A study on PDC of nonlinear systems by T–S Fuzzy Model with comparing pole placement method and linear quadratic regulator

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Abstract: - This paper discuss parallel distributed compensation (PDC) of nonlinear systems by Takagi-Sugeno (T-S) fuzzy model, and presents the design of T-S PDC controller in every detail. The purposed approach is designed both using pole placement method (PPM) and linear quadratic regulator (LQR) optimal control method, and also improved in a way to obtain a better system response by using additional membership functions (MF) for error with different desired pole locations for each MF for PPM and different design parameters for each MF for LQR. The applicability of this new method is also demonstrated on a detailed simulation.

Key-Words: - Parallel Distributed Compensation, Takagi-Sugeno Fuzzy Model, Nonlinear Control

1 Introduction

Conventional PID controllers constitute an important part at control systems due to their simplicity, low cost and effectiveness for linear systems. However conventional PID controllers are usually not effective if the industrial systems contain long dead times, nonlinear structures, higher orders, and uncertainties. Moreover, it has been widely verified that a fuzzy logic type controller can achieve a better performance for complex systems than conventional controllers (Yucelen, 2006).

Shortly, there are two common fuzzy models; mamdani model and Tagaki-Sugeno model. The Takagi–Sugeno (T–S) fuzzy model has taken more consideration especially after 80’s in research community. Main advantage of T-S model is that it can approach a function with using fewer rules. Also there are aforementioned design methodologies of T-S model. Among all of these methodologies, fuzzy parallel distributed compensation design approach is the most attactive due to its conceptually simple and robust view (Wang 1995, Lin 2001, Arslan 2006), and it is suitable with using pole placement method and linear quadratic regulator optimal control .

In this paper, parallel distributed compensation (PDC) of nonlinear systems by Takagi-Sugeno (T-S) fuzzy model is discussed, the designing of T-S PDC controller is given in every detail. Moreover, an improved design approach is proposed to make T-S PDC controller better by adding error fuzzy sets.

The paper is organized in 5 sections. Section 2 briefly gives a general knowledge about Takagi-Sugeno fuzzy model, section 3 describes both pole placement method based and linear quadratic regulator based parallel distributed controller. At section 4, a complete design, a new improvement and a detailed simulation for a nonlinear system is given in 5 steps. Finally the paper is summarized and concluded at section 5.

2 Takagi-Sugeno Fuzzy Model

The fundemental feature of T–S fuzzy model is that it can be used to model an affine nonlinear system (Yesil, 2000) by seperating them to linear local systems or subsystems. The T–S fuzzy model, proposed by Takagi and Sugeno, (Takagi 1985) is defined as in (1), where $k = 1, 2... \quad, z$.

Model Rule k:

IF $p_1(t)$ is M_{1k} and … and $p_v(t)$ is M_{vk} ;
THEN $x'(t) = A_k x(t) + B_k u(t)$ (1)

M_{ij} represents the fuzzy set and z represents the number of IF-THEN rules. x and u represent the state vector and the input vector respectively (Taniguchi 1999). Equation 1 describes fuzzy IF-THEN fules which locally represent linear input-output relations of nonlinear systems (Wang 1995). $p_1(t)$~$p_v(t)$ are the controllable states and/or the functions of the states. The complete fuzzy model can be given by using all linear subsystems. Given a pair of (*x(t),u(t)*), the final output of the fuzzy system is inferred as in (2) and (3) (Takagi 1985, Taniguchi 1999);

$$x'(t) = \frac{\sum_{k=1}^{z} w_k(t)\left\{A_k x(t) + B_k u(t)\right\}}{\sum_{k=1}^{z} w_k(t)} \quad (2)$$

$$w_k(t) = \prod_{j=1}^{s} M_{kj}(x_j(t)) \quad (3)$$

where $M_{kj}(x_j(t))$ is the grade of membership of $x_j(t)$ in M_{kj}.

3 Parallel Distributed Compensation

Parallel distributed compensation or PDC is a design method for T–S fuzzy model. The first thing for verification of controller is to obtain T–S fuzzy model of nonlinear system. For each rule in T–S fuzzy model a linear controller is designed. Because of describing antecedent of T–S model with linear state equations, linear control theory can be used for designing the antecedent of fuzzy controller (Arslan 2006). The controller at the end of design which is formed with fuzzy blending of each linear controller is a complete system controller or in other words is a nonlinear controller.

IF $p_1(t)$ is M_{1k} and … and $p_v(t)$ is M_{vk};
THEN the subsystem is $x'(t) = A_k x(t) + B_k u(t)$,
THEN the controller is $u(t) = -K_k x(t)$ (4)

It is clear that the subsystem and the subsystem's controller are both selected together for each fuzzy rule. Hence the output of the fuzzy nonlinear controller can be given like in equation (5) (Wang 1995).

$$u(t) = -\frac{\sum_{k=1}^{z} w_k(x(t)K_k x(t))}{\sum_{k=1}^{z} w_k(x(t))} = -\sum_{k=1}^{z} \varphi_k(x(t))K_k x(t) \quad (5)$$

There are a variety of methods to select *K* gain matrix, but pole placement and linear quadratic regulator control techniques are the most effective and useful methods. They are both given in the following sections.

3.1. Pole Placement Method

As it is well known, the closed-loop pole locations have a great importance for the behavior of a system. For a nonlinear system, which is modelled by using T–S fuzzy model, each linear subsystem's behavior can be improved by using parallel distributed compensation plus pole placement method. As said before, PDC is the design technique for T–S fuzzy model, and pole placement obtains us to find gain matrix, *K* (Kuo 1999). Before using pole placement method, new closed-loop pole locations for each linear subsystems have to be decided. This method is described in (Yesil, 2000) in detail.

3.2. Linear Quadratic Regulator

Linear quadratic regulator method is an optimal linear quadratic control method, which minimizes the following cost function (Cipriano 1997, Astrom 1984):

$$J = \int_0^{\infty} (x'Qx + u'Ru)dt \quad (6)$$

R and *Q* matrices are performance index matrix and state-cost matrix respectively. The matrices *R* and *Q* determine the relative importance of the error and the limits of the control signal. Before using LQR method, there is no need to decide new closed-loop pole locations but the R and Q matrices. A reasonable choice for these matrices is given by the Bryon's rule (Franklin 2002);

$$Q = \begin{pmatrix} 1/x_{a1}^2 & & 0 \\ & \ddots & \\ 0 & & 1/x_{an}^2 \end{pmatrix} \quad (7)$$

$$R = \begin{pmatrix} 1/u_{a1}^2 & & 0 \\ & \ddots & \\ 0 & & 1/u_{am}^2 \end{pmatrix} \quad (8)$$

where $1/x_{an}^2$ is maximum acceptable value of *n*th state or function of states x_n, and $1/u_{am}^2$ is maximum acceptance limit of *m*th output's control signal. In addition to these two methods, there are a variety of rules that can determine *K* gain matrix in parallel distributed compensation. But these two methods are the most effective methods and gives a systematic way for the TS fuzzy PDC controller design. How to use this two methods are given in detail in section 4.

4 A Detailed Design and Simulation with a Nonlinear System

A nonlinear differantial equation is given in equation 9 and its state space equations is given in equations 10–13.

$$y'' = \sin(y) - y.(y')^2 + u \quad (9)$$

$$x_1 = y \ , \quad x_2 = y' \quad (10)$$

$$\begin{aligned} x' &= f(x) + g(x).u \\ y &= h(x) \end{aligned} \quad (11)$$

$$\begin{bmatrix} x_1' \\ x_2' \end{bmatrix} = \begin{bmatrix} x_2 \\ \sin(x_1) - x_1.x_2^2 \end{bmatrix} + \begin{bmatrix} 0 \\ u \end{bmatrix} \quad (12)$$

$$y = \begin{bmatrix} x_1 & 0 \end{bmatrix} \quad (13)$$

The MATLAB representation of this nonlinear differantial equation is given in figure 1. Nonlinearity in the system can also be understand from its responses to different references, also it is given in figure 2. In other words, this system is a nonlinear system due to $\sin(x_1)$ and $x_1.x_2^2$ terms. Thus, these terms will be important for T–S PDC design in the following section.

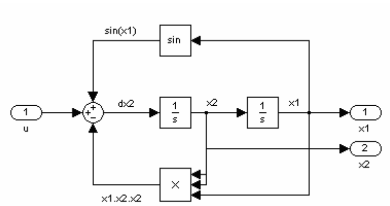

Figure 1. Nonlinear System's MATLAB Model

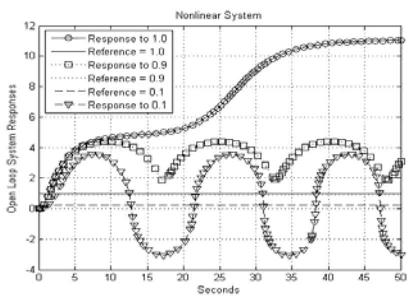

Figure 2. Nonlinear System's Responses to Different References

4.1. Takagi-Sugeno Fuzzy PDC Model of The System and Proposed Controller for The System

In this section constructing T–S fuzzy PDC model of the nonlinear system and design methodology are given in detail step by step.

Step 1 – Finding Membership Functions of $\sin(x_1)$:

First, an interval for this nonlinear term is selected (equation 14). Then it is put to a smaller – bigger equation form to construct membership functions due to interval determined (equation 15 – 16). After this, membership functions are selected as in equations 18 – 20 by using equation 17.

$$\sin(x_1) \quad \in \quad [\pm\pi/3] \quad (14)$$

$$|\sin((\pi/3)/(\pi/3))x_1| \le |\sin(x_1)| \le |x_1| \quad (15)$$

$$|0.8261x_1| \le |\sin(x_1)| \le |x_1| \quad (16)$$

$$\sin(x_1) = M_1^1.(0.8261x_1) + M_1^2.x_1 \quad (17)$$

$$M_1^1 = 1 - M_1^2 \quad (18)$$

$$M_1^2 = \frac{(\sin(x_1)/x_1) - 0.8261}{0.1739} \quad (19)$$

$$M_1^1 = \frac{1 - (\sin(x_1)/x_1)}{0.1739} \quad (20)$$

Graphically, membership functions can be represented as in figure 3. There is no negative part in figure 3, because this term will send to controller's input with its absolute sign due to symmetric membership functions.

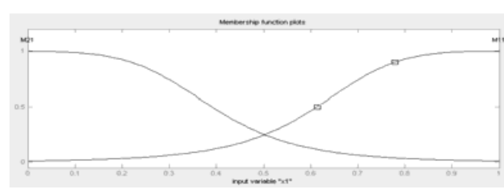

Figure 3. Membership Functions of x_1

Step 2 – Finding Membership Functions of $x_1.x_2^2$:

The same procedure in step 1 is repeated for this nonlinear term too. Related equations are given in 21 – 27, and a most common membership function (triangular form) is selected in figure 4 to construct this term's membership functions.

$$x_1.x_2^2 \quad \in \quad [\pm 2] \quad (21)$$

$$-2.x_2 \le x_1.x_2^2 \le 2.x_2 \quad (22)$$

$$x_1.x_2^2 = M_2^1.0.x_2 + M_2^2.2.x_2 + M_2^3.(-2).x_2 \quad (23)$$

$$M_2^2 = \begin{vmatrix} 1 & x_1.x_2 \ge 2 \\ x_1.x_2 & 0 < x_1.x_2 < 2 \\ 0 & x_1.x_2 \le 0 \end{vmatrix} \quad (24)$$

$$M_2^3 = \begin{vmatrix} 0 & x_1.x_2 \ge 0 \\ -x_1.x_2/2 & -2 < x_1.x_2 < 0 \\ 1 & x_1.x_2 \le -2 \end{vmatrix} \quad (25)$$

$$M_2^1 = 1 - M_2^2 - M_2^3 \quad (26)$$

$$M_2^1 = \begin{vmatrix} 0 & x_1.x_2 \ge 2 \\ 1 - x_1.x_2/2 & 0 < x_1.x_2 < 2 \\ 1 & x_1.x_2 = 0 \\ 1 + x_1.x_2/2 & -2 < x_1.x_2 < 0 \\ 0 & x_1.x_2 \le -2 \end{vmatrix} \quad (27)$$

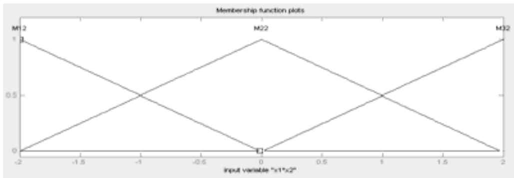

Figure 4. Membership Functions of $x_1.x_2$

Step 3 – The T–S Fuzzy Model of The Nonlinear System:

What was done in step 1 and step 2, can be given with equation 28. To construct a function like in 28, we need 6 T–S fuzzy rules which are given in table 1.

$$f(x) = M_1^1.M_2^1.A_{11}.x + M_1^1.M_2^2.A_{12}.x + M_1^1.M_2^3.A_{13}.x + M_1^2.M_2^1.A_{21}.x + M_1^2.M_2^2.A_{22}.x + M_1^2.M_2^3.A_{32}.x \quad (28)$$

Table 1. T–S Fuzzy Rules

Rule 1: IF $\lvert x_1 \rvert$ is M_1^1 and $x_1 x_2$ is M_2^1, THEN $\dot{x}(t) = A_{11}x(t) + B_{11}u(t)$
Rule 2: IF $\lvert x_1 \rvert$ is M_1^1 and $x_1 x_2$ is M_2^2, THEN $\dot{x}(t) = A_{12}x(t) + B_{12}u(t)$
Rule 3: IF $\lvert x_1 \rvert$ is M_1^1 and $x_1 x_2$ is M_2^3, THEN $\dot{x}(t) = A_{13}x(t) + B_{13}u(t)$
Rule 4: IF $\lvert x_1 \rvert$ is M_1^2 and $x_1 x_2$ is M_2^1, THEN $\dot{x}(t) = A_{21}x(t) + B_{21}u(t)$
Rule 5: IF $\lvert x_1 \rvert$ is M_1^2 and $x_1 x_2$ is M_2^2, THEN $\dot{x}(t) = A_{22}x(t) + B_{22}u(t)$
Rule 6: IF $\lvert x_1 \rvert$ is M_1^2 and $x_1 x_2$ is M_2^3, THEN $\dot{x}(t) = A_{23}x(t) + B_{23}u(t)$

Step 4 – Transfer Functions and Pole Placement Design:

It is a necessity to know a good approximate of a transfer function for each subsystems for pole placement design. For rule 1, state matrix *A* matrix can be given in equation 29. Other state matrices can easily be written like in 29.

State Matrix for Rule 1: $$A_{11} = \begin{bmatrix} 0 & 1 \\ 0.8261 & 0 \end{bmatrix} \quad (29)$$

Transfer functions for these matrices can be found using equation 30, and also again transfer function for Rule 1 is given in equation 31.

$$T(s) = C.\left(s\lambda - A\right)^{-1}.B \quad (30)$$

Transfer Function for Rule 1: $$T_{11}(s) = 1/s \quad (31)$$

It is clear that all *C* and *B* matrices are equal to each other like in equation 12 and 13. 7 error membership functions (where error term is $ex_1 = x_d - x_1$) are added to T–S fuzzy to improve transient response and to obtain a faster response with less overshoot. These functions are given in figure 5 and controller diagram is given in figure 6.

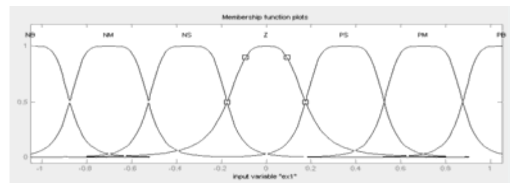

Figure 5. Membership Functions for Error

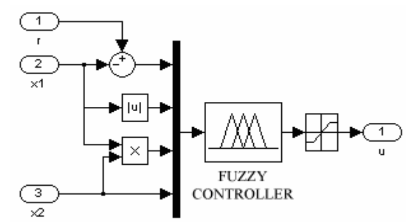

Figure 6. Proposed T–S Controller for The Nonlinear System with Saturation of ± 3

In addition, there will be 6 rules for seven error membership functions, or in other words there will be 42 rules to make system's control better. Some rules are given in table 2.

Table 2. New T–S Fuzzy Rules

Rule 1: If ex_1 is Z and x_1 is M_1^1 and $x_1.x_2$ is M_2^1, Then $u_1 = k_1^1 ex_1 + k_2^1 x_2$
Rule 2: If ex_1 is Z and x_1 is M_1^1 and $x_1.x_2$ is M_2^2, Then $u_1 = k_1^2 ex_1 + k_2^2 x_2$
…
Rule 42: If ex_1 is PB and x_1 is M_1^2 and $x_1.x_2$ is M_2^3, Then $u_{24} = k_1^{24} ex_1 + k_2^{24} x_2$

Necessary parameters for pole placement are given in table 3. Effectivity of adding 7 more membership functions can be seen from this table. First, desired poles' settling time are selected as low as possible, and overshoot of these poles are selected as high as possible to obtain a faster transient response. Then for small values of error (in Z, PS or NS), poles with a low overshoot and a higher settling time are selected to obtain a smooth response when reaching to referance value. The gain values are finally used in new T–S fuzzy rules which are given in table 2.

Table 3. Necessary Parameters for Pole Placement

e(t)	Sub-systems	Settling	%OS	Closed Loop Poles	
Z	All	1.00 s	%1	-4+1.882j	-4-1.8824j
PS, NS	All	1.00 s	%5	-3.5+3.671j	-3.5-3.671j
PM N M	All	0.75 s	%15	-4+6.622j	-4-6.622j
PB, NB	All	0.50 s	%25	-6+13.62j	-6-13.62j

Step 5 – Linear Quadratic Regulator Design:

The nonlinear system given in at the beginning of section 4 has two states and one output. Thus, Q and R matrices become 2x2 and 1x1 matrices respectively due to the Byron's rule.

$$Q = \begin{pmatrix} 1/x_1^2 & 0 \\ 0 & 1/x_2^2 \end{pmatrix} \quad (32)$$

$$R = \left(1/u^2\right) \quad (33)$$

To use advantage of 7 error membership functions like in PPM design, Q and R matrix parameters with error's change is given in table 4.

Table 4. Necessary Parameters for LQR

e(t)	Sub-systems	x1	x2	u (maximum controller signal)
Z	All	0.05	0.1	Equal to reference
PS, NS	All	0.02	0.08	2 times bigger than reference
PM N M	All	0.01	0.06	3 times bigger than reference
PB, NB	All	0.005	0.05	4 times bigger than reference

An advantage of the quadratic optimal control over the pole-replacement method is that selecting x1 << x2 gives a better response for time delays, disturbances and measurement noise. In addition, LQR provides a easier systematic way of computing the state feedback control gain matrix *K* in table 2.

4.2. Simulations

The simulation results for the system given in equation 9 is given in figures 7–12 for different types of controllers. CPID, CPIPD, FPPM, FLQR represent conventional PID controller, conventional PI plus PD controller, T–S PDC pole placement fuzzy controller, and T–S PDC linear quadratic regulator fuzzy controller respectively.

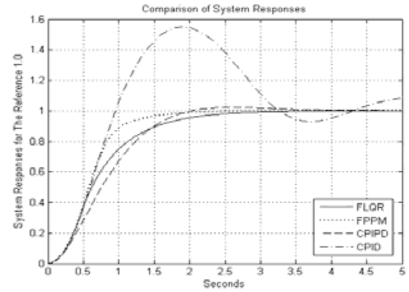

Figure 7. Comparison of System Responses

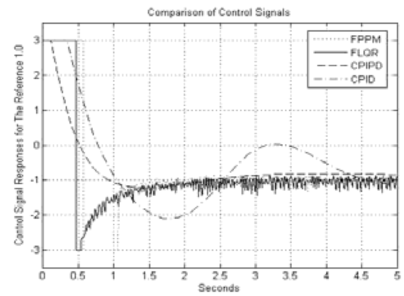

Figure 8. Comparison of Control Signals

Here, all conventional controllers are tuned using a genetic algorithm to obtain best performance. As seen from figure 7, both fuzzy type controllers and conventional PIPD controller give a better response than conventional PID controller. Also FPPM controller gives a bit better response than other controllers. In addition, figure 9 shows these controllers' performance to a step disturbance.

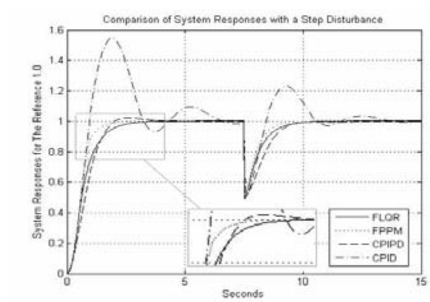

Figure 9. Comparison of System Responses with a Step Disturbance

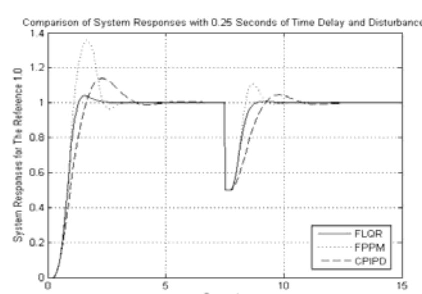

Figure 10. Comparison of System Responses with 0.25 Seconds of Time Delay and a Step Disturbance

In fact, all real industrial systems have time delays, disturbances and measurement noises. It is a necessity to research performance of these controllers for these type of systems. For this reason, figure 10 represents performance of these controller for the same system wıth 0.25 seconds of time delay and a step disturbance. FPPM controller does not give the same good performance on these type of systems. Moreover, CPIPD controller's performance is adequate but it has a 16.7% overshoot. For figure 10, FLQR clearly gets the best performance among all these three controllers. In addition, for a system with 0.50 seconds of time delay and a step disturbance (figure 11), FLQR gets the best performance again.

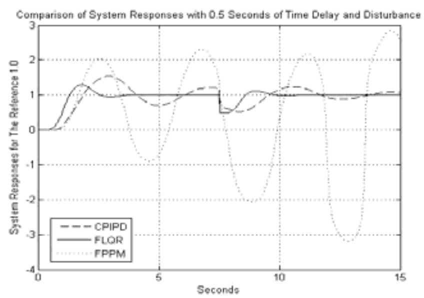

Figure 11. Comparison of System Responses with 0.50 seconds of Time Delay and a StepDisturbance

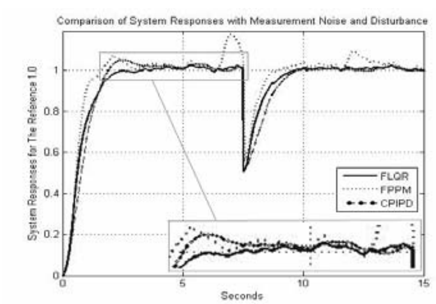

Figure 12. Comparison of System Responses with Measurement Noise and Disturbance

Figure 12 shows controller performances under measurement noise and a step disturbance. FLQR has an better ability to suppress measurement noise than other controllers.

5 Conclusions

In this paper, a new efficient feedback controller using the capabilities of the fuzzy Takagi-Sugeno modeling based parallel distributed compensation and linear quadratic regulator was developed by representing a nonlinear plant with local system fuzzy sets and error (between reference and system's output) fuzzy sets. The proposed method then compared with different controllers in order to verify the effectiveness of this approach for affine nonlinear and/or uncertain systems. Moreover, a complete design method is given in every detail to make the proposed approach more applicable. This new approach has been shown to be very effective through simulations, and it is depicted that this method gives robust system responses for nonlinear systems that include time delay, measurement noise and disturbances. The successful simulations enable to apply this control approach to real time control systems.

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