A Razumikhin-type Stability Criterion for Time-Delay Systems

PooGyeon Park, Byunghoon Kang
Electrical and Computer Engineering Division
Pohang University of Science and Technology
San 31, Hyojadong, Namgu, Pohang, Kyungbuk, 790-784
Korea

Abstract: - In this paper, to concern the delay-dependent stability criterion, we adopt the integral inequality lemma on the LR method. Some variables are introduced to relax the integral term that is generated by the integral inequality lemma. Actually, the form of the criterion shows that, without any model transformations, we can obtain a stability criterion that can be extended to the stabilization problem directly.

Key-Words: - Stability, Razumikhin-type, Time-delay

1 Introduction

Since, the presence of delays can produce severe side effects on system stability and performance, considerable attention has been received during the last few decades [1-5]. And many approaches stem from two main categories: Lyapunov-Krasovskii(LK) method and Lyapunov-Razumikhin(LR) method.

The former has been attracted owing to the structural advantage. In fact, it can expose much delay information in the form of functional-based Lyapunov method. However, its rather complicated form leads us to consider the latter method. The LR method uses functions rather than functionals. Interval-wise fuctionals are converted to instant-wise functions by selecting appropriate representatives, usually supreme values, in each interval. In this way, it can utilize many classic function-based analysis results. Furthermore, its simple structure allows us to extend the results to constraints-related problems, such as saturation problems, easily.

For less conservative results, we usually develop delay dependent criteria. In the LK method, more integral terms are added and their time derivatives are relaxed by applying the integral inequality lemma [2]. Then, based on it , control problems can be developed further. However, in the LR method, it needs some mode transformations which result in conditions that are difficult to extend to the synthesis problem.

In this paper, to concern the delay-dependent stability criterion, we adopt the integral inequality lemma on the LR method. Some variables are introduced to relax the integral term that is generated by the integral inequality lemma. Actually, the form of the criterion shows that, without any model transformations, we can obtain a stability criterion that can be extended to the stabilization problem directly.

2 Main Results

Let us consider the following delayed system:

$$\dot{x}(t) = Ax(t) + A_h x(t-h),\ \ t \ge 0, \tag{1}$$

$$x(t) = \phi(t),\ \ -h \le t \le 0, \tag{2}$$

where $x(t) \in \mathbb{R}^n$ is the state, ϕ is the initial condition, and A and A_h are known constant matrices. Let us define $\chi(t)$ as $\chi(t) \triangleq [x^T(t)\ \ x^T(t-h)\ \ \dot{x}^T(t)]^T$ and the corresponding block entry matrices as $e_1 = [I\ 0\ 0]^T$, $e_2 = [0\ I\ 0]^T$, $e_3 = [0\ 0\ I]^T$ such that the system (1) can be written as

$$0 = (Ae_1^T + A_h e_2^T - e_3^T)\chi(t). \tag{3}$$

We shall consider the following Lyapunov function based on Razumikhin's approach as

$$\mathrm{V}(x(t)) = x^T(t)Px(t). \tag{4}$$

Then, the derivative of V along the trajectory of the system can be written as

$$\dot{\mathrm{V}}(x(t)) = 2\dot{x}^T(t)Px(t) = 2\chi^T(t)e_3Pe_1^T\chi(t), \tag{5}$$

which should be negative whenever

$$x^T(\alpha)Px(\alpha) \le x^T(t)Px(t),\ \ \alpha \in [t-2h,\ t]. \tag{6}$$

Here, we have extended the interval of α appropriate for the following integral inequality lemma. Namely, if

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{12}^T & Y_{22} \end{bmatrix} \ge 0, \tag{7}$$

then we have

$$\begin{aligned} 0 &\le \int_{t-h}^{t} \begin{bmatrix} \chi(t) \\ \dot{x}(\alpha) \end{bmatrix}^T \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{12}^T & Y_{22} \end{bmatrix} \begin{bmatrix} \chi(t) \\ \dot{x}(\alpha) \end{bmatrix} d\alpha \\ &= \chi^T(t)\{hY_{11} + Y_{12}(e_1-e_2)^T + (e_1-e_2)Y_{12}^T\} \\ &\quad \chi(t) + \int_{t-h}^{t} \dot{x}^T(\alpha)Y_{22}\dot{x}(\alpha)d\alpha. \end{aligned} \tag{8}$$

To relax the integral term, we shall introduce free variables λ_1 and λ_2 which satisfy the following condition:

$$\begin{bmatrix} A^T \\ A_h^T \end{bmatrix} Y_{22} \begin{bmatrix} A & A_h \end{bmatrix} < \begin{bmatrix} \lambda_1 P & 0 \\ 0 & \lambda_2 P \end{bmatrix}. \tag{9}$$

Then, in view of (6), we obtain

$$\begin{aligned} &\int_{t-h}^{t} \dot{x}^T(\alpha)Y_{22}\dot{x}(\alpha)\,d\alpha \\ &\le \int_{t-h}^{t} (\lambda_1+\lambda_2)x^T(t)Px(t)\,d\alpha \\ &= h(\lambda_1+\lambda_2)x^T(t)Px(t). \end{aligned} \tag{10}$$

Now, putting all our consideration together, $\dot{\mathrm{V}}(x(t))$ can be upper-bounded by the following quantity:

$$\begin{aligned} \dot{\mathrm{V}}(x(t)) < \chi^T(t)\{&e_3Pe_1^T + e_1Pe_3^T + hY_{11} + \\ &Y_{12}(e_1-e_2)^T + (e_1-e_2)Y_{12}^T + \\ &h(\lambda_1+\lambda_2)e_1Pe_1^T\}\chi(t). \end{aligned} \tag{11}$$

Finally, we shall remove the constraints of the model dynamics itself in (1) by introducing free variables Σ as $0 \equiv \chi^T(t)\Sigma(Ae_1^T + A_he_2^T - e_3^T)\chi(t)$ like [6], and adopt a condition for the constraint (6) as

$$x^T(t-h)Px(t-h) \le x^T(t)Px^T(t), \tag{12}$$

which can be handled through the so called S-procedure [7].

Then, we can conclude the following stability criterion for the system (1).

Theorem 1 The delayed-system (1) is asymptotically stable if there exist scalar variables $\lambda_1, \lambda_2, \lambda_3$ and matrices $P, Y_{11}, Y_{12}, Y_{22}$ and Σ such that the following conditions hold:

$$\begin{aligned} 0 > &\Sigma(Ae_1^T + A_he_2^T - e_3^T) + (Ae_1^T + A_he_2^T - e_3^T)^T\Sigma^T \\ &+hY_{11} + Y_{12}(e_1-e_2)^T + (e_1-e_2)Y_{12}^T + e_1Pe_3^T \\ &+(h\lambda_1 + h\lambda_2 + \lambda_3)e_1Pe^T{}_1 - \lambda_3e_2Pe_2^T + e_3Pe_1^T, \end{aligned}$$

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{12}^T & Y_{22} \end{bmatrix} \ge 0,\ \ \begin{bmatrix} A^T \\ A_h^T \end{bmatrix} Y_{22} \begin{bmatrix} A & A_h \end{bmatrix} < \begin{bmatrix} \lambda_1 P & 0 \\ 0 & \lambda_2 P \end{bmatrix},$$

$$\lambda_1 > 0,\ \lambda_2 > 0,\ \lambda_3 > 0,\ P > 0.$$

3 Simulation Result

Consider the time delay system (1) with the following parameters

$$A = \begin{bmatrix} -2 & 0 \\ 0 & -0.9 \end{bmatrix},\ \ A_h = \begin{bmatrix} -1 & 0 \\ -1 & -1 \end{bmatrix}. \tag{13}$$

With the LR method [8], the system is stable up to 0.749. However, based on Theorem 1, the allowable time delay was found to be 0.996. The simulation used Matlab's LMI toolbox.

4 Conclusion

In this paper, to concern the delay-dependent stability criterion, we adopted the integral inequality lemma on the LR method. Some variables were introduced to relax the integral term that is generated by the integral inequality lemma. Actually, the form of the criterion showed that, without any model transformations, we could obtain a stability criterion that can be extended to the stabilization problem directly.

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