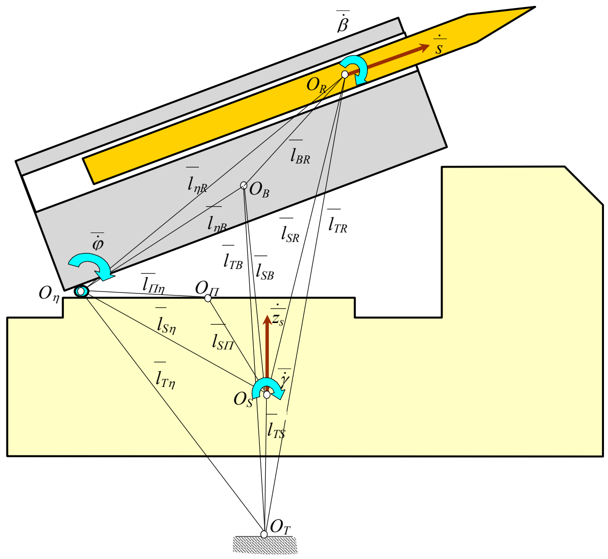

Fig. 9 The calculus diagram for the rocket-launching device system

In our study, the moving rocket system of axes moves in relation to the tilting platform system of axes, whereas the last one moves in relation to the chassis system of axes which has the ground system of axes as a reference system (fixed coordinate system).

Therefore, in this case, seeing the complexity of the considered model, it is necessary to express the velocity and the acceleration for the rigid body motion having as a reference the mobile systems of axes (fig. 10) [2]:

$$\bar{v}_M = \bar{v}_1 + \bar{v}_2 + \bar{v}_3 + \bar{\omega}_1 \times \bar{r}_1 + \bar{\omega}_2 \times \bar{r}_2 + \bar{\omega}_3 \times \bar{r}_3 \qquad (3)$$

$$\begin{aligned}\bar a_M &= \bar a_1 + \bar a_2 + \bar a_3 + \dot{\bar\omega}_1 \times \bar r_1 + \dot{\bar\omega}_2 \times \bar r_2 + \dot{\bar\omega}_3 \times \bar r_3 + \\ &+ \bar\omega_1 \times (\bar\omega_1 \times \bar r_1) + \bar\omega_2 \times (\bar\omega_2 \times \bar r_2) + \\ &+ \bar\omega_3 \times (\bar\omega_3 \times \bar r_3) + 2\bar\omega_1 \times (\bar v_2 + \bar\omega_2 \times \bar r_2) + \\ &+ 2(\bar\omega_1 + \bar\omega_2) \times (\bar v_3 + \bar\omega_3 \times \bar r_3).\end{aligned} \tag{4}$$

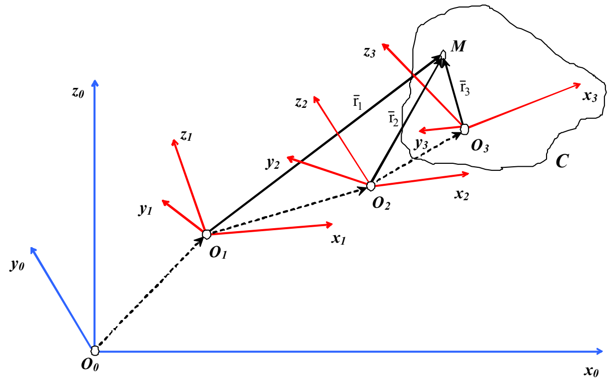

Fig. 10 General systems of axes

Moreover, it was deemed necessary to establish the translations and rotations relative motions of the coordinate systems for all the rocket-launching device system components.

On the basis of the data presented before and using the fundamental theories of the solid mechanics (pulse theorem, etc.) we can determine the vectorial equations that describe the movement of the centre of masse and therefore the movement around it [1].

a) Equations at the moving rocket level

$$\begin{aligned}&M_R \ddot{\bar s} = \bar T + \bar G_R + \bar F_R - M_R \ddot{\bar z}_S - M_R \ddot{\bar\gamma} \times \overline{l_{SR}} + \\ &+ M_R \overline{l_{SR}}\left(\dot\gamma^2\right) - 2M_R \dot{\bar\gamma} \times \dot{\bar s} - M_R \dot{\bar\gamma}\left(\dot{\bar\gamma}\, \overline{l_{SR}}\right) - \\ &- M_R \ddot{\bar\varphi} \times \overline{l_{\eta R}} + M_R \overline{l_{\eta R}}\left(\dot\varphi^2\right) - 2M_R \dot{\bar\varphi} \times \dot{\bar s} - \\ &- M_R \dot{\bar\varphi}\left(\dot{\bar\varphi}\, \overline{l_{\eta R}}\right) + 2M_R \overline{l_{\eta R}}\left(\dot{\bar\gamma}\, \dot{\bar\varphi}\right) - 2M_R \dot{\bar\varphi}\left(\dot{\bar\gamma}\, \overline{l_{\eta R}}\right);\end{aligned} \tag{5}$$

$$\tau_R \dot{\bar\omega}_R + \bar\omega_R \times (\tau_R \bar\omega_R) = \mathbf{M}_R; \tag{6}$$

b) Equations at the tilting platform level

$$\begin{aligned}&0 = \bar F_{Rb} + \bar G_B + \bar R_\Pi - M_B \ddot{\bar z}_S - M_B \ddot{\bar\gamma} \times \overline{l_{SB}} + \\ &+ M_B \overline{l_{SB}}\left(\dot{\bar\gamma}^2\right) - M_B \dot{\bar\gamma}\left(\dot{\bar\gamma}\, \overline{l_{SB}}\right) - M_B \ddot{\bar\varphi} \times \overline{l_{\eta B}} + \\ &+ M_B \overline{l_{\eta B}}\left(\dot{\bar\varphi}^2\right) - M_B \dot{\bar\varphi}\left(\dot{\bar\varphi}\, \overline{l_{\eta B}}\right) + \\ &+ 2M_B \overline{l_{\eta B}}\left(\dot{\bar\gamma}\, \dot{\bar\varphi}\right) - 2M_B \dot{\bar\varphi}\left(\dot{\bar\gamma}\, \overline{l_{\eta B}}\right);\end{aligned} \tag{7}$$

$$\tau_B \dot{\bar\omega}_B + \bar\omega_B \times (\tau_B \bar\omega_B) = \overline{l_{B\eta}} \times \bar R_\Pi + \mathbf{M}_{Rb} + \mathbf{M}_\eta; \tag{8}$$

c) Equations at the vehicle chassis level

$$M_S \cdot \ddot{\bar z}_S = \bar G_S - \bar R_\Pi + \bar F_{a1} + \bar F_{a2} + \bar F_{p1} + \bar F_{p2}; \tag{9}$$

$$\begin{aligned}&\tau_S \dot{\bar\omega}_S + \bar\omega_S \times (\tau_S \bar\omega_S) = -\overline{l_{S\eta}} \times \bar R_\Pi - \\ &- \mathbf{M}_\eta + \overline{l_{SA_1}} \times \bar F_{a1} + \overline{l_{SA_2}} \times \bar F_{a2} + \\ &+ \overline{l_{SP_1}} \times \bar F_{p1} + \overline{l_{SP_2}} \times \bar F_{p2},\end{aligned} \tag{10}$$

where:

- τ_R is the tensor of the rocket moment of inertia in relation to the rocket system of axes $O_R x_R y_R z_R$ ($O_R x_R$ is a symmetrical axis for the rocket),

$$\tau_R = \begin{pmatrix} J_{Rx} & 0 & 0 \\ 0 & J_{Ry} & 0 \\ 0 & 0 & J_{Rz} \end{pmatrix}; \tag{11}$$

- J_{Rx}, J_{Ry}, J_{Rz} are the rockets moments of inertia; the transversal moments of inertia are equal as a result of the rocket symmetry:

$$J_{Ry} = J_{Rz} = J; \tag{12}$$

- $\bar\omega_R$ is the angular velocity of the rocket system of axes, $O_R x_R y_R z_R$, in relation to the ground system of axes, $O_T x_T y_T z_T$, given by the relation:

$$\bar\omega_R = \dot{\bar\gamma} + \dot{\bar\varphi} + \dot{\bar\beta}; \tag{13}$$

- τ_B is the tensor of the tilting platform moments of inertia in relation to the tilting platform system of axes $O_B x_B y_B z_B$, described by the relation:

$$\tau_B = \begin{pmatrix} J_{Bx} & -J_{Bxy} & -J_{Bxz} \\ -J_{Byx} & J_{By} & -J_{Byz} \\ -J_{Bzx} & -J_{Bzy} & J_{Bz} \end{pmatrix}; \tag{14}$$

- $\bar\omega_B$ is the angular velocity of the tilting platform system of axes, $O_B x_B y_B z_B$, in relation to the ground system, $O_T x_T y_T z_T$:

$$\bar\omega_B = \dot{\bar\gamma} + \dot{\bar\varphi}; \tag{15}$$

- τ_S is the tensor of the chassis moments of inertia in relation to the vehicle chassis system $O_S x_S y_S z_S$,

$$\tau_S = \begin{pmatrix} J_{Sx} & -J_{Sxy} & -J_{Sxz} \\ -J_{Syx} & J_{Sy} & -J_{Syz} \\ -J_{Szx} & -J_{Szy} & J_{Sz} \end{pmatrix}; \tag{16}$$

- $\bar\omega_S$ is the angular velocity of the vehicle chassis system, $O_S x_S y_S z_S$, in relation to the ground system, $O_T x_T y_T z_T$,

$$\bar\omega_S = \dot{\bar\gamma}. \tag{17}$$

The components $\mathbf{M}_{\eta y}$ and $\mathbf{M}_{\eta z}$ of the tilting platform and the vehicle chassis connection moment, is given by the counter-balance moment on the tilting platform level, and have the expression:

$$\mathbf{M}_{\eta y} = \mathbf{M}_{\eta y0} - \varphi_y K_{\eta e_y} l_{\varphi y}^2 - \dot\varphi_y K_{\eta v_y} l_{\varphi y}^2, \tag{18}$$

$$\mathbf{M}_{\eta z} = \mathbf{M}_{\eta z0} - \varphi_z K_{\eta e_z} l_{\varphi z}^2 - \dot\varphi_z K_{\eta v_z} l_{\varphi z}^2, \tag{19}$$

where:

- $\mathbf{M}_{\eta y0}$, $\mathbf{M}_{\eta z0}$ are the static connection moments between the tilting platform and the chassis,
- $K_{\eta e_y}$ and $K_{\eta e_z}$ are the elastic coefficients of the balancing system at the tilting platform level for the axes $O_\eta y_\eta$ and $O_\eta z_\eta$;
- $K_{\eta v_z}$ and $K_{\eta v_y}$ are the viscous friction coefficients of the balancing system at the tilting platform level for the axes $O_\eta y_\eta$ and $O_\eta z_\eta$;
- l_{φ_y} and l_{φ_z} are the characteristic lengths for the rotation angles φ_y and φ_z.

The system presented before, formed from 6 vectorial equations (5) - (10) (in the general case it contains 18 scalar equations) and 4 independent vectorial unknown variables $\overline{z}_S$, $\overline{s}$, $\overline{\gamma}$ and $\overline{\varphi}$ (therefore 6 independent scalar unknown variables z_S, $\gamma_x, \gamma_y, \varphi_y, \varphi_z$ and s). In order to calculate these parameters only 6 scalar equations are necessary. The other 12 scalar equations are used to determine the connection forces and moments: $\overline{F}_R$ (2 unknown variables), $\overline{\mathbf{M}}_R$ (3 unknown variables), $\overline{R}_\Pi$ (3 unknown variables), $\overline{\mathbf{M}}_\eta$ (1 unknown variable), $\overline{F}_{p1}$ and $\overline{F}_{p2}$ (3 unknown variables).

4 Conclusion

The solving of the 18 scalar equations demands the writing of all the vector (including forces and moments) expressions in the convenient systems of reference using the transformation matrices in order to switch from one system to another.

The 6 scalar equations are necessary to calculate the 6 unknown variables that describe the movement of the rocket-launching device system during firing z_S, $\gamma_x, \gamma_y, \varphi_y, \varphi_z$ and s, while the other 12 scalar equations allow to compute the evolutions of the connection forces and moments F_{Ry}, F_{Rz}, $\mathbf{M}_{Rx}$, $\mathbf{M}_{Ry}$, $\mathbf{M}_{Rz}$, $R_{\Pi x}$, $R_{\Pi y}$, $R_{\Pi z}$, $\mathbf{M}_{\eta x}$, F_{p1x}, F_{p2x} and F_{py}. Using the expression of these forces and moments in the firsts 6 equations we obtain a system of 6 nonlinear differential second order equations with variable coefficients.

The numerical solving of this system amounts to obtaining the oscillations of the rocket-launching device system during the firing and the evaluation of the dynamic stability. Computing the rocket-launching device system's position variables (z_S, $\gamma_x, \gamma_y, \varphi_y, \varphi_z$ and s) is useful to estimate the influence of the disturbances on the firing accuracy at the moment of launching.

References:

[1] X1. Somoiag P., *Cercetari privind determinarea oscilatiilor la lansare si influenta acestora asupra zborului rachetei*, PhD thesis, Military Technical Academy, Bucharest, 2007.

[2] X2. Voinea R., Voiculescu, D., Ceausu. V., *Mecanica Teoretica*, Educational and Pedagogical Publishing House, 1975.

[3] X3. Voinea R., *Mecanica si vibratii mecanice*, Educational and Pedagogical Publishing House, 1999.

[4] X4. Krylov V., *Computation of Ground Vibrations Generated by Accelerating and Braking Road Vehicles*, Journal of Vibration and Control, SAGE Publications, Vol. 2, No. 3, pages 299-321, 1996.

[5] X5. Soliman A.M.A., Abd Allah S.A., El-Beter A.A., Hamid M.S., *Effect of suspension spring stiffness on vehicle dynamics*, International Journal of Heavy Vehicle Systems 2001 - Vol. 8, No.3/4 pp. 316-334.

[6] X6. Qin Zhua, Mitsuaki Ishitobi., *Chaotic vibration of a nonlinear full-vehicle model*, International Journal of Solids and Structures Volume 43, Issues 3-4, February 2006, pages 747-759.