NEW FIGURES OF MERIT FOR RANGE RESOLUTION RADAR USING HAMMING AND EUCLIDEAN DISTANCE CONCEPTS

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Abstract

For range resolution radar, the usual measure of goodness is obtained from discrimination and merit factor. In addition, the measure of goodness is also expressed with figures of merit, which was proposed earlier. Due to additive noise, it is possible that the transmitted and received signals may not be the same. The return signal is assigned to equivalence classes based on the notion of Hamming distance. The figures of merit are defined in terms of appropriate cross correlation properties averaged over the neighbourhood of the transmitted sequence. For some binary and ternary sequences, the figures of merit are tabulated. They indicate the performance of the deterioration rate as the medium gets noisier. In addition, the Euclidean distance is considered and results were plotted for single errors. Here in this paper, the figures of merit are calculated up to four noise levels and the deterioration performance is evaluated.

***Keywords*: Hamming distance, Euclidean distance, Figures of Merit, and Merit Factor**

1 Introduction

For range resolution radar, a coded waveform or a sequence can be taken as

$$X = x_0, x_1, x_2, \ldots, x_{N-1}$$

with aperiodic autocorrelation

$$r(k) = \sum_{i=0}^{N-1-k} x_i\, x_{i+k} \qquad (1)$$

where $k = 0, 1, 2, \ldots, N-1$

For sequences to be good, the autocorrelation should have very large peak for zero shift with very small side lobes. In other words, r(0) to be very large and r(k≠0) to be ideally zero is required. In this autocorrelation domain, the goodness of a sequence is judged by the discrimination D and merit factor F.

Discrimination D is defined as the ratio of main peak in the autocorrelation to absolute maximum amplitude among side lobes, Moharir [1]. That is

$$D = \frac{r(0)}{\max\limits_{k \neq 0} |r(k)|} \qquad (2)$$

Merit factor F is defined as the ratio of energy in the main peak of the autocorrelation to the energy in the side lobes, Golay [2]. That is

$$F = \frac{r^2(0)}{2\sum_{k=1}^{N-1} r^2(k)} \qquad (3)$$

For the above equation, the factor 2 appears in the denominator as the autocorrelation is an even function. D and F should be as large as possible for sequences to be good.

For binary sequences the alphabet is ± 1 and for ternary sequences it is 0, ± 1. For binary sequences the length of the sequence increases much faster than the discrimination they can achieve, even if Barker criterion is dropped (for example ,to achieve a discrimination of 14, the length must be at least 28). And also their optimal F approaches the values 12.32…. as N increases without bound. But ternary sequences, the above limitations can be overcome.

Further one more parameter called energy efficiency E is defined in the signal domain, by Ackroyd [3],as the ratio of the actual energy in the sequence to the energy in every element in the sequence had the maximum amplitude.

That is

$$E = \frac{\sum_{k=0}^{N-1} x_k^2}{N[\underset{k}{Max}\, x_k^2]} \qquad (4)$$

Ideally E should be 1 which implies that all the elements of the sequence should have the same absolute magnitude as happens in binary sequences with ± 1 as alphabet. This property is referred as *"**constant envelop property**"* whereas for ternary sequences energy efficiency will be always less than 1(less than 100%). This is the only drawback of ternary sequences in addition to its hardware complexity.

2 The Concept of Figures of Merit

To obtain good range resolution, binary or ternary sequences will be used as a coded waveform. The transmitted signal and return signal may not be the replica and also have distortion due to propagation effect and additive noise. In general, the distortion due to propagation effect can be ignored. The additive noise is assumed to be independent of the transmitted signal so that their cross correlation is also negligible.

When the signal is of finite duration then it may be desirable to take cross correlation of the return signal with the delayed versions of the transmitted signal into account without making the assumptions.

The transmitted and the return signals correspond to sequence *X* and *X'*. the return signal obtained from *X* with given number of error, say '*m*'. Let $C^{(m)}(k)$ represents the cross correlation between *X* and *X'*. Then the figure of merit $M^{(m)}$ [4] is defined as

$$M^{(m)} = \frac{\overline{C^{(m)}(0)} - \overline{\max_{k\neq 0}\left| C^{(m)}(k)\right|}}{\overline{C^{(m)}(0)}} \qquad (5)$$

The overhead bars denote averaging over the ensemble of *X'*. The numerator is the difference between the average zero-lag cross correlation and the average of the maximum absolute side lobes. The denominator is the average zero-lag cross correlation. The figures of merit depend on the sequence used for range resolution and its Hamming neighbours defined by a threshold *m* on the Hamming distance.

Here, in particular, when m=0, *X'* is *X* and therefore $C^{(m)}(k)$ is $r(k)$.

Therefore,

$$M^{(0)}=1-(1/D) \qquad (6)$$

Then $M^{(0)}$ is a monotone function of D. Thus if D is an acceptable measure of goodness, so is $M^{(0)}$. However, when D goes to infinity, $M^{(0)}$ becomes unity only, making the latter a non-euphoristic.

Figures of Merit for values of m=4 for binary and ternary sequences are shown in Tables 1 and 2 respectively.

Binary sequences are listed by Golay[5] and the ternary sequences are listed[6,7] on the basis of efficient but incomplete search.. These two categories are made used for determining these new figures of merit. Likewise same concept is applicable to sonar .

This can be extended to multi user environment which is a key task in MIMO(Multi Input and Multi Output) communications.

Table 1: Figures of Merit, $M^{(m)}$ are tabulated for best binary sequences listed by Golay.

Note: The sequences are tabulated in an alphabetically coded format.. If a sequence is of length 3n , it is written as n triplets of elements. There are 27 possible code words for triplets using – and 26 capital letters. They are coded lexicographically with precedence order -1,0,1. Thus -1 -1 -1= - , -1 -1 0= A, …. , 1 1 1= Z. Eight of these , i.e., - , B, F , H , R , T, X and Z are totally represents binary. If the length is 3n+1, the first 3n elements are coded as above and the last element which can only be -1 or 1 is coded as n or p respectively. If the length is 3n+2, one bit prefix and suffix are coded as n or p and the 3n element core as above. The same alphabetical coding procedure is used for both binary and ternary sequences.

N	Alphabetical code for the sequence	Five figures of merit				
		$M^{(0)}$	$M^{(1)}$	$M^{(2)}$	$M^{(3)}$	$M^{(4)}$
9	YXK	0.857	0.710	0.514	0.227	-0.30
	FP-	0.857	0.710	0.505	0.219	-0.29
10	TNUn	0.857	0.738	0.577	0.363	0.023
	UYSn	0.857	0.710	0.552	0.335	-0.02
	UGYp	0.857	0.710	0.552	0.335	-0.02
11	pCXCn	0.889	0.768	0.609	0.456	0.126
12	ZRVF	0.9	0.786	0.666	0.520	0.322
14	pFPBZp	0.917	0.799	0.691	0.576	0.436
	pFXP-n	0.917	0.784	0.687	0.578	0.442
15	ZUPEF	0.909	0.816	0.717	0.612	0.489
16	THEIAp	0.923	0.819	0.732	0.641	0.535
	ZXEKGn	0.923	0.819	0.732	0.641	0.535
17	pIKXYSp	0.846	0.799	0.727	0.641	0.541
	nIOXASn	0.846	0.797	0.723	0.638	0.540
	n-LXDTn	0.846	0.799	0.733	0.652	0.556
	p-KXYTp	0.8	0.787	0.738	0.663	0.571
18	THXARU	0.875	0.813	0.741	0.668	0.586
	ROFRAW	0.933	0.846	0.769	0.688	0.598
	XQ-BGC	0.933	0.846	0.769	0.688	0.598
19	STESZUn	0.867	0.817	0.755	0.686	0.608
	ZAYWEBn	0.867	0.807	0.749	0.683	0.609
20	pXQCKNDn	0.923	0.832	0.766	0.699	0.628
	PHKWQLDn	0.923	0.832	0.766	0.699	0.628
	PBPDTLIn	0.923	0.823	0.757	0.692	0.621
21	ZYIXGOT	0.882	0.840	0.783	0.721	0.655
	VTTXZ-N	0.882	0.851	0.789	0.721	0.649
22	ZNIPOFVn	0.929	0.845	0.783	0.721	0.656
	TLOPIZVp	0.929	0.845	0.783	0.721	0.656
23	pFUUXEU-n	0.947	0.877	0.819	0.761	0.701
	PZRLXDRTn	0.895	0.846	0.796	0.744	0.688
24	SSGNLD-E	0.929	0.839	0.785	0.733	0.678
	YAALLVFU	0.929	0.839	0.785	0.733	0.678
25	FTKKRQI-n	0.905	0.874	0.825	0.774	0.722
	FTSSUER-n	0.857	0.843	0.810	0.767	0.717

Table 2: Figures of Merit, $M^{(m)}$ are tabulated for some good ternary sequences obtained by incomplete but efficient search(alphabetical coding is same as above).

	Alphabetical code for the sequence	Five figures of merit				
		$M^{(0)}$	$M^{(1)}$	$M^{(2)}$	$M^{(3)}$	$M^{(4)}$
9	ZBT	0.667	0.556	0.278	-0.30	-3.04
	ZRT	0.667	0.556	0.272	-0.34	-3.18
11	pXBBn	0.909	0.717	0.475	0.125	-0.58
13	ZXHFp	0.923	0.769	0.581	0.335	-0.03
15	ZXHBF	0.800	0.738	0.612	0.445	0.223
17	pR-XFRp	0.824	0.729	0.639	0.515	0.354
19	XRFZTZn	0.842	0.774	0.695	0.587	0.455
	ZFB-RBn	0.632	0.632	0.588	0.514	0.408
21	X--XFTB	0.857	0.812	0.733	0.634	0.526
23	pX-ZHTTBn	0.783	0.766	0.720	0.647	0.562
	pR-BRXTHn	0.783	0.766	0.716	0.644	0.559
	pXBZXTBBn	0.783	0.749	0.702	0.634	0.552
	pZZ-XFFTn	0.783	0.758	0.714	0.646	0.561
25	ZZ-ZBBBFp	0.88	0.821	0.758	0.687	0.61

3 Inference from the hierarchy of figures of merit

The figures of merit $M^{(1)}$,$M^{(2)}$, $M^{(3)}$ and $M^{(4)}$ illustrate the performance of binary and ternary sequences as follows.

1) For all binary sequences, the figures of merit show a steady deterioration as *m* increases.
2) For both binary and ternary sequences, as the length increases, the deterioration rate decreases as *m* increases.
3) As *m* increases, the figures of merit of ternary sequences show superiority over binary sequences. This is shown in figure 1 for length 25.
4) For some ternary sequences of same length, the figures of merit are same as *m* increases. For instance, for *N*=24, the sequences SSGNLD-E and YAALLVFU have same figures of merit($M^{(0)}$ = 0.9285, $M^{(1)}$ = 0.839 , $M^{(2)}$ = 0.785 , $M^{(3)}$ = 0.733 and $M^{(4)}$ =0.678)
5) For *N*=14, the two ternary sequences pFPBZp and pFXP-n have same $M^{(0)}$ = 0.9166.The former shows better performance at *m*=1 and 2. But the

latter shows better performance at m=3 and 4. The results are shown in fig.2.

6) For N=9, the ternary sequences YXK and FP- ,both have same figures of merit at m=0 and 1 ($M^{(0)}$ = 0.8571, $M^{(1)}$ = 0.710) but at m=2,3 the former shows better performance (fig.3).

From the above, the rate of deterioration can also vary from sequence to sequence with different noise levels. Some sequences are having equivalence of performance at zero noise level and it may breakdown as the medium becomes noisier. For the sequences having superior performance at zero noise level may not posses the same as the noise level increases. Hence, the use of figures of merit has a role to play in choosing good sequences for range resolution radar.

All signal design problems are search oriented and their solution would be time consuming unless good sieves support the search techniques. Therefore, the exhaustive search for binary sequences with large values of $M^{(m)}$, m=0,1,2,3,4 has been pursued. The results have been tabulated for the best two sequences in table 3. From this table, several interesting points can be noted.

N	Alphabetical code for the sequence	Five figures of merit				
		$M^{(0)}$	$M^{(1)}$	$M^{(2)}$	$M^{(3)}$	$M^{(4)}$
5	nBn	0.800	0.333	-1.7	-1.7	0.333
	nFp	0.600	0.333	-1.2	-1.2	0.333
6	-T	0.667	0.458	-0.37	-3.	-0.37
	-R	0.667	0.375	-0.50	-3.2	-0.5
7	-Xp	0.857	0.543	-0.06	-2.49	-2.49
	BHp	0.714	0.514	-0.06	-2.54	-2.54
8	nBHn	0.750	0.563	0.188	-0.82	-3.77
	nRHn	0.750	0.542	0.170	-0.84	-3.8
9	-TR	0.778	0.635	0.300	-0.33	-3.18
	-RT	0.778	0.587	0.317	-0.27	-2.98
10	BT-n	0.800	0.650	0.385	-0.06	-1.25
	-TBp	0.800	0.650	0.381	-0.06	-1.26
11	nBXXp	0.909	0.717	0.475	0.125	-0.58
	nTZRp	0.818	0.677	0.444	0.099	-0.6
13	-BRTn	0.923	0.769	0.581	0.335	-0.03
	-HBFp	0.846	0.734	0.581	0.358	-0.01

Table 3: Figure of Merit for some binary sequences with good $M^{(m)}$ are obtained by exhaustive search (alphabetical coding is same as above).

For N=5, the first sequence (nBn) is of Barker and has superior performance to the non-Barker sequence(nFp) at m=0. But at m=1and 4, both the sequences are having same performance while the Barker sequence is inferior at m=2 and 3. For N=6, the sequences –T and –R , both have the same performance level at m=0. But the equivalence breaks down as the noise level increases. It is interesting to note that both the sequences replicate the performance of m=2 at m=4. For N=11, the deterioration rate of the first Barker sequence (nBXXp) from m=0 to m=2 is higher than that for the non-Barker sequence (nTZRp). However, the deterioration rate at higher noise levels for both the sequences is same. For n=13 , the performance of the first Barker sequence (-BRTn) is superior to that of the non-Barker sequence (-HBFp) at lower noise levels. But the situation is reversed at higher noise levels. However, the performance level of both the sequences is same at m=2. Later at m=4, the figure of merit approaches to zero. It is apparent from tables 1 and 3 that the performance of the sequences listed by Golay[5] based on merit factor, for N=9 are inferior to those obtained through exhaustive search based on the figures of merit[4}. The performances of figures of merit for N=5,6,11and 13 are shown in figures 4 through 7 respectively.

4. Soft decision using Euclidean distance

The Figures of Merit evaluated thus far uses hard decision that uses the Hamming distance to measure the similarity between the received and transmitted waveform.

The soft decision [9] uses Euclidean distance to measure the similarity between the received and transmitted waveform. This is necessary since the received waveform is not a stream of 0 and 1 anymore, but an array of real values.

If the $c=(c_1, c_2, c_3, \ldots, c_N)$ is a transmitted waveform (with $c_i = \{\pm 1\}$) and received waveform $r=(r_1, r_2, \ldots, r_N)$ Euclidian distance is

$$E(r,c) = \sqrt{\sum_{i=1}^{N} (r_i - c_i)^2} \quad (6)$$

where N represents length of the code

For binary sequences of lengths N=9,11, and 13 the plots were drawn for soft decision Vs Hard decision and are shown in Fig.8, Fig.9 and Fig. 10 respectively.

5. Conclusions

Based on the results obtained for different sequences, the performance of the sequences can be evaluated. The sequences can be ranked accordingly based on the performance of the sequence at different noise levels. The figures of merit group out the sequences with better resistance to increasing noise levels as compared to the known sequences. The Euclidean distance concept is out performing the Hamming distance concept for different binary lengths. The Euclidian distance concept can be extended for binary higher lengths and ternary sequences to find out good waveforms in range resolution radar. The figures of merit also provide useful information for setting up adaptive, diversity-combinatorial and robust pulse compression schemes for range resolution radar. This can also be extended for monogenic signatures [8].

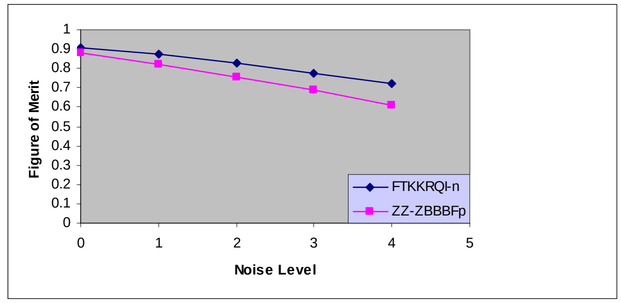

Figure1: Noise level versus figure of merit for binary and ternary sequences of length 25

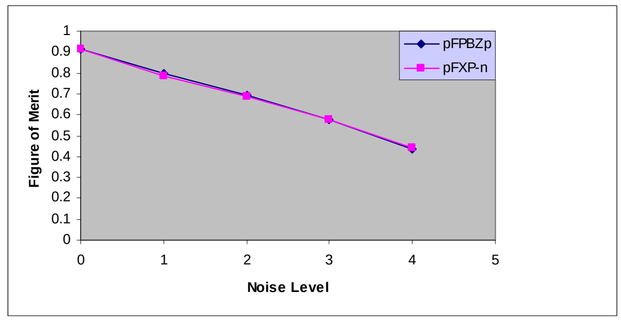

Figure2: Noise level versus figure of merit for ternary sequences of length 14

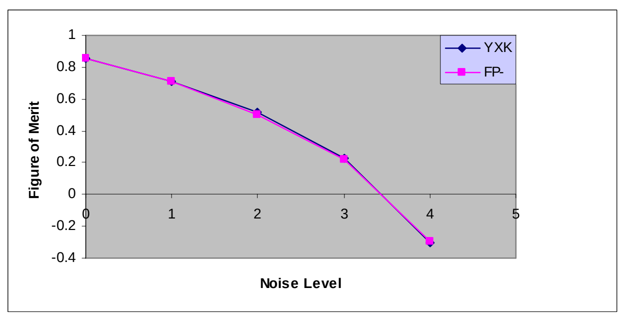

Figure3: Noise level versus figure of merit for ternary sequences of length 9

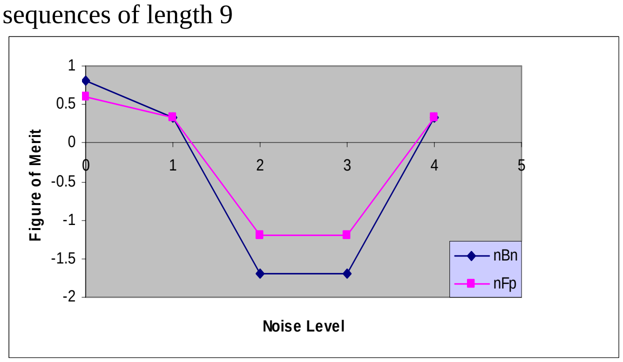

Figure4: Noise level versus figure of merit for binary sequences of length 5

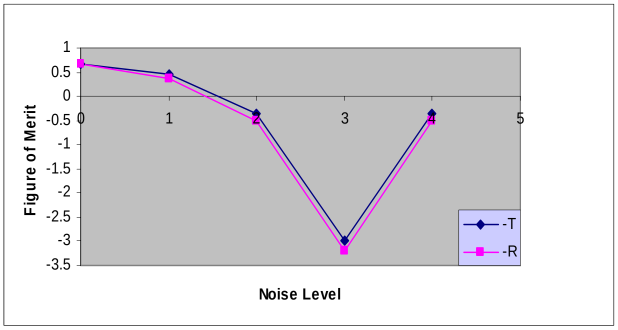

Figure5: Noise level versus figure of merit for binary sequences of length 6

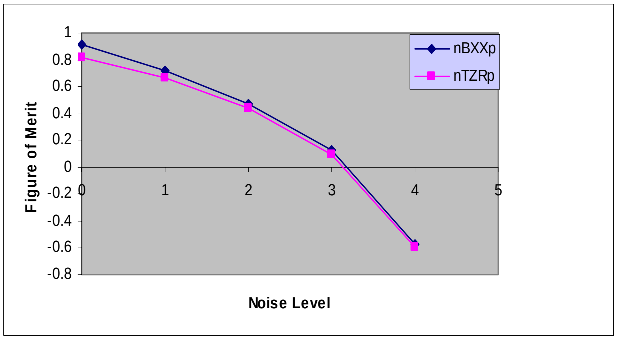

Figure6: Noise level versus figure of merit for binary sequences of length 11

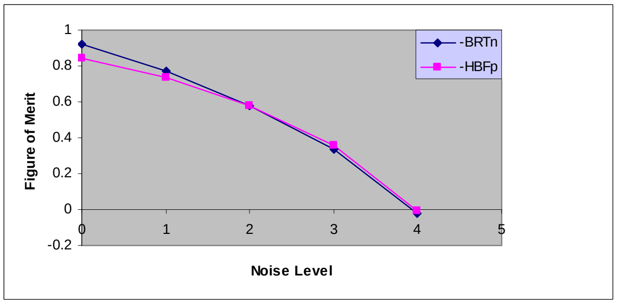

Figure7: Noise level versus figure of merit for binary sequences of length 13

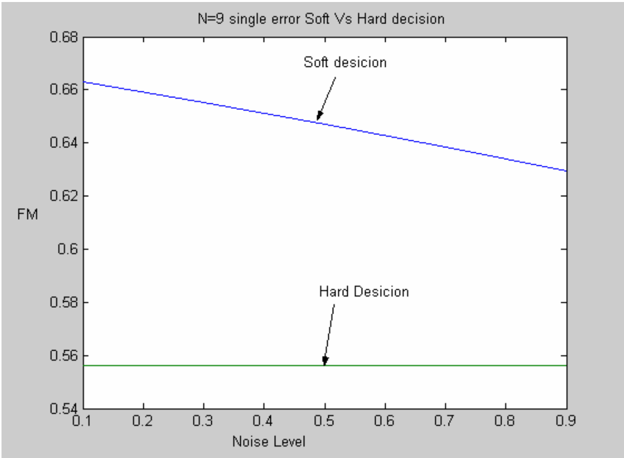

Fig8. N=9 Soft Vs Hard decision

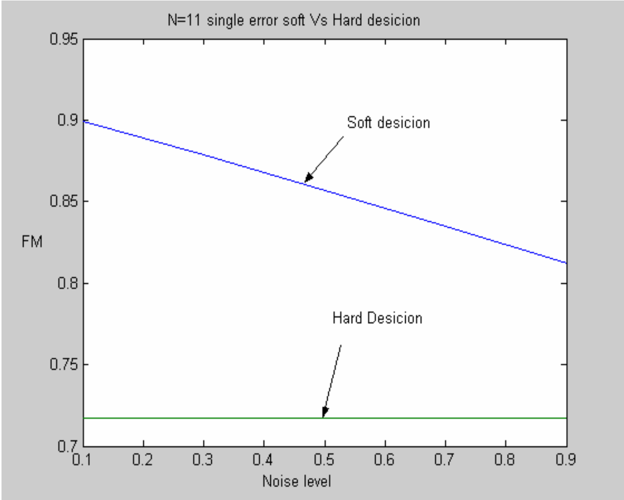

Fig.9 N=11 soft Vs hard decision

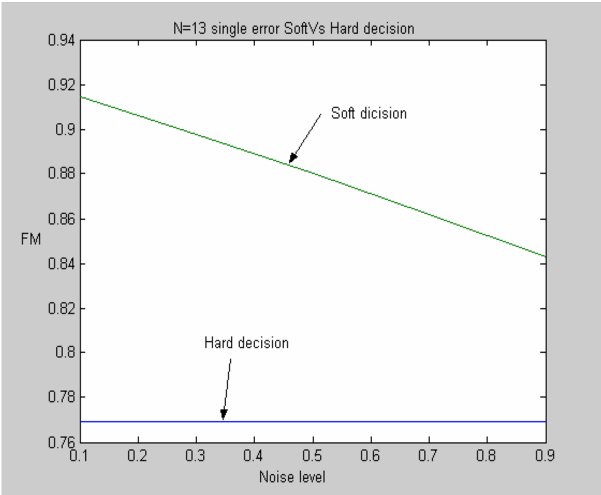

Fig10. N=13 Soft Vs Hard decision

References

[1] P.S.Moharir, Signal Design, Int J electron, vol 41, pp 381-398, 1976.

[2] M.J.E.Golay, Hybrid low autocorrelation sequences, Trans IEEE, vol IT-21,pp 460-462, 1975.

[3] M.H.Ackroyd, Amplitude and phase modulated pulse trains for radar, Radio & Electr Engr, vol 41, pp 541-552, 1971.

[4] P.S.Moharir, K.Raja Rajeswari & K.Venkata Rao, New figures of Merits for Pulse Compression Sequences, J Inst Electron Telecomn Engrs, vol 38, pp 209-216, 1992.

[5] M.J.E.Golay, Sieves for low autocorrelation binary sequences, Trans IEEE, vol IT-23,pp 43-51, 1977.

[6] P.S.Moharir, S.K.Verma & K.Venkata Rao, Ternary pulse compression sequences, J Inst Electron Telecomn Engrs, vol 31, pp 1-8, 1985.

[7] K.Venkata Rao, Coded waveform design for radar, Ph D Thesis, University of Roorkee, Roorkee, 1984.

[8] P.S.Moharir, K.Venkata Rao & S.K.Verma, Monogenic function range resolution radar, Proc IEE, Pt F, vol 134, pp 620-628, 1987.

[9] John G Proakis Digital Communications 4th Edition McGraw Hill Publications