Modeling of switching operations using fault matrix method

MARTIN WOLTER, BERND R. OSWALD
Institute of Electric Power Systems
Leibniz University of Hannover
Appelstr. 9a, 30167 Hannover
GERMANY
http://www.iee.uni-hannover.de

Abstract: - Switching operations in energy supply networks are either modeled by adding or removing artificial nodes which results in state dependent grid topology or by setting the switch impedance to high or low value. This procedure is not very accurate and can cause numerical problems.

In this paper a more skillful method of modeling switches by systematically modifying coefficients in the grid admittance matrix is presented.

Key-Words: - Switching operation, admittance matrix, Power system, Fault matrix method, Transmission lines

1 Introduction

The conventional method of fault calculation is intended to calculate currents and voltages at the fault location using symmetrical components. Therefore elements of $\underline{\boldsymbol{Y}}_{\mathrm{KK}}^{-1}$ are needed which cannot easily be calculated due to singularity of $\underline{\boldsymbol{Y}}_{\mathrm{KK}}$ in grids without neutral earthing. Additionally complexity increases extensively in case of multiple coexistent faults.

An often used simple way of modeling shunt faults is adding admittance to the fault location. In case of unresistant short-circuits the admittance gets infinite and has to be replaced by a predefined high value in calculation programs.

On the other hand series faults and switches are modeled by adding or removing additional nodes to the grid which results in switch state dependent and variably sized grid matrices and decreases calculation performance or by approximating an open switch by inserting high impedances.

It is obvious that the conventional method is unsatisfactory particularly with regard to simulate switching operations. In this paper a new approach to model switches based on the fault matrix method is introduced.

2 Fault Matrix Method

The fault matrix method was developed to calculate any balanced or unbalanced single or coexistent faults on power transmission systems. It features a systematic and simple algorithm to be used in steady-state or dynamic modeling and offers an exact and well arranged simulation of resistant and unresistant series and shunt faults by only modifying coefficients in the terminal admittance matrix without changing grid topology.

In this paper simulation of switch operations are focused which are modeled as series faults. Calculation result is the modified admittance matrix. Fault currents and voltages are not of interest now and can later be calculated i.e. during power flow simulation.

3 Three-phase equipment and grid model

All transformers and lines can be modeled as three-phase quadrupoles as seen in Fig. 1. Its terminals A and B are logically and physically connected to grid nodes.

The logical connection is manifested in the nodal-terminal-incidence-matrix $\boldsymbol{K}_{\mathrm{KT}}$ which defines grid topology by attaching terminals to nodes. $\boldsymbol{K}_{\mathrm{KT}}$ is not subject to change during switching operations.

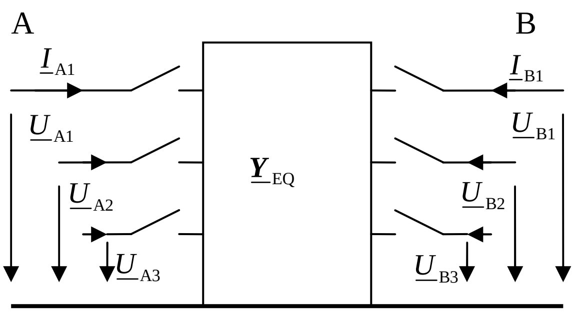

Fig. 1: three-phase quadrupole

The physical behavior of the equipment including the influence of its switches is defined by its admittance-matrix $\underline{\boldsymbol{Y}}_{\mathrm{EQ}}$.

$$\begin{bmatrix} \underline{\boldsymbol{i}}_{\mathrm{A}} \\ \underline{\boldsymbol{i}}_{\mathrm{B}} \end{bmatrix} = \begin{bmatrix} \underline{\boldsymbol{Y}}_{\mathrm{AA}} & \underline{\boldsymbol{Y}}_{\mathrm{AB}} \\ \underline{\boldsymbol{Y}}_{\mathrm{BA}} & \underline{\boldsymbol{Y}}_{\mathrm{BB}} \end{bmatrix} \begin{bmatrix} \underline{\boldsymbol{u}}_{\mathrm{A}} \\ \underline{\boldsymbol{u}}_{\mathrm{B}} \end{bmatrix} = \underline{\boldsymbol{Y}}_{\mathrm{EQ}} \, \underline{\boldsymbol{u}}_{\mathrm{EQ}} \tag{1}$$

All admittance matrices are collected to the terminal-admittance-matrix $\underline{\boldsymbol{Y}}_{\mathrm{T}}$.

$$\underline{\boldsymbol{Y}}_{\mathrm{T}} = \begin{bmatrix} \underline{\boldsymbol{Y}}_{\mathrm{EQ,1}} & \cdots & \boldsymbol{0} \\ \vdots & \ddots & \vdots \\ \boldsymbol{0} & \cdots & \underline{\boldsymbol{Y}}_{\mathrm{EQ,x}} \end{bmatrix} \tag{2}$$

The nodal-admittance-matrix can now be built

$$\underline{\boldsymbol{Y}}_{\mathrm{KK}} = -\boldsymbol{K}_{\mathrm{KT}} \, \underline{\boldsymbol{Y}}_{\mathrm{T}} \, \boldsymbol{K}_{\mathrm{KT}}^{\mathrm{T}} \tag{3}$$

4 Calculation of switch state dependent admittance matrices

As mentioned above the intention of fault matrix method is to modify admittance matrices. To model switching operations a switch state matrix $\boldsymbol{F}$ of the same size as $\underline{\boldsymbol{Y}}_{\mathrm{T}}$ is necessary and is build in the same manner since for each terminal a switch state matrix $\boldsymbol{F}_{\mathrm{EQ,T}}$ is needed. A one on its main diagonal represents a closed switch at the corresponding terminal whereas a zero indicates an open one. Table 1 shows matrices of some common switch states.

Table 1: Switch state matrices

state	switch state matrix
all closed	$\boldsymbol{F}_{\mathrm{EQ,T}} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
1LO: L1	$\boldsymbol{F}_{\mathrm{EQ,T}} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
2LO: L2, L3	$\boldsymbol{F}_{\mathrm{EQ,T}} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$
3LO	$\boldsymbol{F}_{\mathrm{EQ,T}} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$$\boldsymbol{F} = \begin{bmatrix} \boldsymbol{F}_{\mathrm{EQ,1}} & \cdots & \boldsymbol{0} \\ \vdots & \ddots & \vdots \\ \boldsymbol{0} & \cdots & \boldsymbol{F}_{\mathrm{EQ,x}} \end{bmatrix} \tag{4}$$

With these state matrices and the voltages across the switches the voltage constraint can be defined.

$$\begin{bmatrix} \boldsymbol{F}_1 & & \\ & \ddots & \\ & & \boldsymbol{F}_{\mathrm{x}} \end{bmatrix} \begin{bmatrix} \underline{\boldsymbol{u}}_{\mathrm{F,1}} \\ \vdots \\ \underline{\boldsymbol{u}}_{\mathrm{F,x}} \end{bmatrix} = \boldsymbol{0} \tag{5}$$

According to equation (5) the second constraint concerning terminal currents is given in equation (6).

$$\begin{bmatrix} \boldsymbol{E} - \boldsymbol{F}_1^{\mathrm{T}} & & \\ & \ddots & \\ & & \boldsymbol{E} - \boldsymbol{F}_{\mathrm{x}}^{\mathrm{T}} \end{bmatrix} \times \\ \times \left\{ \begin{bmatrix} \underline{\boldsymbol{i}}_{\mathrm{EQ,1}} \\ \vdots \\ \underline{\boldsymbol{i}}_{\mathrm{EQ,x}} \end{bmatrix} - \begin{bmatrix} \underline{\boldsymbol{Y}}_{\mathrm{F,1}} & & \\ & \ddots & \\ & & \underline{\boldsymbol{Y}}_{\mathrm{F,x}} \end{bmatrix} \begin{bmatrix} \underline{\boldsymbol{u}}_{\mathrm{F,1}} \\ \vdots \\ \underline{\boldsymbol{u}}_{\mathrm{F,x}} \end{bmatrix} \right\} = \boldsymbol{0} \tag{6}$$

Terminal currents are assumed to be zero at open switches so switch admittances $\underline{\boldsymbol{Y}}_{\mathrm{F}}$ are negligible during further investigations.

Equation (1) is extended with the voltage constraint resulting in

$$\begin{bmatrix} \underline{\boldsymbol{i}}_{\mathrm{EQ,1}} \\ \vdots \\ \underline{\boldsymbol{i}}_{\mathrm{EQ,x}} \end{bmatrix} = \begin{bmatrix} \underline{\boldsymbol{Y}}_{\mathrm{EQ,1}} & & \\ & \ddots & \\ & & \underline{\boldsymbol{Y}}_{\mathrm{EQ,x}} \end{bmatrix} \left\{ \begin{bmatrix} \underline{\boldsymbol{u}}_{\mathrm{EQ,1}} \\ \vdots \\ \underline{\boldsymbol{u}}_{\mathrm{EQ,x}} \end{bmatrix} - \begin{bmatrix} \underline{\boldsymbol{u}}_{\mathrm{F,1}} \\ \vdots \\ \underline{\boldsymbol{u}}_{\mathrm{F,x}} \end{bmatrix} \right\} \tag{7}$$

or more concisely

$$\underline{\boldsymbol{i}}_{\mathrm{T}} = \underline{\boldsymbol{Y}}_{\mathrm{T}} \left\{ \underline{\boldsymbol{u}}_{\mathrm{T}} - \underline{\boldsymbol{u}}_{\mathrm{F}} \right\} \tag{8}$$

Now terminal currents in equation (6) are eliminated by inserting equation (7). Using the auxiliary matrices

$$\underline{\boldsymbol{Y}}_{\mathrm{F1}} = \begin{bmatrix} \boldsymbol{F}_1^{\mathrm{T}} & & \\ & \ddots & \\ & & \boldsymbol{F}_{\mathrm{x}}^{\mathrm{T}} \end{bmatrix} \begin{bmatrix} \underline{\boldsymbol{Y}}_{\mathrm{EQ,1}} & & \\ & \ddots & \\ & & \underline{\boldsymbol{Y}}_{\mathrm{EQ,x}} \end{bmatrix} \\ \underline{\boldsymbol{Y}}_{\mathrm{F2}} = \begin{bmatrix} \underline{\boldsymbol{Y}}_{\mathrm{EQ,1}} & & \\ & \ddots & \\ & & \underline{\boldsymbol{Y}}_{\mathrm{EQ,x}} \end{bmatrix} \begin{bmatrix} \boldsymbol{F}_1 & & \\ & \ddots & \\ & & \boldsymbol{F}_{\mathrm{x}} \end{bmatrix} \tag{9}$$

and

$$\underline{\boldsymbol{Z}}_{\mathrm{FF}} = \left[\underline{\boldsymbol{Y}}_{\mathrm{F1}} - \underline{\boldsymbol{Y}}_{\mathrm{T}} + \underline{\boldsymbol{Y}}_{\mathrm{F2}}\right]^{-1} \begin{bmatrix} \boldsymbol{E} - \boldsymbol{F}_1^{\mathrm{T}} & & \\ & \ddots & \\ & & \boldsymbol{E} - \boldsymbol{F}_{\mathrm{x}}^{\mathrm{T}} \end{bmatrix} \tag{10}$$

the voltages across the switches can be calculated.

$$\begin{bmatrix} \underline{\boldsymbol{u}}_{\mathrm{F,1}} \\ \vdots \\ \underline{\boldsymbol{u}}_{\mathrm{F,x}} \end{bmatrix} = \underline{\boldsymbol{Z}}_{\mathrm{FF}}\, \underline{\boldsymbol{Y}}_{\mathrm{T}} \begin{bmatrix} \underline{\boldsymbol{u}}_{\mathrm{EQ,1}} \\ \vdots \\ \underline{\boldsymbol{u}}_{\mathrm{EQ,x}} \end{bmatrix} \tag{11}$$

Equation (11) is inserted into equation (7) resulting in

$$\begin{aligned} \underline{\boldsymbol{i}}_{\mathrm{T}} &= \underline{\boldsymbol{Y}}_{\mathrm{T}} \underline{\boldsymbol{u}}_{\mathrm{T}} - \underline{\boldsymbol{Y}}_{\mathrm{T}} \underline{\boldsymbol{Z}}_{\mathrm{FF}} \underline{\boldsymbol{Y}}_{\mathrm{T}} \underline{\boldsymbol{u}}_{\mathrm{T}} \\ &= \left\{\boldsymbol{E} - \underline{\boldsymbol{Y}}_{\mathrm{T}} \underline{\boldsymbol{Z}}_{\mathrm{FF}}\right\} \underline{\boldsymbol{Y}}_{\mathrm{T}} \underline{\boldsymbol{u}}_{\mathrm{T}} \end{aligned} \tag{12}$$

containing the modified admittance matrix $\underline{\boldsymbol{Y}}'_{\mathrm{T}}$:

$$\underline{\boldsymbol{Y}}'_{\mathrm{T}} = \left\{\boldsymbol{E} - \underline{\boldsymbol{Y}}_{\mathrm{T}}\, \underline{\boldsymbol{Z}}_{\mathrm{FF}}\right\} \underline{\boldsymbol{Y}}_{\mathrm{T}} \tag{13}$$

The nodal admittance matrix can now be calculated according to equation (3)

$$\underline{\boldsymbol{Y}}'_{\mathrm{KK}} = -\boldsymbol{K}_{\mathrm{KT}}\, \underline{\boldsymbol{Y}}'_{\mathrm{T}}\, \boldsymbol{K}_{\mathrm{KT}}^{\mathrm{T}} \tag{14}$$

One can see that it is possible to engage or disengage an arbitrary number of switches without increasing calculation effort.

5 Single-phase model

Assuming a balanced power system and balanced switching a single-phase system model is used to simplify calculation.

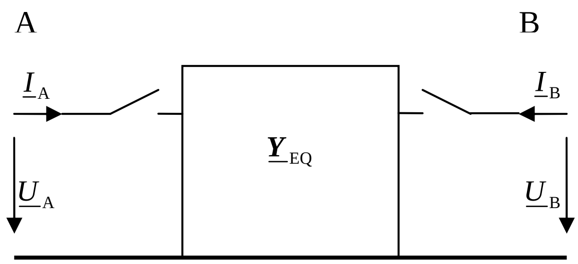

Fig. 2: single-phase quadrupole

Equation (1) is simplified to

$$\begin{bmatrix} \underline{I}_{\mathrm{A}} \\ \underline{I}_{\mathrm{B}} \end{bmatrix} = \begin{bmatrix} \underline{Y}_{\mathrm{AA}} & \underline{Y}_{\mathrm{AB}} \\ \underline{Y}_{\mathrm{BA}} & \underline{Y}_{\mathrm{BB}} \end{bmatrix} \begin{bmatrix} \underline{U}_{\mathrm{A}} \\ \underline{U}_{\mathrm{B}} \end{bmatrix} = \underline{\boldsymbol{Y}}_{\mathrm{EQ}}\, \underline{\boldsymbol{u}}_{\mathrm{EQ}} \tag{15}$$

and the terminal switch state matrix is reduced to either 1 if the switch is closed or 0 if it is disengaged.

Calculation of terminal admittance matrices is done analogously as described above. Due to only two switches per quadrupole the number of possible equipment admittance matrices is reduced to four, shown in Table 2.

Table 2: equipment admittance matrices

Switch A	Switch B	$\boldsymbol{F}_{\mathrm{EQ}}$	$\underline{\boldsymbol{Y}}'_{\mathrm{EQ}}$
closed	closed	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} \underline{Y}_{\mathrm{AA}} & \underline{Y}_{\mathrm{AB}} \\ \underline{Y}_{\mathrm{BA}} & \underline{Y}_{\mathrm{BB}} \end{bmatrix}$
open	closed	$\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 0 & 0 \\ 0 & \underline{Y}_{\mathrm{BB}} - \dfrac{\underline{Y}_{\mathrm{AB}}\,\underline{Y}_{\mathrm{BA}}}{\underline{Y}_{\mathrm{AA}}} \end{bmatrix}$
closed	open	$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$	$\begin{bmatrix} \underline{Y}_{\mathrm{AA}} - \dfrac{\underline{Y}_{\mathrm{AB}}\,\underline{Y}_{\mathrm{BA}}}{\underline{Y}_{\mathrm{BB}}} & 0 \\ 0 & 0 \end{bmatrix}$
open	open	$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

6 Simple Example

Given is a simple balanced grid with four nodes shown in Fig. 3 with four identical lines. Their parameters are displayed in table 3 and the grid matrices in equation (16) ff. A load of 100 MW is attached to node 4. Node 1 is used as slack.

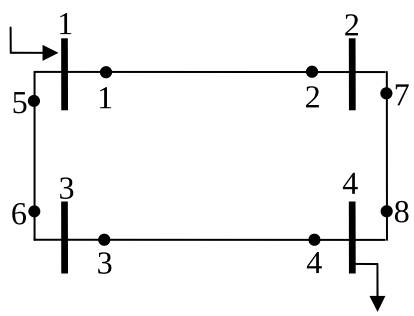

Fig. 3: simple example grid

Table 3: line parameters

length	10 km
r'	0.2 Ω/km
x'	0.4 Ω/km
c'	10 $\mathrm{nF/km}$

$$\boldsymbol{K}_{\mathrm{KT}} = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{bmatrix} \quad (16)$$

$$\underline{\boldsymbol{Y}}_{\mathrm{T},i} = \begin{bmatrix} 0.1-\mathrm{j}0.2 & -0.1+\mathrm{j}0.2 \\ -0.1+\mathrm{j}0.2 & 0.1-\mathrm{j}0.2 \end{bmatrix} \frac{1}{\Omega} \quad (17)$$

$$\underline{\boldsymbol{Y}}_{\mathrm{T}} = \begin{bmatrix} \underline{\boldsymbol{Y}}_{\mathrm{T},1} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \underline{\boldsymbol{Y}}_{\mathrm{T},2} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \underline{\boldsymbol{Y}}_{\mathrm{T},3} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \underline{\boldsymbol{Y}}_{\mathrm{T},4} \end{bmatrix} \quad (18)$$

$$\underline{\boldsymbol{Y}}_{\mathrm{KK}} = \begin{bmatrix} -2+\mathrm{j}4 & 1-\mathrm{j}2 & 1-\mathrm{j}2 & 0 \\ 1-\mathrm{j}2 & -2+\mathrm{j}4 & 0 & 1-\mathrm{j}2 \\ 1-\mathrm{j}2 & 0 & -2+\mathrm{j}4 & 1-\mathrm{j}2 \\ 0 & 1-\mathrm{j}2 & 1-\mathrm{j}2 & -2+\mathrm{j}4 \end{bmatrix} \frac{0.1}{\Omega} \quad (19)$$

A power flow calculation returns the following results.

$$\boldsymbol{u}_{\mathrm{K}} = \begin{bmatrix} 110 \\ 109.05 \\ 109.05 \\ 108.11 \end{bmatrix} \mathrm{kV}, \quad \boldsymbol{i}_{\mathrm{T}} = \begin{bmatrix} 266.97 \\ 267 \\ 267 \\ 267.01 \\ 266.97 \\ 267 \\ 267 \\ 267.01 \end{bmatrix} \mathrm{A} \quad (20)$$

Now the switch at terminal 4 is opened. This results in the following changes to the grid matrices.

$$\boldsymbol{F} = \begin{bmatrix} 1 & & & & & & & \\ & 1 & & & & & & \\ & & 1 & & & & & \\ & & & 0 & & & & \\ & & & & 1 & & & \\ & & & & & 1 & & \\ & & & & & & 1 & \\ & & & & & & & 1 \end{bmatrix} \quad (21)$$

$$\underline{\boldsymbol{Y}}_{\mathrm{T},2} = \begin{bmatrix} \mathrm{j}3.142 & 0 \\ 0 & 0 \end{bmatrix} \frac{10^{-5}}{\Omega} \quad (22)$$

$$\underline{\boldsymbol{Y}}_{\mathrm{KK}} = \begin{bmatrix} -2+\mathrm{j}4 & 1-\mathrm{j}2 & 1-\mathrm{j}2 & 0 \\ 1-\mathrm{j}2 & -2+\mathrm{j}4 & 0 & 1-\mathrm{j}2 \\ 1-\mathrm{j}2 & 0 & -1+\mathrm{j}2 & 0 \\ 0 & 1-\mathrm{j}2 & 0 & -1+\mathrm{j}2 \end{bmatrix} \frac{0.1}{\Omega} \quad (23)$$

The new power flow calculation results can be seen below.

$$\boldsymbol{u}_{\mathrm{K}} = \begin{bmatrix} 110 \\ 107.94 \\ 110.02 \\ 105.99 \end{bmatrix} \mathrm{kV}, \quad \boldsymbol{i}_{\mathrm{T}} = \begin{bmatrix} 544.58 \\ 544.67 \\ 2 \\ 0 \\ 3.99 \\ 1.99 \\ 544.67 \\ 544.71 \end{bmatrix} \mathrm{A} \quad (24)$$

7 Conclusion

The fault matrix method offers a quick, simple and systematic way to model power transmission system switches by modifying terminal admittance matrix coefficients. Main advantages compared to conventional methods are

- The grid topology is switch state independent.
- Open switches are not approximated by high impedances.
- Numerical problems are bypassed.
- Calculation complexity is not affected by the number of coexistent open switches.
- Need of only one algorithm to be able to model all kinds of balanced and unbalanced faults and switch states.

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