Modeling and Simulation of the Fuzzy Relay Type Controller for Solving the Double Integrator Control Problems

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Abstract: - In this paper, we present the modeling and simulation results of the fuzzy relay type controllers for solving the double integrator control problems. We also give the comparative analysis of determined and fuzzy relay type controllers with leading hysteresis. Unlike determined controller, the fuzzy relay type law does not use the derivative of an input signal. Therefore, the performance of the fuzzy relay type controller with the noisy input is much high. On the other hand, fuzzy system with the fuzzy relay type controller is more robust than classical one – this is the main advantage of the fuzzy controllers. Simulation of the fuzzy relay type controller is executed by using tools in MATLAB.

Key-Words: - Double Integrator, Noise, Membership Function, Robustness, Determined Controller, Fuzzy Relay Type Controller

1 Introduction

Many continuous dynamic controlled objects can be described by the double integrator model with transfer function $G(s) = 1/s^2$ [1]. It is known, that continuous-discrete control systems are not stationary in continuous time [2]. For their analysis and synthesis the special methods, which have complex mathematical models, are used [3], [4], [5]. At the same time, the research methods of stationary continuous-discrete systems are well developed and a hybrid problem is frequently reduced to one of stationary cases by using different type approximations. Modern control systems of such objects use the digital controllers (regulators) which have many advantages in comparison with analog ones (for example, absence of parameter drift, easy adjustment, etc.). Thus, whole system (object + controller) is the hybrid continuous-discrete system, in which the separate parts are described by differential and difference equations.

The double integrator is widely used for modeling of the mechanical, hydraulic and electrical objects, for example, in DC motor controls [1], [2], [3], in pointing a plane or a rocket to a target in the simple interception problems [8], [11], [12], [13], in soft landing on the Moon problems [5], [6], [7], [8], etc.. Besides, the double integrator is successfully used as a test object for demonstration of the relay control law features.

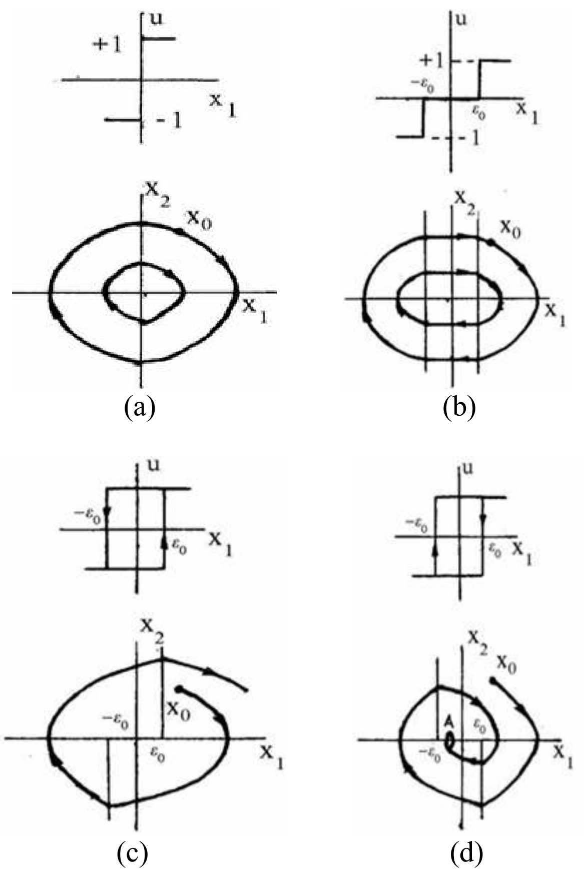
There are many publications concerning the double integrator problems. However, there still exist some important problems in this area. Practically in all existing works, the problems related to the double integrator are solved in determined mode without taking into account noise imposed on object coordinates. In [9], [14], the optimum control problem for minimizing of energy losses is solved under the conditions of different uncertainty. For example, such uncertainties as the changes in object parameters, initial conditions, control restrictions, etc., usually take place during system functioning. The synthesis problem of control is solved in real time under variable and known restrictions. A solution is obtained in the discontinuous class of control systems closed by relay one. However, in real conditions, the parameters of the object and the full vector of the current state are usually known. An identifier and a state observer are included to the system for their definition.

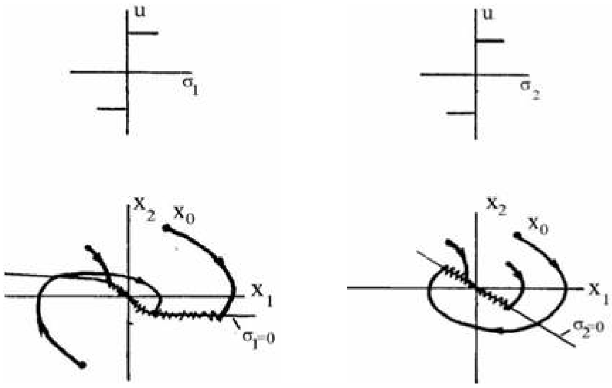
In the present paper, we consider the determined object, but "struggle" with various type of uncertainties is entrusted to the fuzzy relay controller. Because the derivative of the input signal, which is very sensitive to noise, is also used for formation of the determined relay control laws with the hystereses, in this paper, the noise robust problems (robust related to noise) of the fuzzy type relay controllers are investigated [16], [10]. Thus, robustness means the small change at the output of the closed control system during small change in the parameters of control object. The systems having the robust feature are called as the robust (rough) systems. This paper also gives the comparative analysis of proposed models. Finally, the proposed fuzzy relay type controller model is simulated using the tools in MATLAB.

2 Definition of the Problem

First of all, let us consider the main control properties of the double integrator based on the determined relay controllers. For the analysis and comparison of the system dynamics we will use the phase plane method. The specificity of the given object is as following: the phase trajectories are the symmetric parabolas for $u = \pm 1$, however, they are the straight lines for $u = 0$ [1]. During the usage of relay controllers with symmetric static characteristics, the symmetry of object trajectory leads to appearance of a limiting cycle, the amplitude of which depends on the initial conditions. For stabilization of object (in other words, putting and keeping it at the origin of coordinates), it is necessary to use such controller laws which could break down the symmetry of phase trajectories stuck on lines of switching. In Fig.1, the basic relay characteristics and corresponding phase portraits are shown for the double integrator that presented in coordinates:

$$x_1 = y,\ x_2 = \dot{y}:$$
$$\dot{x}_1 = x_2, \quad \dot{x}_2 = bu,\ x_0 = (x_{10}, x_{20})^T$$





(e) (f)

Fig.1 Static Characteristics and Phase Portraits of Basic Relay Laws for Double Integrator

As we see from Fig.1 a, b, the limited cycles (auto-oscillations) arise from any initial condition for both the conventional and relay characteristics with an insensitive zone for any $x_1 < |\varepsilon_0|$. In case of a relay characteristic with lag behind hystereses (Fig.1, *c*), unstable oscillation process takes place, however, with leave behind hystereses, stable oscillation process occurs. In the last case, auto oscillations are appeared with small amplitude in zone $x_1 < |\varepsilon_0|$ (see letter *A* in Fig.1, *d*). Similar stabilization, because of the small amplitudes of auto-oscillations, is considered as satisfactory. The relay control law with the specified static characteristics can be represented in the form:

$$u = \begin{cases} +1 & for \quad x_1 \geq \varepsilon_0, \\ \mp sign(dx_1) & for \quad -\varepsilon_0 < x_1 < \varepsilon_0, \\ -1 & for \quad x_1 \leq -\varepsilon_0. \end{cases} \qquad (1)$$

As defined in (1), a derivative dx_1, which characterizes the sign of derivative $dx_1 / dt = x_2$ or the direction of movement, is used for definition of control in a zone of uncertainty $x_1 < |\varepsilon_0|$. As has been mentioned above, dx_1 is very sensitive to various types of noise and due to this reason dx_1 is the basic violation source of roughness of similar determined type controllers.

Relay laws, which provide the movement in the sliding mode, present a special interest (Fig.1, *e, f*). This mode allows us to obtain a stable quality of transient processes during any change of object parameters over a wide range (in this case b is the coefficient of amplification). The main reason of the stability is following. The system in the origin of coordinates moves according to $\sigma_1 = x_2|x_2| + \delta_0 x_1 = 0$ (parabola) or $\sigma_2 = x_2 + cx_1 = 0$ (straight line) established "hard" for any coefficient of amplification b. Here δ_0 and c are adjustment parameters for switching lines, the values of these parameters are chosen under condition for satisfaction of the particular quality parameter. It is possible to set $\delta_0 = 0.05, c = 2$ during simulation process.

3 Fuzzy Relay Type Controller Modeling

Fuzzy controllers are used to control the consumer products, such as washing machines, video cameras, and rice cookers, as well as industrial processes, such as cement kilns, underground trains, and robots. A fuzzy controller can include empirical rules, and that is especially useful in operator controlled plants[15,17]. Let us consider a typical fuzzy controller:

1. *If error is Neg and change in error is Neg then output is NB*
2. *If error is Neg and change in error is Zero then output is NM*

….

The set of rules is called a *rule base*. The rules are similar to *if-then* format and, formally, *if* side is called *condition* and *then* side is called *conclusion*. The input value *"Neg"*, the output value *"NB"*, and *"NM"* are *linguistic terms* which mean *Negative, Negative Big* and *Negative Medium*, respectively. A computer is able to execute the rules, to compute a control signal depending on the measured input *error* and to change *error* [17].

Membership functions, which allow us to design the fuzzy relay type controller based leading hysteresis, are presented in Fig.2, *a* and *b*.

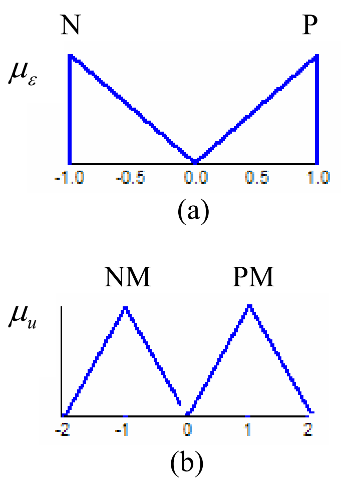


Fig.2 Linguistic Terms and Membership Functions for Fuzzy Sets

In [16], the conventional type relay controller is obtained (Fig.1, *a*).

Fig.2 shows the linguistic terms and the membership functions corresponding to fuzzy rules:

IF ε *=P THEN u=PM; IF* ε *=N THEN u=NM;*

where, *P*, *N* – *positive* and *negative* values, respectively; *PM*, *NM* are *positive-medium* and *negative- medium*, respectively.

The proposed fuzzy controller differs from the determined type controller (1) with leading hysteresis. In order to obtain the similar hysteresis, the fuzzy controller does not use the derivative dx_1/dt (or the variation $dx_1 = x_{1k} - x_{1k-1}$ for digital realization) as input variable. Due to this novelty, we obtain high noise stability by proposed fuzzy controller.

4 Fuzzy Relay Type Controller Simulation

Tuning of the fuzzy controller (or changing the parameters μ_x, μ_u of the membership functions and their ranges) is realized by supplying a sinusoidal harmonious signal $x = \sin(\omega t)$ to its input. The obtained static characteristic of a fuzzy controller with leading type of the hysteresis and with a zone $\varepsilon_0 \approx 0.075$ and amplitude $u \approx \pm 1$ is shown in Fig.3.

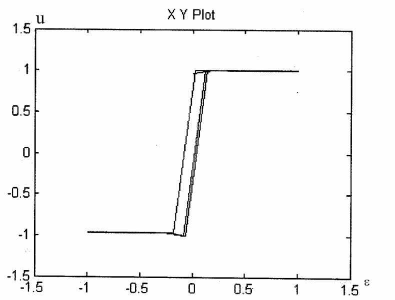


Fig.3 Static Characteristic for Relay Type Fuzzy Controller

Character of the hysteresis function can be defined by observing the movement of the representing point (x, u) at XY plane. If distinguishing of the representing point is difficult or inconvenient then it is possible to go to the time domain plane by observing the phase of switching of the relay and compare it with the sinusoid. In Fig.4, the block diagram of the proposed and designed automatic control system is shown.

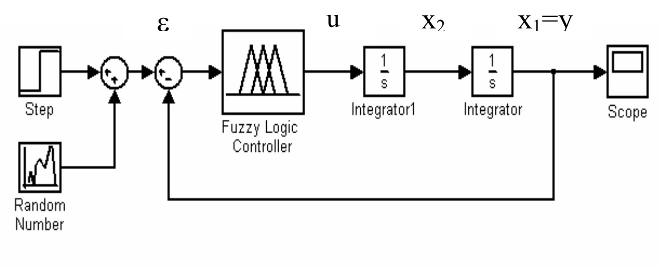
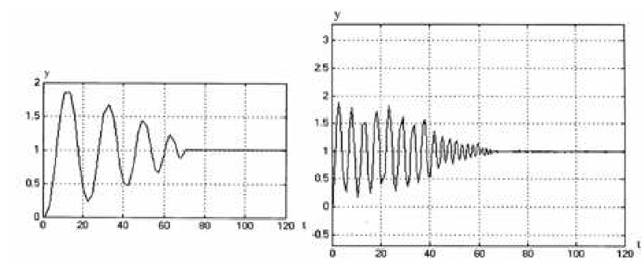


Fig.4 Block Diagram of Automatic Control System

In Fig.5 *a* and *b,* the transient responses for the determined controller (Fig.1, 2) and for its relay type fuzzy analogue are shown when the noise is absent and $\varepsilon_0 = \pm 0.075$. As we see from Fig.5, the response and over control amplitudes are almost the same for both cases, however the oscillation of the fuzzy control system is considerably high. The frequency of oscillation is mainly related to the inclination of the fuzzy controller characteristic (Fig.3). It should be noted, that straightening and increase in width of the zone of the fuzzy characteristic were not studied in this paper. But this part of research is important for reducing the oscillation by changing the parameters of membership functions. We are going to research them as future work. Besides, because the determined relay characteristic with the leading hysteresis in MATLAB is unforeseen we designed it by standard blocks according to the algorithm (1) for sign "+" at the operator sign.



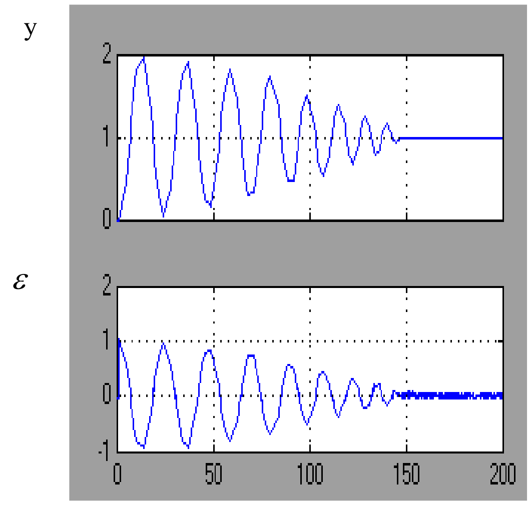
(a) (b)

Fig.5 Transient Characteristics for ξ =0:

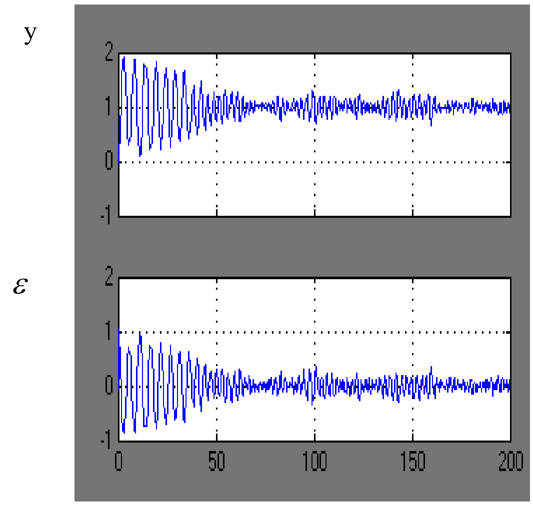
a) for Determined Type Controller; b) for Relay Type Fuzzy Controller

In Fig.6 *a* and *b,* the transient responses $y(t)$, $\varepsilon(t)$ and $u(t)$ are shown when the noise influence $\xi(t)$ effects the reference signal. The noise was modeled by Gausses type of noise with parameters $m = 0$ and $\sigma = 0.05$.

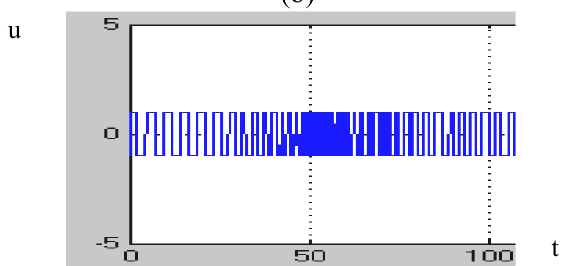
In a stochastic variant, the determined controller leads to substantial increase of response time (about 2,5 times). The fuzzy controller does not affect so much on change of dynamic parameters; however in the steady state mode it is observed the amplification of noise. The noise immunity can be improved in a static mode by choosing the suitable fuzzyfications methods, too.



(a)



(b)



(c)

Fig.6. Step Responses for Input with Noise Influence: a) for Determined Controller; b) for Relay Type Fuzzy Controller; c) for Control Variable

The relay type fuzzy controller obtained by means of the proposed system can be realized on any programmed controller. It is possible to use Motorola 68HC11, 68HC12, Intel MCS-96, etc. as the real microcontrollers supporting Fuzzy logic.

5 Conclusion

In summary, it is possible to come to the following conclusions:

1) Thus, the obtained results partially confirm the research statement in [16] about the robustness of fuzzy controllers in respect to the determined analogues.

2) System with a fuzzy type controller is the more robustness than system with classical one. The robustness is a basic advantage of the fuzzy type controllers.

3) The given research has shown an opportunity of use of various kinds of membership function for fuzzy type controller (triangular, trapezoidal membership functions, etc.).

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