Rigorous Analysis of Resonance Wavelength in Optical fiber Gratings

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Abstract: In this paper, the exact dispersion formulations have been used to obtain the resonance wavelength in a typical three-layer optical fiber grating. Furthermore, the variation of the resonance wavelength is determined versus external index and fiber parameters. The results obtained by approximate formulations in previous works show a good agreement with our obtained results.

Key-Words: - Optical fiber grating, long period fiber grating (LPFG), resonance wavelength

1 Introduction

Optical fiber phase gratings formed by ultraviolet irradiation have developed rapidly in recent years. Numerous applications have been demonstrated that utilize fiber gratings as mirrors, in which a forward-propagating mode guided by the fiber core couples to a backward propagating mode of the same type [1] and as mode converters, in which one type of guided core mode couples to a different type [2] Fiber gratings can also function as loss filters by enabling the guided core mode to couple to radiation modes of the fiber [3] which are effectively extinguished by leakage of light away from the fiber. When the cladding is surrounded by a medium with a refractive index lower than that of the glass, such as air, the core mode may couple to fiber cladding modes [4]. The transmission spectrum of a typical LPFG consists of a number of rejection bands at specific wavelengths (the resonance wavelengths), each of which corresponds to the coupling between the guided mode and a particular cladding mode [5]. LPFGs have been used widely as filters and gain flatteners for erbium-doped fiber amplifiers (EDFAs) [6].Because the resonance wavelengths of an LPFG are sensitive to a number of physical parameters, such as strain, temperature, and the refractive index of the surrounding medium [7], various kinds of LPFG sensors based on the measurement of the shift in the resonance wavelength have been demonstrated [8, 9]. The fundamental guided mode propagating along the core of a single-mode fiber is coupled to forward-propagating cladding modes through a grating photo-written along the core of the fiber. Because the cladding of a single-mode fiber supports many cladding modes, a LPG has many resonance peaks. It was recently reported that the spectral resonance peaks of a LPG are shifted by the refractive-index change of the ambient around the cladding [10].

In this paper, by using rigorous formulations at the first time, the resonance wavelengths of the LPFG for several modes have been obtained. In Section 2, the applied theory and formulations for computing the resonance wavelength are presented. In Section 3, for several configurations of one LPFG the resonance wavelength is calculated versus ambient index and fiber parameter. Finally, this paper is concluded in Section 4.

2 Theory and Formulation

The phase-matching condition of a LPG[2] is generally given as a function of mode indices of the core, ${n_{ol}}^{co}$, and the cladding, ${n_{op}}^{cl}$:

$$\lambda_p = (n_{ol}^{co} - n_{op}^{cl})\Lambda \qquad (1)$$

where Λ is the period of the grating, p is the mode number, and λ_p is the wavelength of the pth-order resonance peak. The effective indices of the core and the cladding modes can be found as functions of the propagating wavelength, the geometry of the fiber, and the refractive indices of the core and the cladding materials. Since we are interested mainly in low Δ fibers, where $\Delta = (n_1\text{-}n_2)/n_1$ is the normalized core–cladding index difference, the linearly polarized (LP) approximation should be sufficient to describe a mode guided by the fiber core. In particular, the exact dispersion relation that we solve to obtain the LP_{01} mode effective index is:

$$V\sqrt{1-b}\,\frac{J_1(V\sqrt{1-b})}{J_0(V\sqrt{1-b})}=V\sqrt{b}\,\frac{K_1(V\sqrt{b})}{K_0(V\sqrt{b})} \quad (2)$$

where J is a Bessel function of the first kind, K is a modified Bessel function of the second kind, $V=(2\pi/\lambda)a_1\sqrt{n_1^2-n_2^2}$ is the V number of the fiber at a wavelength λ, b is the normalized effective index, given by $b=(n_{eff}^2-n_2^2)/(n_1^2-n_2^2)$, and the rest of the parameters are as defined in Fig. 1.

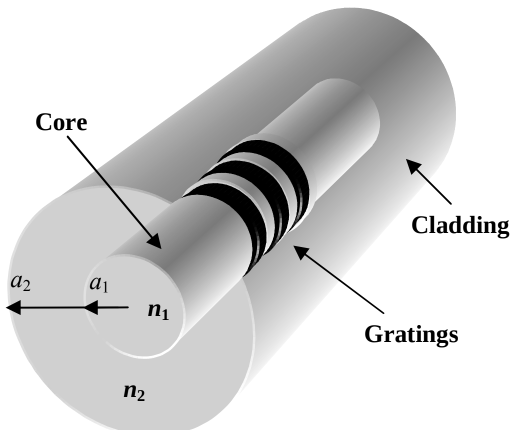


Fig. 1. Fiber optic grating schematic.

The exact modes for such a three-layer fiber have been detailed in [11]. Our analysis follows that of [11], but we include an explicit dispersion relation here in a clearer, ready-to-program form. The dispersion relation for a cladding mode with azimuthal dependence exp(±*ilβ*), for which by definition $n_3 < n_{\text{eff}} < n_2$, is given by:

$$\zeta_0=\zeta_0' \quad (3)$$

The following definitions have been used:

$$\sigma_1 \equiv il\,n_{eff}/Z_0,\ \sigma_2 \equiv il\,n_{eff}/Z_0$$
$$u_{21}\equiv\frac{1}{u_2^2}-\frac{1}{u_1^2},\ u_{32}\equiv\frac{1}{w_3^2}+\frac{1}{u_2^2} \quad (6)$$

where,

$$u_j^2 \equiv (2\pi/\lambda)^2(n_j^2-n_{eff}^2) \qquad j\in(1,2)$$
$$w_3^2 \equiv (2\pi/\lambda)^2(n_{eff}^2-n_3^2) \quad (7)$$

$$J\equiv\frac{J_l'(u_1a_1)}{u_1J_l(u_1a_1)},\ K\equiv\frac{K_l'(w_3a_2)}{w_3K_l(w_3a_2)} \quad (8)$$

$$p_l(r)\equiv J_l(u_2r)N_l(u_2a_1)-J_l(u_2a_1)N_l(u_2r)$$
$$q_l(r)\equiv J_l(u_2r)N_l'(u_2a_1)-J_l'(u_2a_1)N_l(u_2r)$$
$$r_l(r)\equiv J_l'(u_2r)N_l(u_2a_1)-J_l(u_2a_1)N_l'(u_2r) \quad (9)$$
$$s_l(r)\equiv J_l'(u_2r)N_l'(u_2a_1)-J_l'(u_2a_1)N_l'(u_2r)$$

In (8) and (9) the prime notation indicates differentiation with respect to the total argument and N is a Bessel function of the second kind, or the Neumann function. The dispersion relation given by (3)–(9) is straightforward to solve numerically.
we limit the analysis to untilted gratings or, more generally, to gratings that consist of a circularly symmetric index perturbation in any transverse plane of the fiber, the only nonzero coupling coefficients between the core mode and the cladding modes involve cladding modes of azimuthal order l = 1.

3. Discussion and Results

3.1. External index

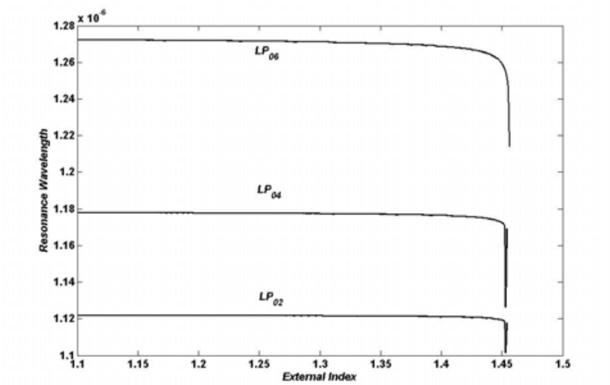
Assume the fiber grating as shown in Fig. 1 with following parameters:
n_1 = 1.4617, n_2 = 1.457, a_1 = 3.6 um, a_2 = 62.5 um and the grating pitch is Λ = 400 um. For bare fibers in air, the external index (the index of the medium that surrounds the fibers) can be taken as n_3 = 1. By coating the fibers with a suitable polymeric material or emerging them in an index-matching liquid, the external index can be brought close to the cladding index. Therefore, it is desirable to investigate the resonance wavelengths for any mode which propagates in cladding versus ambient index. As illustrated in Fig. 2, the resonance wavelength for all modes decreases with increase of ambient index. The minimum resonance wavelength occurs at the ambient indexes close to the cladding index. Moreover, when the ambient index reaches to cladding index, the modes propagate in surrounding medium we do not have any guided mode in cladding media. As shown in Figs. 2(a) and 2(b), the reduction of the resonance wavelength for each even and odd mode when the ambient index is close to cladding index is larger for higher modes. Also, the obtained results demonstrated in Fig. 2 have a reasonable agreement with which reported in [12].

3.2. Fiber parameter

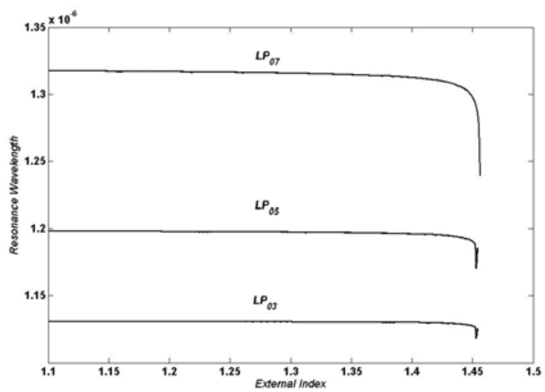
The normalized core-cladding index difference (Δ) has important role in design and fabrication of the

$$\zeta_0' = \sigma_1 \frac{u_2\left(\frac{u_{32}}{a_2}J - \frac{n_3^2 u_{21}}{n_2^2 a_1}K\right)P_1(a_2) + \frac{u_{32}}{a_2}q_1(a_2) + \frac{u_{21}}{a_1}r_1(a_2)}{u_2\left(\frac{n_3^2}{n_2^2}JK + \frac{\sigma_1\sigma_2 u_{21}u_{32}}{n_1^2 a_1 a_2}K\right)P_1(a_2) - \frac{n_3^2}{n_1^2}Kq_1(a_2) + Jr_1(a_2) - \frac{n_2^2}{n_1^2 u_2}s_1(a_2)} \tag{4}$$

$$\zeta_0 = \frac{1}{\sigma_2}\frac{u_2\left(JK - \frac{\sigma_1\sigma_2 u_{21}u_{32}}{n_1^2 a_1 a_2}K\right)P_1(a_2) - Kq_1(a_2) + Jr_1(a_2) - \frac{1}{u_2}s_1(a_2)}{-u_2\left(\frac{u_{32}}{n_2^2 a_2}J - \frac{u_{21}}{n_1^2 a_1}K\right)P_1(a_2) + \frac{u_{32}}{n_1^2 a_2}q_1(a_2) + \frac{u_{21}}{n_1^2 a_2}r_1(a_2)} \tag{5}$$



(a)



(b)

Fig. 2. Resonance wavelength versus external index for all modes, (a) even modes.(b) odd modes.

optical fiber. Hence, as indicated in Fig. 3, the effect of Δ has been investigated for LP_{07} mode for aforementioned structure versus external index. It is obvious that by increasing this fiber parameter, Δ, the resonance wavelength decreases sensitively. The 0.0008 increase in Δ imposes 550 nm discrepancy in the wavelength resonance. Likewise, if ambient index is close to cladding index, the reduction in resonance wavelength is less than when the higher Δ is assumed.

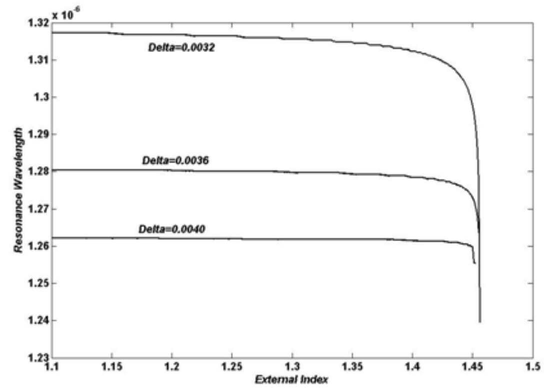


Fig. 3. Resonance wavelength of LP_{07} mode versus external index for different Δ.

4. Conclusion

The analysis of resonance wavelength using exact dispersion relations has been investigated in this paper. The effects of the external index and normalized core-cladding index difference have been calculated. It was observed that for all modes (even or odd) when external index is near the cladding index, the variation of resonance wavelength will be very sensitive. Also, the resonance wavelength changes drastically by smooth variation of the fiber parameter (Δ). Therefore, the external index and Δ is very important factors that should be considered in fiber grating design and fabrication.

References:

[1] G. Meltz, W. W. Morey, and W. H. Glenn, “Formation of bragg gratings in optical fibers by a transverse holographic method,” *Opt. Lett.*, Vol. 14, pp. 823–825, 1989.

[2] K. O. Hill, B. Malo, K. A. Vineberg, F. Bilodeau, D. C. Johnson, and I. Skinner, “

Efficient mode conversion in telecommunication fibre using externally written gratings," *Electron. Lett*., Vol. 26, pp. 1270–1272, 1990.

[3] V. Mizrahi and J. E. Sipe, "Optical properties of photosensitive fiber phase gratings," *J. Lightwave Technol.*, Vol. 11, pp. 1513–1517, 1993.

[4] A. M. Vengsarkar, P. J. Lemaire, J. B. Judkins, V. Bhatia, T. Erdogan, and J. E. Sipe, "Long-period fiber gratings as band-rejection filters," *J. Lightwave Technol*., Vol. 14, pp. 58–65, 1996.

[5] A. M. Vengsarkar, P. J. Lemaire, J. B. Judkins, V. Bhatia, T. Erdogan, and J. E. Sipe, "Long-period fiber gratings as band-rejection filters," *J.Lightwave Technol.*, Vol. 14, pp. 58–65, 1996.

[6] M. K. Pandit, K. S. Chiang, Z. H. Chen, and S. P. Li, "Tunable long-period fiber gratings for EDFA gain and ASE equalization," *MicrowaveOpt. Technol. Lett.*, Vol. 25, pp. 181–184, 2000.

[7] X. Shu, L. Zhang, and I. Bennion, "Sensitivity characteristics of longperiod fiber gatings," *J. Lightwave Technol.*, Vol. 20, pp. 255–266, 2002.

[8] V. Bhatia and A. M. Vengsarkar, "Optical fiber long-period grating sensors," *Opt. Lett.*, Vol. 21, pp. 692–694, 1996.

[9] M. N. Ng, Z. Chen, and K. S. Chiang, "Temperature compensation of long-period fiber grating for refractive-index sensing with bending effect," *IEEE Photon. Technol. Lett.*, Vol. 14, pp. 361–363, 2002.

[10] H. J. Patrick, A. D. Kersey, F. Bucholtz, K. J. Ewing, J. B. Judkins, and A. M. Vengsarkar, in *Conference on Lasers and Electro-Optics*, Vol. 11 of 1997 OSA Technical Digest Series (Optical Society of America, Washington, D.C., 1997), p. 420.

[11] C. Tsao, *Optical Fibre Waveguide Analysis*, Oxford, New York, 1992.

[12] B. H. Lee, Y. Liu, S. B. Lee and S. S. Choi, "Displacement of the resonance peaks of a long-period fiber grating induced by a change of ambient refractive index," *Opt. Lett.*, Vol. 22, No. 23, pp. 1769–1771, 1997.