Sliding Modes in Finite-Time Control Systems with Variable Structure

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Abstract: - This paper proposes a method for the organization of real sliding movements, which are caused by finite switching frequency of control systems. In one of two proposed structures for the organization of sliding modes, the control is formed according to the principle of feedback connections; in the other proposed structure, the control is formed according to the principle of discrete control where the discrete levels are defined by crossing (hitting) points on a hyperplane. The proposed method allows users to solve practically oriented problems of organization of sliding movements while taking into account real boundaries existing in systems. The proposed models are simulated using MATLAB/SIMULINK.

Key-Words: - Variable Structure Control, Point Sliding Modes, Sliding Movements, Finite Switching Frequency

1 Introduction

Many control, identification and estimation problems in conditions of parametrical uncertainty find effective solutions through creation of sliding modes [1]-[8]. In these modes, after some period of time, a movement of the system takes place in a small vicinity of the given hyperplane, whose position does not depend on parameters of objects. Therefore it is possible to obtain stable dynamic parameters, despite a change of object parameters over a wide range. The organization of such a mechanism of movement is reached due to high-frequency switching of separate structures at the moments of flight of a representative point of the given plane.

In control systems, because of inertia properties of the actuators and objects, an application of high-frequency intermittent control is always limited. For this reason, a reduction of the control switching frequency in sliding modes and its coordination with frequency characteristics of real objects has been a problem in all periods of development of the theory and practice of Variable Structure Control (VSC). With the purpose of to avoid strict mathematical formalization and to improve the system serviceability, designers have started to use an indistinct sliding mode [9].

Reduction of frequency by itself does not create a special difficulty. The basic problem consists of providing the closeness of a phase trajectory to a sliding surface during change of object parameters over a wide range.

In this paper, we propose a completely novel method of organization of the sliding mode. We have named this sliding mode as «Point Sliding Mode Control» (PSMC). In [10], PSMC has been used for stabilization of second order non-stationary objects. In this study, the results of [10], also taking in account [11], are distributed on n^{th} order objects with Single Input and Single Output (SISO) and all possible modes in finite-time VSC systems are also investigated.

2 State of Problem

Let us consider a completely controlled object described in the state space model,

$$\dot{x}(t) = Ax(t) + Bu\ ,\ y = x_1, \qquad (1)$$

where, $x \in R^n$ is the measured state vector; u is the scalar control variable; A is the n-by-n system matrix; B is the n-by-1 input matrix; y is the observable output.

The problem is to find control variable $u(x,t)$, which will force the object (1) to change its state from any initial condition $x_0 \in R^n$ at the moment of time $t = 0$ into a point $x = 0$ for finite time $T(x_0)$ and establish the system in this point for $t \geq T(x_0)$:

$$u(x,t)=\begin{cases} -K_1^T x & for \quad \sigma(x)>0, \\ R(t)K_2 x_s^* & for \quad \sigma(x)<0, \end{cases} \qquad (2)$$

where $K_1^T=(k_1,k_2,...,k_n)^T$ is a vector coefficient of amplification of the first (linear) structure; $K_2=(k_{ij}),i,j=1,2,...,n-1$ is a matrix coefficient of amplification of the second structure; $R(t)=(1,t,...,t^{n-2})$ is a vector of nonlinear functions; $\sigma(x)=s_1 s_2$ is a switching function; $s_1=(c_1,1)x=0$, $s_2=(c_2,1)x=0$ are switching hyperplanes; $c_1=(c_{11},c_{12},...,c_{1,n-1})$, $c_2=(c_{21},c_{22},...,c_{2,n-1})$ are vectors of angular coefficients; x_s^* is the constant vector formed from the first *n-1* coordinate of a crossing points x_s of a phase trajectory on a hyperplane $s_1=0$; $x_s=(x_s^{*T},x_{n.s})^T\in\{x:s_1=0\}$. If $x_0\in\{x:\sigma\le 0\}$, then $x_s:=x_0$.

3 Solution Models

We do not discuss in detail the techniques for definition of adjustment parameters K_1, K_2, c_1 and c_2, which are described in [12].

The vector coefficient of amplification K_1 is determined from a condition of crossing the phase trajectories from $x_0\in\{x:\sigma>0\}$ on $s_1=0$. This requirement can be executed on oscillatory trajectories such as a steady or unstable spiral or on conservative ellipsoids. For such trajectories the values of a characteristic matrix $D=A-BK^T$ are in a complex-connected $p_t=\alpha_t\pm j\omega$.

Proceeding from this, K_1 is defined on the basis of the solution of a problem of modal control (a problem of distribution of poles and zeros) with the determinant equation:

$$\det(pI-D)=\begin{cases} \prod_{i=1}^{n_1}\left[\left(p-h_i\right)^2+\omega_i^2\right], \; for\ even\ n, \\ (p-p_1)\prod_{i=1}^{n_2}\left[\left(p-h_i\right)^2+\omega_i^2\right], \; for\ odd\ n, \end{cases} \qquad (3)$$

where $n_1=n/2$, $n_2=(n-1)/2$.

Elements K_1 are defined for given values α_i, ϖ_i and p_1 as a result of the solution of a system of algebraic equations made on a basis (3), by equating the coefficients at identical degrees of the operator p. Parameters α_i, ϖ_i and p_1 can be determined with taking into account the restrictions on K_1.

Matrix coefficient of amplification K_2 and c_1 are defined from a condition of crossing of a phase trajectory $x(t)$ from any point of a hyperplane $s_1=0$ at coordinate origin $x(t_s)=0$:

$$K_2=-H^{-1},\; c_1=-h_n H^{-1}, \qquad (4)$$

where,

$$\Phi(t_s)=\left(-\frac{H}{h_n}\right)=\int_0^{t_s} e^{-A\tau}BR(\tau)\mathrm{d}\tau \qquad (5)$$

Here, h_n is the last row in n-by-$(n-1)$ matrix Φ; $t_s\in(0,\infty)$ is the given time of movement of the system in the second structure, i.e. at $\sigma<0$. Parameter t_s can be determined in view of the speed and restrictions on K_2.

In practical tasks, frequently, $n=2$. Thus, the control (2) becomes:

$$u=\begin{cases} -(k_1,k_2)x & for \quad s_1x_1>0, \\ k_{11}x_{1s}=const & for \quad s_1x_1<0. \end{cases} \qquad (6)$$

As we see from (6), a programming control $u=u_2=k_{11}x_{1s}$ is transformed into constant control, whose amplitude is corrected depending on the axis x_{1s} of the crossing points of a trajectory on a line of switching $s_1=cx_1+x_2=0$. Hence, we get the relay-linear finite-time control with a variable level of a relay signal.

4 Simulation of Modes in Finite-Time Systems with Variable Structure

We will demonstrate various modes for the double integrator,

$$\dot{x}_1=x_2,\dot{x}_2=bu, \quad y=x_1, \qquad (7)$$

with the control algorithm (6).

The modeling scheme of the stabilization system using SIMULINK is shown in Fig.1.

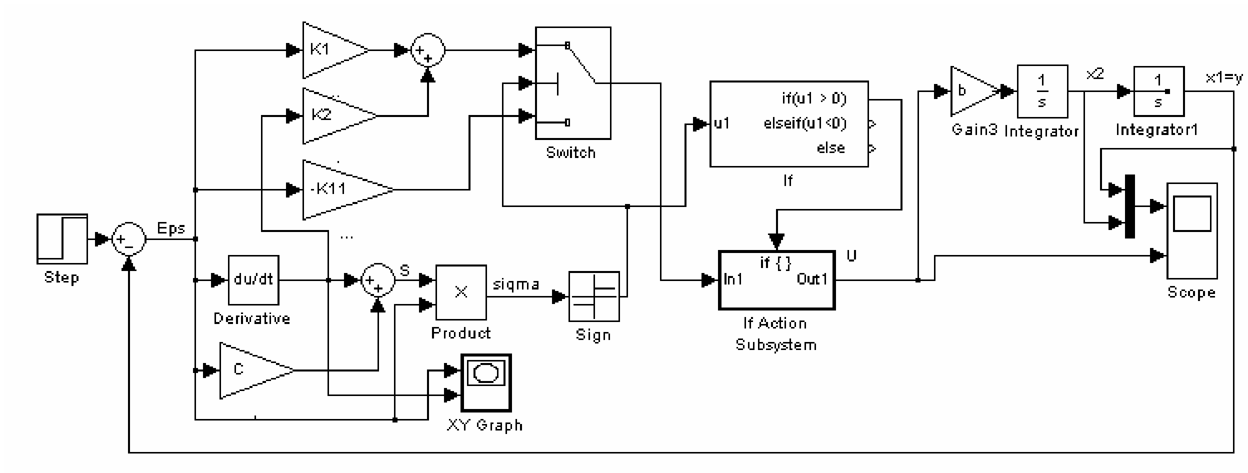


Fig.1 The modeling scheme of the stabilization system

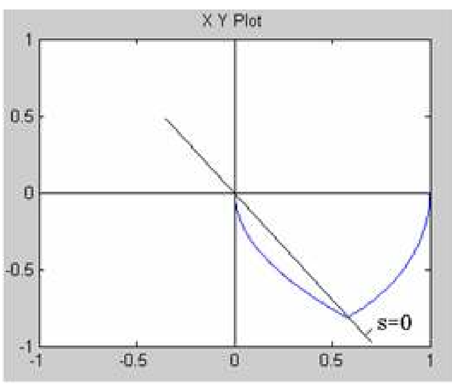
4.1 Finite-Time Mode

In this mode, according to (4), there is a strict equality between parameters c_1 and K_2 of adjustment of the second structure:

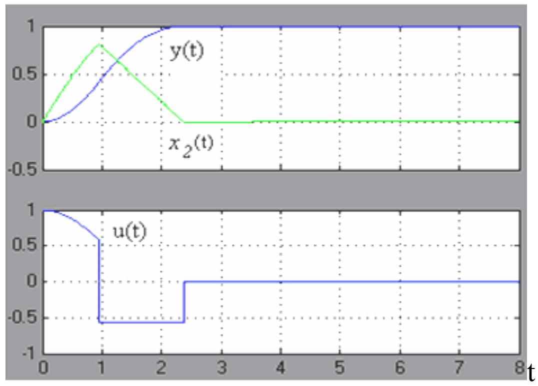
$$c_1 = h_n K_2 \tag{8}$$

At satisfaction (8), the given point $x = 0$ is reached for finite time and for minimal number of switching, independent of the system order, namely, from an initial condition $x_0 \in \{x : \sigma \geq 0\}$ - for one, and from $x_0 \in \{x : \sigma \leq 0\}$ - for two switching.

Choosing $k_1 = 1$ and $k_2 = 0$ on basis (3) for the first structure, we shall have ellipsoids. The condition (8) for the double integrator becomes: $c^2 = 2bk_{11}$. From here, at $b = 1$, we choose $k_{11} = 1$ and $c = \sqrt{2}$. Thus the phase trajectories will consist of parabolas. The transitive characteristics and a phase portrait of the stabilization system in finite-time mode are shown in Fig.2 a and b, respectively.



(a)



(b)

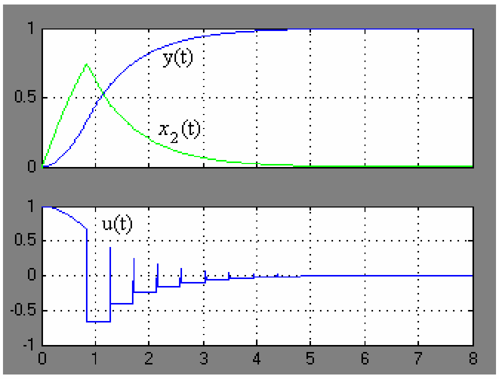
Fig.2 Dynamic characteristics of system in finite-time mode

4.2 Point Sliding Mode

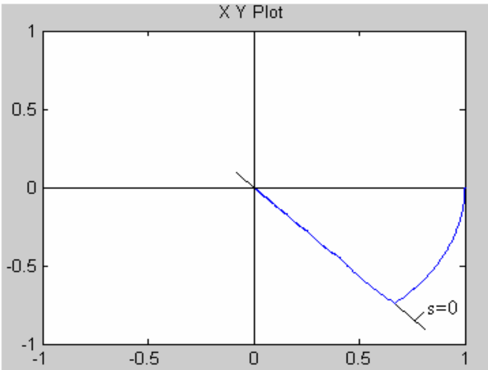
The sliding movement obtained in this mode differs in that the breaks in control on a plane $s_1 = 0$ take place in the isolated points placed in equal time intervals θ. Thus, the trajectory between points of switching remains in a small vicinity of the plane $s_1 = 0$. Necessarily, the step of movement in Point Sliding Mode (PSM) can be adjusted on an interval $\theta \in (0, t_s)$.

For the double integrator (7), $\theta = 2(c^2 - bk_{11})/(bck_{11})$. The parameter $\mu = \theta / t_s$ can be characterized as "completeness" of a point sliding mode $\mu \in [0,1]$. For example, in finite-time mode, $\theta = t_s \Rightarrow \mu = 1$. According to the increase in frequency of switching, μ is decreased. The condition of execution of PSMC for the double integrator (7) looks like: $c^2 / 2b < k_{11} < c^2 / b$.

The transitive characteristics and a phase portrait of the stabilization system in PSMC are shown in Fig.3 a and b, respectively, for $b=1$, $k_1=1$, $k_2=0$, $k_{11}=1$, $c=1{,}1$; $\theta=0.38$ sec, $\mu=0.27$.



(a)



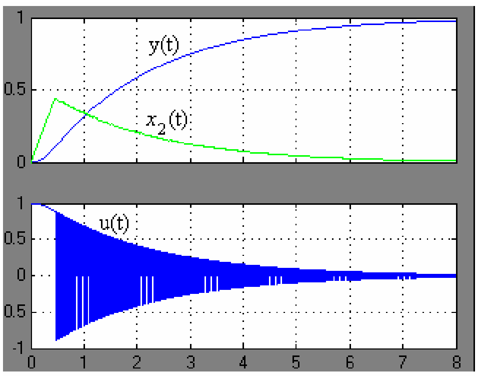
(b)

Fig.3 Dynamic characteristics of system in PSM

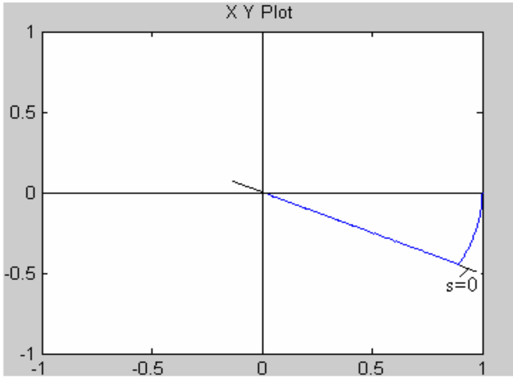
As we see from Fig.3 b, a trajectory does practically not differ from line $s=0$ after crossing it.

At $\theta<0$, PSM does not differ from a usual sliding mode. Thus, the obtained $k_{11}>c^2/b$ coincides with a condition followed from [2].

The dynamic characteristics of PSM are shown in Fig.4 a and b for $\theta<0$. The system was simulated for $b=1$, $k_1=1$, $k_2=0$, $k_{11}=1, c=0.5 \Rightarrow \theta=-3$.



(a)



(b)

Fig.4 Dynamic characteristics of system in a usual sliding mode

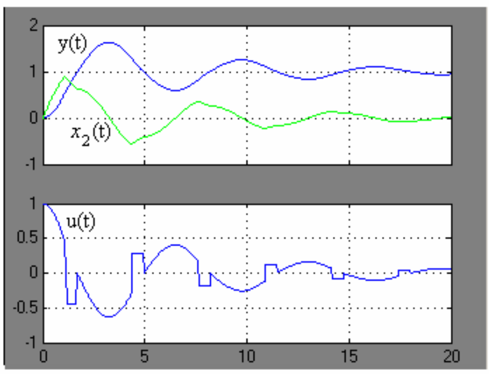
For $b=1$, $k_{11}=1$, $c=\sqrt{2}$, time $\theta=t_s=\sqrt{2} \Rightarrow \mu=1$. An increase of amplification coefficient b of object twice reduces the "completeness" PSM up to $\mu=0$. On further increases of b, the parameter $\mu<0$ and PSM is estimated as a usual sliding mode. Keeping the control switching frequency at the given level, i.e. providing a condition $\mu=const>0$, can be provided by fine tuning parameters k_{11} and (or) c.

As is shown from Fig.4, at the values of regulator parameters chosen here, PSM practically does not differ from a usual sliding mode.

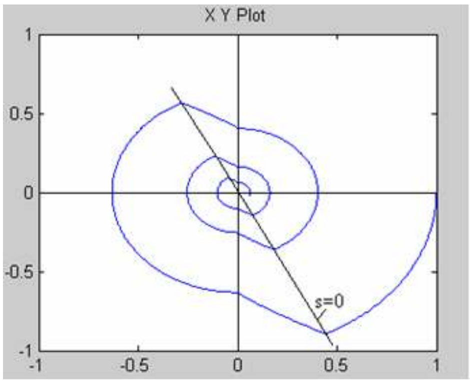
4.3 Switching Mode

This mode practically does not differ from a switching mode that takes place in the usual VSC and this mode is characterized by oscillatory transient process. In this case, the parameters of system (7) and (6) satisfy a condition $0 \le k_{11} < c^2/2b$.

The dynamic characteristics of a switching mode are shown in Fig.5, for $b=1$, $k_1=1$, $k_2=0$, $k_{11}=1$, $c=2$.



(a)



(b)

Fig.5 Dynamic characteristics of system in a switching mode

The analysis of system speed on a basis (6) shows that at identical coefficients of amplification the greatest speed $t_k \approx 2.3$ sec is reached for finite time mode. Depending on the control switching frequency increase, the system speed is reduced. Thus, in case of a sliding mode with $\theta = -3$, $t_k \approx 8$ sec (Fig.4).

5 Conclusion

In summary, it is possible to come to the following conclusions:

1. The proposed method of organization of sliding modes allows us to expand essentially set of combined phase structures in VSC.
2. The proposed point sliding mode allows us to reduce control switching frequency and, hence to expand the area of practical application of VSC systems.
3. The systems with the point sliding mode can be used for control objects with nonstationary and nonstable parameters. This mode can be implicated to such objects as electromechanical, technical, electronics, etc. systems from different areas of techniques and technologies.
4. During the changes of system parameters over a wide range, the control switching frequency can be supported at a constant level by fine tuning of regulator parameters.
5. Modeling researches of the system using the SIMULINK package have produced a number of the positive results of important practical interest.

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