

A Fuzzy TOPSIS Approach with Subjective Weights and Objective Weights

TIEN-CHIN WANG¹HSIEN-DA LEE^{1,2}CHUAN-CHENG WU³

¹Department of Information Management
I-Shou University, Taiwan

²Department of Information Management
Fortune Institute of Technology
Kaohsiung, Taiwan

³Department of Industrial Engineering and Management
I-Shou University, Taiwan

Abstract: - Multiple criteria decision making (MCDM) is widely used in ranking one or more alternatives from a set of available alternatives with respect to multiple criteria. Inspired by MCDM which are methods to systematically evaluate alternatives, we propose a new Fuzzy TOPSIS for evaluating outsourcing software companies both using subjective and objective weights. While crisp data are inadequate to model the real life situations in MCDM, decision makers (DM) use linguistic variables to describe the important degrees of specific attributes. We also present the same TOPSIS method by using an entropy-based weighting approach. With our method, we provide decision makers more information to make more subtle decisions.

Key-Words: - TOPSIS, MCDM, Subjective weights, Objective weights, Entropy

1. Introduction

As the downsizing trend has dominated MIS departments in business corporations, how to balance the effectiveness and efficiency has become a critical issue to every MIS department director. Outsourcing can be a solution for MIS department to leverage its work force and work burden. As the digital enterprises emerge, selecting appropriate software providers becomes more and more important. In software selection, many aspects have to be carefully evaluated. Usually, software application packages are evaluated in terms of software attributes such as investment costs, corporation images, advanced functions, training time, documentations, easy of use and the vendor support [5]. The decision process of selecting an appropriate information system usually has to take many factors into considerations, for instance, organizational needs and goals, risks, benefits, limited resources, compatibilities between current systems and future systems, etc. Several qualitative and quantitative criteria may affect mutually when evaluate alternatives, which may make the selection process complex and challenging. Due to the vagueness of human being's thought, the selection is often based on inadequate information or personal judgments. Decision makers may find it

hard to identify the best choice due to the lack of systematic methods to deal with multi-criteria problems. This paper formulates the outsourcing software company evaluation as a MCDM model, and then presents an effective approach by modifying TOPSIS for solving this problem.

2. Multiple Criteria Decision Making (MCDM)

The decision problem of selecting information system can be described as a complex, multi-objective task, based on uncertain data. Therefore it can be defined as a multi-criteria decision making problem under uncertainty, whose solution requires reliable methods that can incorporate all risk factors in a methodical manner. A software selection problem can be calculated as a multiple criteria decision making problem in which alternatives are the software packages to be selected and criteria are those attributes under consideration.

In traditional MCDM, alternative rating and weights are measured in crisp numbers. Classical MCDM methods require the determination of alternatives rating and criteria weights are made

which depend on decision makers' (DM) judgments/preferences. Crisp values are commonly used to represent those ratings and weights. However, in practice, alternative ratings and criteria weights could not be assessed precisely, which may come from various sources, including (1) unquantifiable information, (2) incomplete information, (3) unobtainable information, and (4) partial ignorance [4]. Under many circumstances where performance rating and weights can not be given precisely, the fuzzy set theory is introduced to model the uncertainty of human judgments and such problems is known as fuzzy multiple criteria decision making (FMCDM). Bellman and Zadeh [6] first introduced fuzzy set theory into MCDM as an approach to effectively dealing with the inherent imprecision, vagueness and ambiguity of the human decision making process. Since then, many researchers have been working on the process with uncertain data. Multi-criteria decision making (MCDM) has been widely used in selecting or ranking a finite set of decision alternatives characterized by multiple and usually conflicting criteria (attributes). Tremendous efforts have been spent and significant advances have been made towards the development of numerous MCDM methodologies for solving different types of problems such as election forecasting [3], selecting robots for manufacturing companies [1] and military threat assessment [2].

2.1 Fuzzy MCDM with Linguistic Terms

In fuzzy MCDM [7], performance ratings and weights are usually represented by fuzzy numbers. An alternative is calculated by aggregating all criteria weights and alternatives ratings, where alternatives with a higher utility are preferred. Since software quality is basically determined by subjective perceptions and feelings towards each of the evaluated criterion, the fuzzy MCDM (Fuzzy Multiple Criteria Decision Making, FMCDM) approach can be more suitable to explain how customers make decisions to select the best software for organizations.

While crisp data are inadequate to model the real life situations in MCDM, we use linguistic variables to specifically describe the degrees of a criterion. In order to facilitate the making of subjective assessment by the DM using fuzzy numbers, two sets of linguistic terms are used for assessing criteria weights and performance rating on each qualitative criterion respectively. A linguistic variable is a variable which apply words or sentences in a natural or artificial language to describe its degree of value, and we use this kind of expression to compare each

criteria by linguistic variables in a fuzzy environment as "extremely important", "very important", "important", "very unimportant", and "extremely unimportant" with respect to a fuzzy five level scale. The triangular fuzzy numbers are used to represent the approximate value, denoted as (a_1, a_2, a_3) , where $1 \leq a_1 \leq a_2 \leq a_3 \leq 9$. Table 1 defines the linguistic terms and shows the membership functions of these linguistic terms.

Table1. Linguistic Terms and Membership Function

Linguistic Term	Extremely Unimportant	Very Unimportant	Important	Very Important	Extremely Important
Membership Function	(1,1,3)	(1,3,5)	(3,5,7)	(5,7,9)	(7,9,9)

3. Fuzzy TOPSIS Approach with Subjective and Objective Weights

A software selection problem can be formulated as multiple criteria decision making problem in which alternatives are the software to be selected and criteria are the software attributes under consideration. Among many famous MCDM methods, Technique for Order Performance by Similarity to Ideal Solution (TOPSIS) was first developed by Hwang and Yoon [8]. It bases upon the concept that the chosen alternative should have the shortest distance from the Positive Ideal Solution (PIS), i.e., the solution that maximizes the benefit criteria and minimizes the cost criteria; and the farthest from the Negative Ideal Solution (NIS), i.e., the solution that maximizes the cost criteria and minimizes the benefit criteria.

3.1 Subjective and Objective Weighting

This paper formulates the outsourcing software company evaluation as a MCDM model, and then presents an effective approach by modifying TOPSIS for solving this problem. To ensure that the evaluation result will be affected by the weighting approaches, subjective weighting and objective weighting methods are both utilized in the comparison. The use of subjective weighting is based on decision maker's expertise and judgment, nevertheless the objective weighting is based on mathematical computation. The approach with objective weighting is particularly applicable for situations where reliable subjective weights cannot be obtained [10].

Many objective weighting measures had been proposed by researchers. Shannon's entropy concept [11] is well suited for measuring the relative contrast

intensities of attributes to represent the average intrinsic information transmitted to the DM [12]. The standard deviation (S.D.) method calculates objective weights by $w_i = \sigma_i / \sum_{k=1}^m \sigma_k$ where σ_i is the standard deviation of the performance rating vectors. Among those measures, Shannon's entropy gains most attention. The Shannon entropy is a measure of uncertainty in information formulated in terms of probability theory. Entropy weight is a parameter that describes how much different alternatives approach one another in respect to a certain attribute. The greater the value of the entropy, the smaller the entropy weight, then the smaller the different alternatives in this specific attribute, and the less information the specific attribute provides, and the less important this attribute becomes in decision making process.

The steps of fuzzy TOPSIS algorithm can be expressed as follows:

Step 1: Construct a decision matrix

Assume there m alternatives (software products) $A_i (i=1,2,\dots,m)$ to be evaluated against n selection criteria $C_j (j=1,2,\dots,n)$. Subjective assessments are to be made by DM to determine (a) the weighting vector $W = (w_1, w_2, \dots, w_n)$ and (b) the decision matrix $X = \{x_{ij}, i=1,2,\dots,m; j=1,2,\dots,n\}$, using the linguistic terms given in Table 1. The weighting vector W represents the relative importance of n selection criteria $C_j (j=1,2,\dots,n)$ for the problem. The decision matrix $X = \{x_{ij}, i=1,2,\dots,m; j=1,2,\dots,n\}$ represents the utility ratings of alternative A_i with respect to selection criteria C_j . Given the weighting vector W and decision matrix X , the objective of the problem is to rank all the alternatives by giving each of them an overall utility with respect to all selection criteria. The decision matrix can be expressed as follows [13].

$$X = \begin{matrix} & C_1 & C_2 & \dots & C_n \\ \begin{matrix} A_1 \\ A_2 \\ \dots \\ A_m \end{matrix} & \begin{bmatrix} x_{11} & x_{12} & \dots & x_{1n} \\ x_{21} & x_{22} & \dots & x_{2n} \\ \dots & \dots & \dots & \dots \\ x_{m1} & x_{m2} & \dots & x_{mn} \end{bmatrix} \end{matrix} \quad (1)$$

$$W = [w_1 \quad w_2 \quad \dots \quad w_n]$$

Step 2: In this step, we both use subjective weighting method and entropy-based objective weighting method. (a). Subjective: Determine the DM's weights for each criterion

$$\tilde{W}_j = \frac{1}{n} \sum_{j=1}^n \tilde{w}_j^e, \quad j=1,2,\dots,n \quad (2-1)$$

(b). Objective: In order to determine objective weights by the entropy measure, the decision matrix needs to be normalized for each criterion $C_j (j=1,2,\dots,n)$ to obtain the projection value of each criterion P_{ij} :

$$p_{ij} = \frac{x_{ij}}{\sum_{i=1}^m x_{ij}} \quad (2-2)$$

After normalized the decision matrix, we can calculate the entropy values e_j as

$$e_j = -k \sum_{j=1}^n p_{ij} \ln p_{ij} \quad (2-3)$$

k is a constant, let $k = (\ln(m))^{-1}$

The degree of divergence d_i of the intrinsic information of each criterion $C_j (j=1,2,\dots,n)$ may be calculated as

$$d_j = 1 - e_j \quad (2-4)$$

The value d_j represents the inherent contrast intensity of c_j . The higher the d_j is, the more important the criterion c_j is for the problem. The objective weight for each criterion can be obtained.

$$W_j = \frac{d_j}{\sum_{k=1}^n d_k} \quad (2-5)$$

Step 3: Calculate the aggregate weights for each criterion W_j as follows:

$$\tilde{X}_{ij} = \frac{1}{n} \sum_{e=1}^n \tilde{x}_{ij}^e, \quad i=1,2,\dots,m \quad (3)$$

Step 4: Obtain the decision matrix to identify the j th criteria with respect to i th alternative.

$$\tilde{R} = \begin{bmatrix} \tilde{r}_{ij} \\ \dots \\ \dots \end{bmatrix}_{m \times n} \quad (4)$$

Step 5: Normalize the decision matrix in order to make each criterion value is limited between 0 and 1, so that each criterion is comparable. The initial data with respect to each criterion will be normalized by dividing the sum of criterion values. For fuzzy data

denoted by triangular fuzzy number as (a_{ij}, b_{ij}, c_{ij}) , the normalized values for benefit-related criteria and cost-related criteria are calculated as follows.

$$\tilde{r}_{ij} = \left(\frac{a_{ij}}{c_j^+}, \frac{b_{ij}}{c_j^+}, \frac{c_{ij}}{c_j^+} \right), \quad j \in B \quad (5-1)$$

$$\tilde{r}_{ij} = \left(\frac{a_j^-}{c_{ij}^-}, \frac{a_j^-}{b_{ij}^-}, \frac{a_j^-}{a_{ij}^-} \right), \quad j \in C \quad (5-2)$$

$$c_j^+ = \max_i c_{ij} \quad \text{if } j \in B$$

$$a_j^- = \min_i a_{ij} \quad \text{if } j \in C$$

Step 6: Calculate the overall performance evaluation for each alternative by multiplying the aggregate weights for each normalized criterion.

$$\tilde{V} = \left[\tilde{v}_{ij} \right]_{m \times n}, \quad i = 1, 2, \dots, m \quad j = 1, 2, \dots, n \quad (6-1)$$

$$\tilde{v}_{ij} = \tilde{r}_{ij} \otimes \tilde{w}_j \quad (6-2)$$

Step 7: Determine the positive ideal solution A^+ and the negative ideal solution A^- . Sort the weighted normalized values for each criterion in descending order.

$$A^+ = \left(\tilde{v}_1^+, \tilde{v}_2^+, \dots, \tilde{v}_n^+ \right) \quad (7-1)$$

$$A^- = \left(\tilde{v}_1^-, \tilde{v}_2^-, \dots, \tilde{v}_n^- \right) \quad (7-2)$$

Step 8: Calculate the distance from the positive ideal solution and the negative ideal solution for each alternative. According to Bojadziev and Bojadziev [9], the distance between two triangular fuzzy numbers $A_1 = (a_1, b_1, c_1)$ and $A_2 = (a_2, b_2, c_2)$ is calculated as

$$d(A_1, A_2) = \sqrt{\frac{1}{3} [(a_1 - a_2)^2 + (b_1 - b_2)^2 + (c_1 - c_2)^2]} \quad (8)$$

$$d_i^+ = \sum_{j=1}^k d \left(\tilde{v}_{ij}, \tilde{v}_j^+ \right), \quad i = 1, 2, \dots, m$$

$$d_i^- = \sum_{j=1}^k d \left(\tilde{v}_{ij}, \tilde{v}_j^- \right), \quad i = 1, 2, \dots, m$$

Step 9: Calculate the closeness coefficient (CC). And rank each CC of each alternative in descending order. The alternative with the highest CC value will be the best choice.

$$CC_i = \frac{d_i^-}{d_i^+ + d_i^-}, \quad i = 1, 2, \dots, m \quad (9)$$

4. Problem Solution

In this section, an example is provided. A computer center in a university desires to select a new information system in order to improve work productivity. After preliminary screening, four alternatives A_1, A_2, A_3, A_4 have remained in the candidate list. Four experts, D_1, D_2, D_3, D_4 , form a committee to act as decision makers. There are four criteria need to be considered:

- (1) Costs of hardware/software investment (C_1)
- (2) Contribution to organization performance (C_2)
- (3) Effort to transform from current system (C_3)
- (4) Outsourcing software developer reliability (C_4)

The proposed fuzzy TOPSIS method is applied to solve this problem, and the computational procedure is summarized as follows:

Step 1: The research uses the linguistic variables developed by Chen & Hwang [7] as Fig.1 and Table 2 for each criterion. We use triangular fuzzy number to express importance of each criterion. The linguistic terms range from “very low” to “very high”. The specific term “Very” is utilized to stress the degree of each criterion.

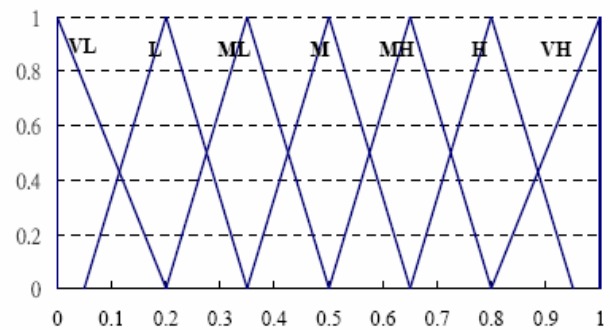


Fig. 1. The fuzzy linguistic variables for each criterion

Table 2. Fuzzy linguistic terms and their correspondent fuzzy numbers for each criterion

Importance	Abbreviation	Fuzzy Number
Very Low	VL	(0, 0, 0.2)
Low	L	(0.05, 0.2, 0.35)
Medium Low	ML	(0.2, 0.35, 0.5)
Medium	M	(0.35, 0.5, 0.65)
Medium High	MH	(0.5, 0.65, 0.8)
High	H	(0.65, 0.8, 0.95)
Very High	VH	(0.8, 1, 1)

Step 2: Same as step 1. Decide each alternative’s linguistic term and fuzzy number in Fig. 2. and Table 3.

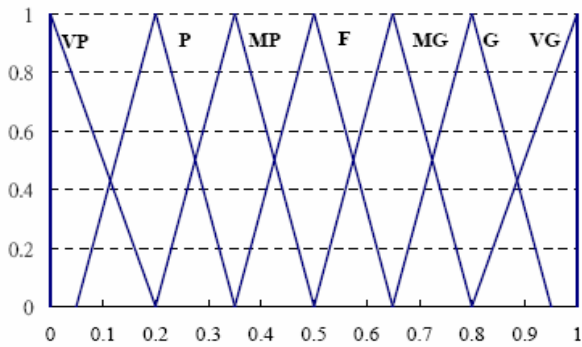


Fig. 2. The fuzzy linguistic variables for each alternative

Table 3. Fuzzy linguistic terms and their correspondent fuzzy numbers for each alternative

Performance	Abbreviation	Fuzzy Number
Very Poor	VP	(0, 0, 0.2)
Poor	P	(0.05, 0.2, 0.35)
Medium Poor	MP	(0.2, 0.35, 0.5)
Fair	F	(0.35, 0.5, 0.65)
Medium Good	MG	(0.5, 0.65, 0.8)
Good	G	(0.65, 0.8, 0.95)
Very Good	VG	(0.8, 1, 1)

Step 3: Each DM may rate each criterion's weight with respect to linguistic term. That means, each expert may apply his/her own expertise to judge how important a criterion is. The result is shown in Table 4.

Table 4. Each criterion weight in linguistic term

	D1	D2	D3	D4
C1	H	VH	H	VH
C2	VH	ML	VH	H
C3	MH	VH	M	H
C4	ML	L	MH	VH

Step 4: According to (2), the aggregated fuzzy rating and fuzzy weight of each criterion are shown in Table 5.

Table 5. Each criterion weight fuzzy number

Criterion	Fuzzy Number
C1	(0.725,0.900,0.975)
C2	(0.613,0.788,0.863)
C3	(0.575,0.738,0.850)
C4	(0.388,0.550,0.663)

Besides subjective weights, we apply entropy method to calculate objective weight for each criterion. According to Table 4, we derive its crisp projection for each criterion.

Table 6. Each criterion projection value

	D1	D2	D3	D4
C1	0.83	0.92	0.78	0.94
C2	0.92	0.35	0.93	0.74
C3	0.62	0.90	0.45	0.72
C4	0.31	0.24	0.68	0.93

According to (2-3)(2-4)(2-5), we calculate e_j , d_j and w_j respectively. The Table 7 shows the entropy-based weighting result.

Table 7. Entropy-based weights

	e_j	d_j	w_j
C1	0.3486	0.6513	0.3917
C2	0.5298	0.4702	0.2828
C3	0.7120	0.2880	0.1730
C4	0.7468	0.2531	0.1522

From above table, we may clearly identify that C1 is the most important criterion during the MCDM process.

Step 5: Each DM rates each alternative with respect to each criterion. Since the judgments would be partially depends on personal preference, DM's recommendation is applied fuzzy linguistic terms. By applying Eqs.(3), (4), and (5), the original DM rating table and normalized fuzzy decision matrix are shown in Table 8 and Table 9 respectively.

Table 8. The initial DM rating

C	C1				C2				C3				C4			
DM	D1	D2	D3	D4	D1	D2	D3	D4	D1	D2	D3	D4	D1	D2	D3	D4
A1	F	G	G	F	MG	P	F	MG	G	VG	F	G	F	F	MP	F
A2	P	F	P	F	VP	P	F	P	G	F	G	G	P	F	VG	F
A3	G	F	G	MG	F	G	G	F	VG	VG	G	F	F	G	G	F
A4	VG	G	G	MG	G	F	G	G	G	G	MG	G	G	VG	F	F

Table 9. Normalized decision matrix

	C1	C2	C3	C4
A1	(0.541,0.703,0.865)	(0.400,0.571,0.743)	(0.671,0.849,0.973)	(0.385,0.569,0.754)
A2	(0.216,0.378,0.541)	(0.129,0.257,0.443)	(0.630,0.795,0.959)	(0.477,0.677,0.815)
A3	(0.500,0.662,0.824)	(0.571,0.743,0.914)	(0.712,0.904,0.986)	(0.615,0.800,0.985)
A4	(0.703,0.878,1.000)	(0.657,0.829,1.000)	(0.671,0.836,1.000)	(0.662,0.862,1.000)

Step 6: According to (6), calculating each alternative's overall rating and is shown in Table 10.

Table 10. Alternative overall rating

	C1	C2	C3	C4
A1	(0.392,0.632,0.843)	(0.245,0.450,0.641)	(0.386,0.626,0.827)	(0.149,0.313,0.499)
A2	(0.157,0.341,0.527)	(0.079,0.203,0.382)	(0.362,0.586,0.815)	(0.185,0.372,0.540)

A3 (0.363,0.596,0.804) (0.350,0.585,0.789) (0.410,0.667,0.838) (0.238,0.440,0.652)
 A4 (0.509,0.791,0.975) (0.403,0.655,0.863) (0.386,0.616,0.850) (0.256,0.474,0.663)

Step 7: According to Eqs.(7) and (8), we calculate each alternative's PIS and NIS. Then calculate the closeness coefficient (CC) and determine the best alternative. The result is shown in Table 11.

Table 11. Closeness coefficient table

Alternative	d^+	d^-	CC	Rating
A1	2.619	2.676	0.505	3
A2	3.138	2.119	0.403	4
A3	2.350	2.963	0.558	2
A4	2.135	3.224	0.602	1

As the result from Table 11, the order of rating among those alternatives is $A_4 \succ A_3 \succ A_1 \succ A_2$, The best alternative would be A_4 .

5. Conclusion

The main purpose of this paper is to develop a fuzzy TOPSIS method to select information systems appropriately for an organization from available alternatives. The problem has been described as a multi-criteria decision making method under uncertainty, prompting the need for the method to handle imprecise judgments from decision makers. A new fuzzy modification of the TOPSIS method based on subjective weights and objective weights is applied to calculate weights of criteria and overall rating scores of the alternatives. While crisp data are inadequate to model the real life situations in MCDM, we modified available procedures in the TOPSIS technique when decision makers use linguistic variables. With our method, we provide decision makers more information to make more subtle decisions.

References:

[1] T. C. Chu and Y. C. Lin, A fuzzy TOPSIS method for robot selection, *International Journal of Manufacturing Technology*, 21, 2003, pp.284-290.
 [2] Q. Changwen and H. You, A method of heat assessment using multiple attribute decision making, *Frontiers in Education Conferees*, Vol.3, November 1998, pp.992-997.
 [3] G. F. Roycs and R. C. Bastos, Fuzzy MCDM in election prediction, *IEEE International*

Conference on Systems, Mon. and Cybernetics, Vol.5, Oct. 2001, pp.3258-3263.
 [4] Chung-Hsing Yeh, Hepu Deng, An Algorithm for Fuzzy Multi-Criteria Decision Making, *IEEE International Conference on Intelligent Processing Systems*, 1997, pp.1564-1568.
 [5] T. J. Stewart, A critical survey on the status of multiple criteria decision making: theory and practice, *OMEGA*, Vol.20, No.516, 1992, pp.569-586.
 [6] R. E. Bellman and L. A. Zadeh: Decision-Making in a Fuzzy Environment, *Management Science*, Vol.17, 1970, pp.141-164.
 [7] S. J. Chen and C. L. Hwang, *Fuzzy Multiple Attribute Decision Making*, (Lecture Notes in Economics and Mathematical System Series 375). Springer-Verlag. New York, 1992.
 [8] C. L. Hwang and K. S. Yoon, *Multiple attribute decision making: methods and applications*, Berlin, Germany: Springer-Verlag, 1981.
 [9] G. Bojadziev and M. Bojadziev, Fuzzy sets, fuzzy logic, applications, *Advances in Fuzzy Systems - Applications and Theory*, Vol.5, World Scientific, 1995.
 [10] H. Deng, C. H. Yeh and R. J. Willis, Inter-company comparison using modified TOPSIS with objective weights, *Computers & Operations Research*, Vol.27, 2000, pp.963-973.
 [11] C. E. Shannon and W. Weaver, *The Mathematical Theory of Communication*, Urbana: The University of Illinois Press, 1947.
 [12] M. Zeleny, *Multiple criteria decision making*, New York: Springer, 1996.
 [13] C. T. Chen, Extensions of the TOPSIS for group decision-making under fuzzy environment, *Fuzzy Sets and Systems*, Vol.114, 2000, pp.1-9.