LONGITUDINAL DISPLACEMENT AUTOPILOT DESIGN for BOEING 747-400 by ROOT-LOCUS

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Abstract: - In this paper, displacement autopilot has been designed for Boeing 747-400 and appropriate S_{amp} and S_{rg} values have been obtained. Moreover root locus diagrams are also drawn. Finally pitch angle time response is analyzed.

Key-Words: - Displacement Autopilots, Root-Locus Design, Boeing 747-400

1 Introduction

The longitudinal stability derivatives of Boeing 747-400 were calculated in the previous chapter. The calculations of displacement autopilot with vertical gyro and pitch rate gyro in the flying condition of Boeing 747-400 are done by the help of the longitudinal transfer functions for elevator servo and longitudinal transfer functions for elevator servo that have determined in the first chapter. Block diagram of the aircraft with a displacement autopilot with vertical gyro and pitch rate feedback is drawn. The transfer functions both for inner and outer loops are found for the Fig. 1.1.

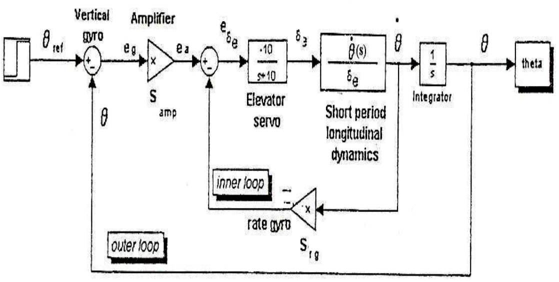

Figure 1.1 Displacement Autopilot with Pitch Rate Feedback for Damping

The root locus for inner loop is drawn by using MATLAB as the rate gyro sensitivity; S_{rg} is increased from zero to infinity. The values of S_{rg} corresponding to damping ratios of 0.8, 0.7 and 0.6 and corresponding natural and damped frequencies are obtained. Similarly, the root locus for outer loop is drawn as the amplifier sensitivity; S_{amp} is increased from zero to infinity. Lastly Routh-Hurwitz criterion is applied to the stability of closed-loop flight control system. Finally, pitch angle response to the -6 degree input pulse elevator deflection is calculated and then interpreted with the help of MATLAB-SIMULINK. For all corresponding calculations MATHEMATICA programming is used.

Autopilot systems are very important both for stable and unstable aircrafts. In order to response correctly to the pilot's command and the condition of the aircraft, autopilot systems should be designed with great care. The simplest form of autopilot, which is the firstly to be discovered, is the displacement-type autopilot **[1].** This type of autopilot is designed to hold the aircraft in straight and level flight with little or no maneuvering capability. The aircraft is initially trimmed to straight and level flight for displacement-type autopilot and the autopilot is engaged. When the pitch altitude varies from the reference, a voltage is then produced by the vertical gyro. This voltage is amplified and fed to elevator servo. The elevator causes the aircraft to pitch about the Y axis by servo positioning, and then the aircraft is returned to desired pitch altitude.

2 Displacement Autopilot Design of the Aircraft

A typical displacement autopilot with pitch rate feedback for damping is shown in Fig.1.1 .

It is obvious from the figure that, the ($\frac{\dot{\theta}(s)}{\delta e}$) transfer function is needed to be firstly calculated in order to design the appropriate autopilot for the aircraft Boeing 747-400.

The short-period approximation transfer function ($\frac{\theta(s)}{\delta e}$), from **[2]**:

$$\frac{\theta(s)}{\delta_e(s)} = \frac{-0,844535-1,68964s}{s(1.58874+1.175235s+s^2)} \quad \textbf{(21)}$$

3 Inner Loop Transfer Function

In Automatic control we derived the transfer function of a simple feedback system. In the figure below a feedback system similar to our inner loop is shown.

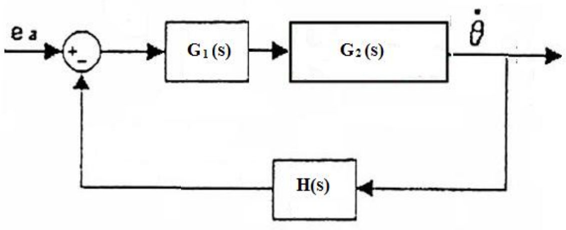

Figure 4.3 Block diagram of a similar, simple feedback system

By simplifying the inner loop transfer function:

$$\frac{\dot{\theta}}{e_a} = \frac{8.44535 + 16.8964s}{15.8874 + 13.341s + 11.1752s^2 + s^3 + (8.44535 + 16.8964s)S_{rg}} \quad (3.1)$$

4 Outer Loop Transfer Function

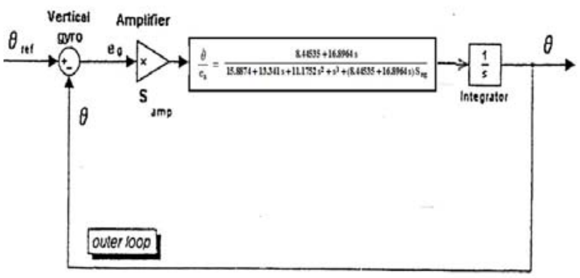

Figure 4.1 Block diagram of the outer loop

Transfer function of the outer loop, $\frac{\theta}{\theta_{ref}}$ is found as in the Equation 4.1.

$$-(10(-0.844535-1.68964s)S_{amp}) \Big/ \left(s(10+s)(1.58874+1.17523s+s^2)\left(1-\frac{10(-0.844535-1.68964s)S_{rg}}{(10+s)(1.58874+1.17523s+s^2)}\right)\right.$$
$$\left(1-(10(-0.844535-1.68964s)S_{amp}) \Big/ \left(s(10+s)(1.58874+1.17523s+s^2)\left.\left.\left(1-\frac{10(-0.844535-1.68964s)S_{rg}}{(10+s)(1.58874+1.17523s+s^2)}\right)\right)\right)\right) \quad (4.1)$$

5 Root Locus for Inner Loop

The open loop transfer function is found as in the Equation 5.1.

$$\frac{8.44535 + 16.8964s}{15.8874 + s(13.3411 + s(11.1752 + s))} \quad (5.1)$$

By using the transfer functions, root locus diagram is drawn with the help of MATLAB and then the rate gyro sensitivity, S_{rg} and natural frequency, w_n are found for the given damping ratios. Finally the damped frequencies are calculated.

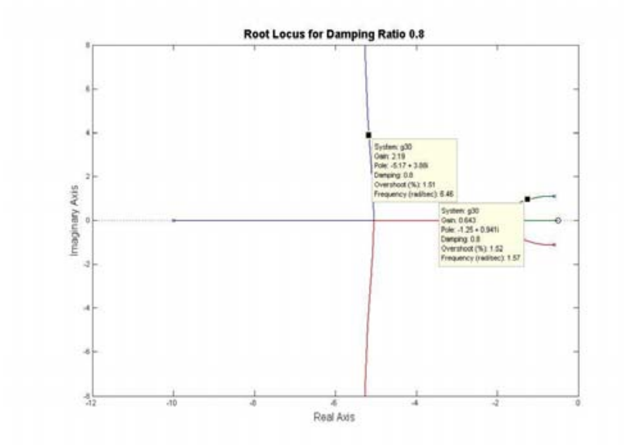

Figure 5.1 Root Locus Diagram for Inner Loop with $\zeta = 0.8$

- **Calculations For the Point on Figure 5.1 where S_{rg1}= 0.643**

S_{rg1}= 0.643

w_{n1}= 1.57 rad/sec

$$w_{d1} = 1.57\sqrt{1-0.8^2} = 0.942 rad/\sec$$

$$\frac{10.\,(0.844535 + 1.68964\,s)\,S_{amp}}{s\,(21.3178 + 24.2055\,s + 11.1752\,s^2 + s^3) + (8.44535 + 16.8964\,s)\,S_{amp}} \quad (5.2)$$

By assuming that the amplifier sensitivity is equal to 1 and then simplifying the Equation 5.2, Equation 5.3 is obtained:

$$\frac{8.44535 + 16.8964\,s}{8.44535 + 38.2142\,s + 24.2055\,s^2 + 11.1752\,s^3 + s^4} \quad (5.3)$$

And then the root locus of the outer loop, which is defined with the transfer function is drawn as follows:

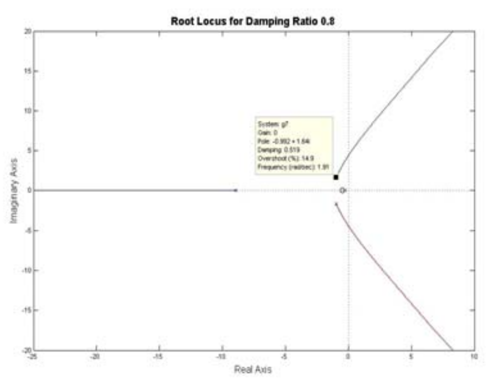

Figure 5.2 Root Locus Diagram for Inner Loop with $\zeta = 0.8$,

S_{rg1}= 0.643, S_{amp}=1

- **Calculations For the Point on Figure 5.2 where S_{rg2}= 2.19**

S_{rg2}= 2.19

w_{n2}= 6.47 rad/sec

$$w_{d2} = 6.47\sqrt{1-0.8^2} = 3.882 rad/\sec$$

By assuming that the amplifier sensitivity is equal to 1 the following equation is obtained:

$$\frac{8.44535 + 16.8964\,s}{8.44535 + 51.2792\,s + 50.3443\,s^2 + 11.1752\,s^3 + s^4} \quad \textbf{(5.4)}$$

The root locus of the outer loop is drawn as follows:

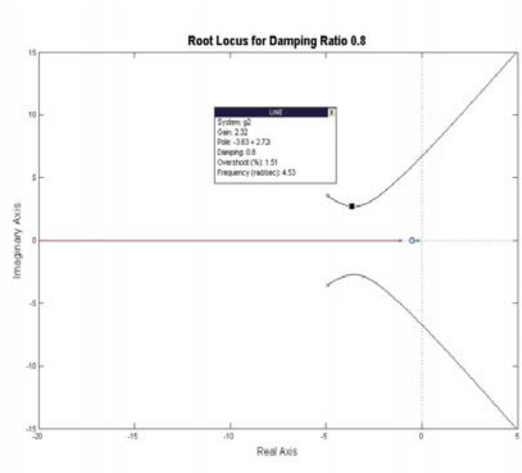

Figure 5.3 Root Locus Diagram for Inner Loop with $\zeta = 0.8$, S_{rg1}= 2.19, S_{amp}=1

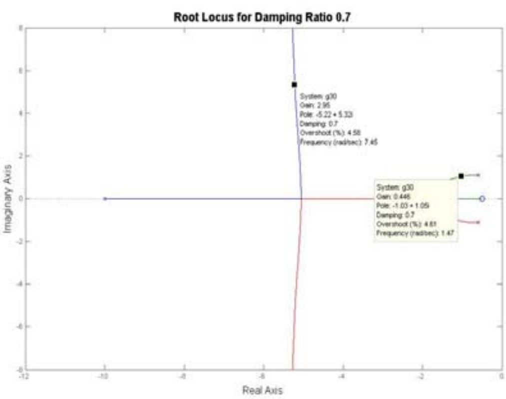

Figure 5.4 Root Locus Diagram for Inner Loop with $\zeta = 0.7$

- **Calculations For the Point on Figure 5.4 where S_{rg1}= 0.447**

S_{rg1}= 0.447

w_{n1}= 1.47 rad/sec

$$w_{d1} = 1.47\sqrt{1-0.7^2} = 1.04979 rad/\sec$$

By assuming S_{amp}=1 and the root locus is drawn as follows:

$$\frac{10.\,(0.844535 + 1.68964\,s)}{8.44535 + 16.8964\,s + s\,(19.6625 + 20.8938\,s + 11.1752\,s^2 + s^3)} \quad \textbf{(5.5)}$$

By simplifying the Equation 5.5, Equation 5.6 is obtained:

$$\frac{8.44535 + 16.8964\,s}{8.44535 + 36.5589\,s + 20.8938\,s^2 + 11.1752\,s^3 + s^4} \quad \textbf{(5.6)}$$

The root locus of the outer loop, which is defined with the transfer function given in Equation 5.6, is drawn as follows:

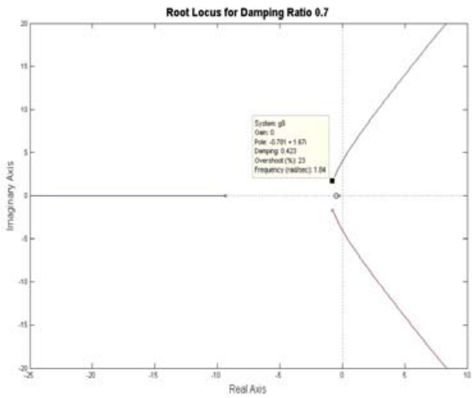

Figure 5.5 Root Locus Diagram for Inner Loop with $\zeta = 0.7$ S_{rg1}= 0.447, S_{amp}=1

- **Calculations For the Point on Figure 5.5 where S_{rg2}= 2.95**

S_{rg2}=2.95

w_{n2}= 7.45 rad/sec

$$w_{d2} = 7.45\sqrt{1-0.7^2} = 5.32036 rad/\sec$$

$$\frac{8.44535 + 16.8964\,s}{8.44535 + 57.6976\,s + 63.1856\,s^2 + 11.1752\,s^3 + s^4} \quad \textbf{(5.7)}$$

The root locus of the outer loop, which is defined with the transfer function is drawn as follows:

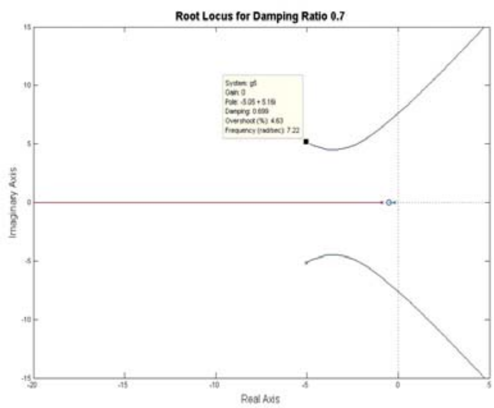

Figure 5.6 Root Locus Diagram for Inner Loop with $\zeta = 0.7$ S_{rg1}= 2.95, S_{amp}=1

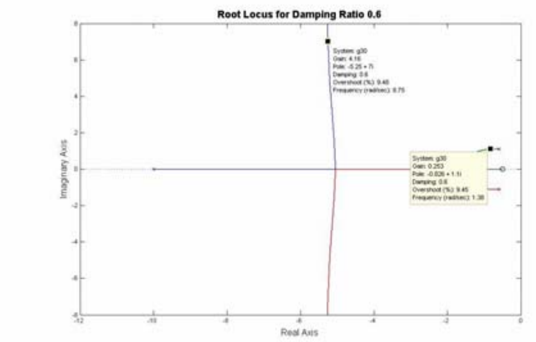

Figure 5.7 Root Locus Diagram for Inner Loop

- **Calculations For the Point on Figure 5.7 where S_{rg1}= 0.252**

  S_{rg1}= 0.252

  w_{n1}= 1.37 rad/sec

$$w_{d1} = 1.37\sqrt{1-0.6^2} = 1.096 rad/\sec$$

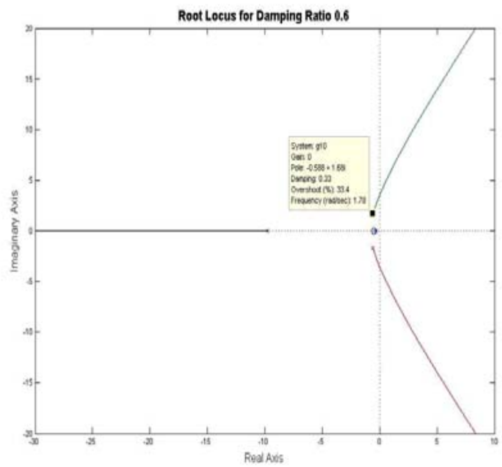

Figure 5.8 Root Locus Diagram for Inner Loop with $\zeta = 0.7$

S_{rg1}= 0.252, S_{amp}=1

- **Calculations For the Point on Figure 5.8 where S_{rg2}= 4.16**

  S_{rg2}=4.16

  w_{n2}= 8.75 rad/sec

$$w_{d2} = 8.75\sqrt{1-0.6^2} = 7.0 rad/\sec$$

$$(10.(0.844535+1.68964s)S_{amp})/$$
$$(s(51.0201+83.6303s+11.1752s^2+s^3)+(8.44535+16.8964s)S_{amp})$$

(5.8)

By assuming S_{amp}=1 and simplifying the equation given above, the root locus for the equation is drawn as follows:

$$\frac{10.\,(0.844535 + 1.68964\,s)}{8.44535 + 16.8964\,s + s\,(51.0201 + 83.6303\,s + 11.1752\,s^2 + s^3)}$$

(5.9)

$$\frac{8.44535 + 16.8964\,s}{8.44535 + 67.9165\,s + 83.6303\,s^2 + 11.1752\,s^3 + s^4}$$

(5.10)

The root locus of the outer loop, which is defined with the transfer function given is drawn as follows:

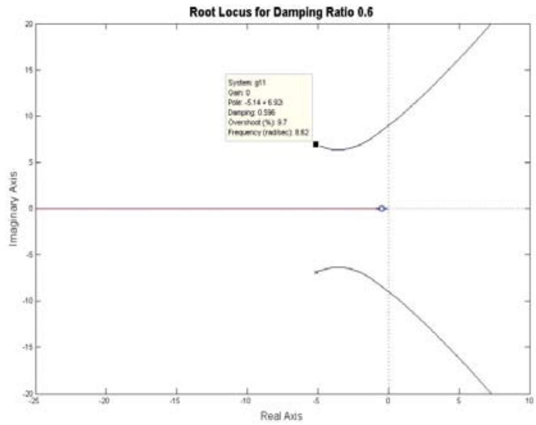

Figure 5.9 Root Locus Diagram for Inner Loop with $\zeta = 0.6$

S_{rg1}= 4.16, S_{amp}=1

6 Root Locus for Outer Loop

After the root locus for inner loop is drawn and then S_{rg} values are found for given damping ratios, the root locus diagram for outer loop is drawn by using determined S_{rg} values. By the way, the amplifier sensitivity (S_{amp}) values are obtained as follows.

- **For S_{rg1}=0.643:**

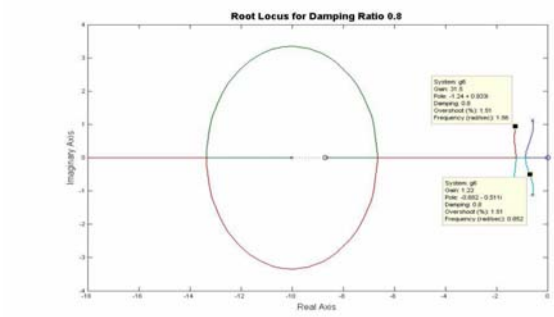

Figure 6.1 Root Locus Diagram for Outer Loop with $\zeta = 0.8$ and S_{rg1}=0.643

According to Figure 6.1, the S_{amp} values corresponding to S_{rg1} are found S_{amp1}=31.5 and S_{amp2}=1.22.

♦ **For S_{rg2}=2.19:**

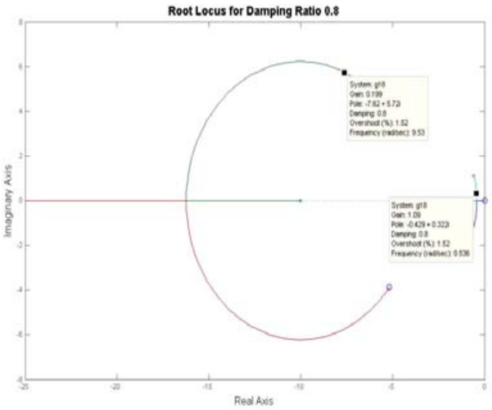

Figure 6.2 Root Locus Diagram for Outer Loop with $\zeta = 0.8$ and S_{rg2}=2.19

According to Figure 6.2, the S_{amp} values corresponding to S_{rg2} are found S_{amp1}=1.09 and S_{amp2}=0.199.

♦ **For S_{rg1}=0.447**

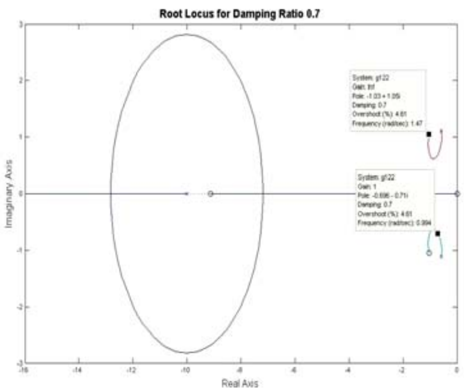

Figure 6.3 Root Locus Diagram for Outer Loop with $\zeta = 0.7$ and S_{rg1}=0.447

According to Figure 6.3, the S_{amp} values corresponding to S_{rg1} are found S_{amp1}=1 and $S_{amp2}=\infty$.

♦ **For S_{rg2}=2.95:**

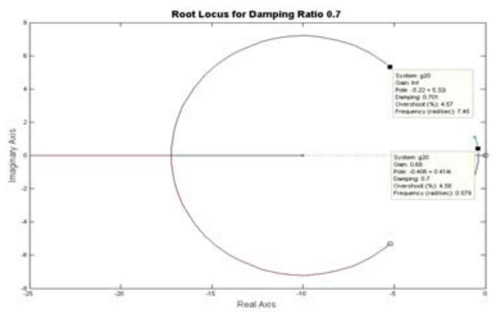

Figure 6.4 Root Locus Diagram for Outer Loop with $\zeta = 0.7$ and S_{rg2}=2.95

According to Figure 6.4, the S_{amp} values corresponding to S_{rg2} are found $S_{amp1}=\infty$ and S_{amp2}=0.68.

♦ **For S_{rg1}=0.252:**

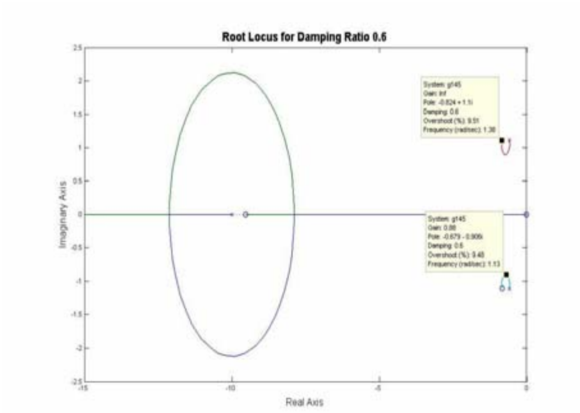

Figure 6.5 Root Locus Diagram for Outer Loop with $\zeta = 0.7$ and S_{rg1}=0.252

According to Figure 6.5, the S_{amp} values corresponding to S_{rg1} are found $S_{amp1}=\infty$ and S_{amp2}=0.879.

♦ **For S_{rg2}=4.16:**

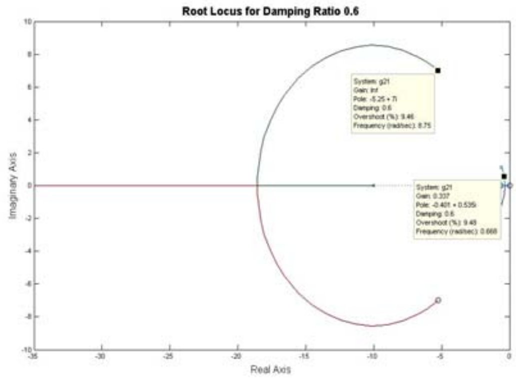

Figure 6.6 Root Locus Diagram for Outer Loop with $\zeta = 0.6$ and S_{rg2}=4.16

According to Figure 6.6, the S_{amp} values corresponding to S_{rg2} are found $S_{amp1}=\infty$ and S_{amp2}=0.337.

7 Stability Analysis

After the appropriate Srg and Samp values corresponding to them are determined, it is necessary to test their stability in order to choose the most appropriate amplifier sensitivity corresponding to the necessary rate gyro sensitivity. As it is explained previously, Routh-Hurwitz criterion is used to determine each system's stability.

		Appropriate for the designed autopilot
$\zeta=0.8$	**S_{rg1}=0.643 S_{amp1}=31.5**	—
	S_{rg1}=0.643 S_{amp2}=1.22	√
	S_{rg2}=2.19 S_{amp1}=1.09	√
	S_{rg2}=2.19 S_{amp1}=0.199	√
$\zeta = 0.7$	**S_{rg1}=0.447 $S_{amp1}=\infty$**	—
	S_{rg1}=0.447 S_{amp2}=1	√
	S_{rg2}=2.95 $S_{amp1}=\infty$	—
	S_{rg2}=2.95 S_{amp2}=0.68	√
$\zeta = 0.6$	**S_{rg1}=0.252 $S_{amp1}=\infty$**	—
	S_{rg1}=0.252 S_{amp2}=0.879	√
	S_{rg2}=4.16 $S_{amp1}=\infty$	—
	S_{rg2}=4.16 S_{amp2}=0.337	√

8 Pitch Angle Time Response of the Flight Control System of the Aircraft

The pitch angle time response can be determined with the help of outer loop transfer function that had already been found out for the flight control system. In order to calculate the value of the pitch angle for -6 degree, one of the values for Sr_g and S_{amp} are selected as follows:

$$S_{rg}=0.643$$

$$S_{amp}=1.22$$

Then the outer loop transfer function is obtained as given in the following and then by simplifying it becomes as given:

θ (t) is found as given:

$$0.299276e^{-8.98921t}-(0.220914-0.65582i)e^{(-0.94884-1.76278i)t}-(0.220914+0.65582i)e^{(-0.94884+1.76278i)t}+0.142552e^{-0.28834t}$$

For $t \to \infty$

$$\theta(\infty) = 0$$

Finally, pitch angle time response for -6° input pulse elevator deflection is drawn.

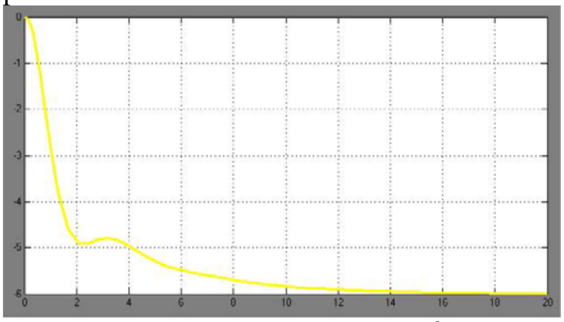

Figure 8.1 Pitch Angle Time Response to the -6° Input Pulse Elevator Deflection

From Fig. 8.1 the designed autopilot system with pulse elevator deflection input would approach to -6° pitch angle in approximately 16 seconds.

9 Conclusion

The transfer functions for inner loop and then the outer loop are obtained with the help of automatic control principles. Rate gyro sensitivity, S_{rg} and amplifier sensitivity are still unknown for the designed autopilot system. The root locus for open-inner loop is drawn with the help of MATLAB and then the appropriate S_{rg} values are found for given damping ratios. Next, the outer loop transfer functions are obtained for the corresponding S_{rg} values. The stabilities of the appropriate flight control systems are checked by using Ruth-Hurwitz criterion. Finally the pitch angle time response of the designed autopilot to the -6 degree input pulse elevator deflection is drawn.

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[1] Blakelock, J., H., Automatic Control of the Aircraft and Missiles, John Wiley & Sons, 1965

[2] Kosar, Durmaz, Jafarov, Longıtudınal Dynamıcs Analysıs Of Boeıng 747-400 (WSEAS proceedings).