Flexible Jacket Matrices for Cooperative Multi-Agent Network

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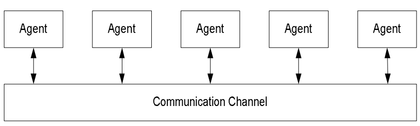
Abstract: - Currently, due to the growing needs in Communications of Multi-Agent Network, DFT and DCT orthogonal transform which is used in communication systems with a fixed size of 2^p (p is a prime) and 2^n respectively do not meet the requirements of future service. We propose the cocyclic Jacket matrices, mathematically let $A = (a_{kl})$ be a matrix, if $A^{-1} = (a_{kl}^{-1})^T$, then the matrix A is a Jacket matrix, which has a flexible matrix size construction with Kronecker construction method and simple element inverse. it is very important in the communication of the multi-agent network because it can provide the agents different data rate and data length.

Key-Words: - DFT , DCT , Kronecker product, cocyclic Jacket matrices , cooperative multi-agent systems.

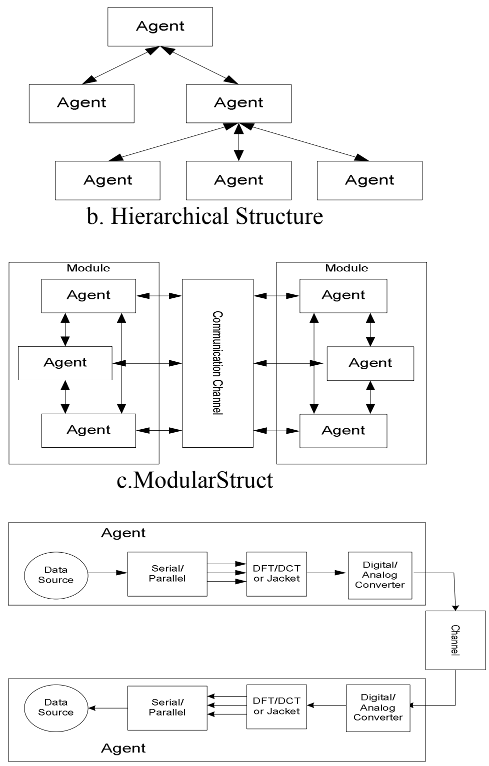
1 Introduction

Recently, Multi-Agent system (MAS) has received considerable attention [1-5]. In generally, MAS consists of lots of autonomous agents that need to communicate and share information with each other to make a right decision automatically. In multi-agent systems, groups of agents must coordinate effectively in order to solve problems, allocate tasks across a distributed organization, collectively distribute knowledge and information, and achieve collective goals. The organizational structure of a multi-agent system dictates the interactions among the agents, and can play a significant role in the overall performance of a society of agents.The performance of the Multi-Agent system also depends on the performance of the communication systems between the agents. In this paper, we would like to propose Jacket matrices which are important in the communication systems, and prove the flexiability of Jacket matrices.

2 Structure of the Multi-Agent Network



a. Flat Structure



d. Communication between two agents

Fig. 1. Structure of the multi-agent network

Multi-agent network is a distributed system, and the coordination is achieved by communication, different network structure such as flat structure, hierarchical structure, modular structure are shown in Fig1.(a,b,c), and communication between two agents is shown in Fig1.(d). It is obviously that, the data length transmitted between two agents depends on the size of the DFT/DCT orthogonal transform [6-8]. We proposed Jacket transform with simple element inverse and flexible size which is always better than the DFT/DCT which has a fixed size.

3 Element-Wise Inverse Jacket Matrices

Let a square matrix $[J]_{m\times m} = [J_{ij}]_{m\times m}$. If its inverse matrix is obtained simply by an element-wise inverse, i.e., like $[J]^{-1}_{m\times m} = \frac{1}{C}[1/J_{ij}]^T{}_{m\times m}$, for $1 \le i, j \le m$, where C is a nonzero constant, then we call matrix $[J]_{N\times N}$ a Jacket matrix [13-18], such as

$$[J]_m = \begin{bmatrix} j_{0,0} & j_{0,1} & \cdots & j_{0,m-1} \\ j_{1,0} & j_{1,1} & \cdots & j_{1,m-1} \\ \vdots & \vdots & & \vdots \\ j_{m-1,0} & j_{m-1,1} & \cdots & j_{m-1,m-1} \end{bmatrix},$$

and its inverse is

$$[J]^{-1}_m = \frac{1}{C}\begin{bmatrix} 1/j_{0,0} & 1/j_{0,1} & \cdots & 1/j_{0,m-1} \\ 1/j_{1,0} & 1/j_{1,1} & \cdots & 1/j_{1,m-1} \\ \vdots & \vdots & & \vdots \\ 1/j_{m-1,0} & 1/j_{m-1,1} & \cdots & 1/j_{m-1,m-1} \end{bmatrix}^T,$$

where C is the normalized value for this matrix, and T is the transpose.

4 Fast Cocyclic Jacket Matrices With Any Size

We now define a nth vector over $GF(p)$ as $\vec{i} = (i_0, i_1, \cdots, i_{n-1}) \in V_p^n$, where $0 \le i_k \le p-1$, $0 \le k \le n-1$, and i is the decimal index expressed by $i = \sum_{k=0}^{n-1} i_k p^{n-1-k}$, where $0 \le i \le p^n - 1$. Using the same definition we can get $\vec{j} \in V_p^n$, the operation "$\circ$" means elements multiply and then modular p, mathematically, it can be shown as following,

$$\vec{i} \circ \vec{j} = (i_0, i_1, \cdots, i_{n-1}) \circ (j_0, j_1, \cdots, j_{n-1}) \quad (1)$$
$$= \langle i_0 \times j_0 \rangle_p + \langle i_1 \times j_1 \rangle_p + \cdots + \langle i_{n-1} \times j_{n-1} \rangle_p .$$

We can use the index mapping to construct cocyclic Jacket matrices of order p^n as the following theorem.

Theorem 1. Let $J_p = [\omega^{i_0 \circ j_0}]$ $0 \le i_0, j_0 \le p-1$ be a matrix of order p, where $\omega = \exp(2\pi\sqrt{-1}/p)$, and $i_0 \circ j_0 = \langle i_0 \times j_0 \rangle_p$, then the matrix of order $N = p^n$ given by

$$J_{p^n} = [\omega^{\vec{i} \circ \vec{j}}] \qquad (0 \le i, j \le N-1) \quad (2)$$

$$= \underbrace{J_p \otimes J_p \otimes \cdots \otimes J_p}_{n} . \quad (3)$$

is a cocyclic Jacket matrix, and the symbol "$\otimes$" represents the Kronecker product.
Its factorization is expressed a

$$J_{p^n} = A^1_{p^n} A^2_{p^n} A^3_{p^n} \cdots A^n_{p^n} , \quad (4)$$

where $A^i_{p^n} = I_{p^{i-1}} \otimes J_p \otimes I_{p^{n-i}} \quad (1 \le i \le n)$, (5)

and I_N is the identify matrix.

Proof: Before proving, we first introduce a proposition which is much helpful during the proof.

Proposition 1. If a matrix can be written as the following formula

$$[J]_{p_1^{e_1} p_2^{e_2} \cdots p_s^{e_s}} = [J]_{p_1^{e_1}} \otimes [J]_{p_2^{e_2}} \otimes \cdots \otimes [J]_{p_s^{e_s}} , \quad (6)$$

where $[J]_{p_i^{e_i}} = [J]_{p_i} \otimes [J]_{p_i} \otimes \cdots \otimes [J]_{p_i}$, $[J]_{p_i}$ is a cocyclic Jacket matrix for p_i is a prime number, $i = 1, 2, \cdots, s.$ Then the matrix $[J]_{p_1^{e_1} p_2^{e_2} \cdots p_s^{e_s}}$ is also a cocyclic Jacket matrix. The proof of this proposition can be found in[9].

The matrix of Eq.(6) is a cocyclic Jacket matrix, since

$$\sum_{j \in V_p^n} \omega^{\vec{i} \circ \vec{j} - \vec{i}' \circ \vec{j}} = \begin{cases} N & (\vec{i} = \vec{i}') \\ 0 & (\vec{i} \ne \vec{i}') \end{cases} . \quad (7)$$

Comparing with (3) and (4), obviously it is a Jacket matrix. Then we will prove it is also a cocyclic Jacket matrix, as we know $J_p = [\omega^{i_0 \circ j_0}]$, with the operation $i_0 \bullet j_0 = \langle i_0 + j_0 \rangle_p$, which means elements add and then modular p. $C = \{1, \omega, \cdots, \omega^{p-1}\}$ with traditional multiplication, the rows and columns are indexed by the elements of G under the increasing order $\{0,1,\cdots,(p-1)\}$. Let $i_0, i_0', j_0, j_0' \in G$, base on the form (1), we have

$$\varphi(i_0,0)=\varphi(0,i_0)=\varphi(0,0)=1 \quad . \tag{8}$$

$$\varphi(i_0,j_0)=\omega^{\langle i_0\times j_0\rangle_p} . \tag{9}$$

$$\varphi(i_0,j_0\bullet j_0')=\omega^{\langle i_0(j_0+j_0')\rangle_p} . \tag{10}$$

$$\varphi(i_0,j_0)\varphi(i_0',j_0')=\omega^{\langle i_0\times j_0+i_0'\times j_0'\rangle_p} \quad . \tag{11}$$

Therefore, for any $g,h,k\in G$, we have

$$\varphi(g,h)\varphi(g\bullet h,k)=\omega^{\langle gh\rangle_p}\times\omega^{\langle (g+h)k\rangle_p} = \omega^{\langle gh+(g+h)k\rangle_p} , \tag{12}$$

also,

$$\varphi(g,h\bullet k)\varphi(h,k)=\omega^{\langle g(h+k)\rangle_p}\times\omega^{\langle hk\rangle_p} = \omega^{\langle g(h+k)+hk\rangle_p} , \tag{13}$$

since $\langle gh+(g+h)k\rangle_p=\langle g(h+k)+hk\rangle_p$, so we have $\varphi(g,h)\varphi(g\bullet h,k)=\varphi(g,h\bullet k)\varphi(h,k)$.

Therefore, J_p is a cocyclic matrix. On the other hand we have

$$\begin{aligned} J_{p^n} &= [\omega^{\vec{i}\circ\vec{j}}]=[\omega^{(i_0 i_1\cdots i_n)\circ(j_0 j_1\cdots j_n)}] \\ &= \omega^{i_0\circ j_0+i_1\circ j_1+\cdots+i_{n-1}\circ j_{n-1}} . \\ &= \underbrace{J_p\otimes J_p\otimes\cdots\otimes J_p}_{n} . \end{aligned} \tag{14}$$

where $0\le i_k, j_k\le p-1$. From (18) we can get

$$J_{p^{n+1}}=J_{p^n}\otimes J_p . \tag{15}$$

Using Proposition 1 It's easy to see that J_{p^n} is also a cocyclic matrix. So J_{p^n} is a cocyclic Jacket matrix, the proof of Eq.(6) and Eq.(7) is completed. Then we will introduce a similar way given in [11] to prove Eq.(8) and Eq.(9).We use induction on the index n, when $n=1$, it is clearly true:

$$\begin{aligned} J_{p^1} &= A_{p^1}^1 = I_{p^0}\otimes J_p\otimes I_{p^0} \\ &= [I]_{1\times 1}\otimes J_p\otimes [I]_{1\times 1} \\ &= J_p . \end{aligned} \tag{16}$$

Assume the hypothesis is true for n, and then show it must therefore hold for $n+1$. For $1\le i\le n$ we obtain the following from the hypothesis:

$$\begin{aligned} A_{p^{n+1}}^i &= I_{p^{i-1}}\otimes J_p\otimes I_{p^{n+1-i}} = I_{p^{i-1}}\otimes J_p\otimes(I_{p^{n-i}}\otimes I_p) \\ &= (I_{p^{i-1}}\otimes J_p\otimes I_{p^{n-i}})\otimes I_p = A_{p^n}^i\otimes I_p , \end{aligned}$$

and

$$A_{p^{n+1}}^{n+1}=I_{p^n}\otimes J_p\otimes I_{p^0}=I_{p^n}\otimes J_p \quad . \tag{17}$$

We can write,

$$\begin{aligned} J_{p^{n+1}} &= A_{p^{n+1}}^1 A_{p^{n+1}}^2 A_{p^{n+1}}^3\cdots A_{p^{n+1}}^n A_{p^{n+1}}^{n+1} \\ &= (A_{p^n}^1\otimes I_p)(A_{p^n}^2\otimes I_p)(A_{p^n}^3\otimes I_p) \\ &\quad \cdots(A_{p^n}^n\otimes I_p)(I_{p^n}\otimes J_p) && (18) \\ &= (A_{p^n}^1 A_{p^n}^2 A_{p^n}^3\cdots A_{p^n}^n I_{p^n})\otimes J_p && (19) \\ &= J_{p^n}\otimes J_p . && (20) \end{aligned}$$

In this process, (23) coming from (22) based on the formula $(A\otimes B)(C\otimes D)=(AC)\otimes(BD)$ which can be found in [6],[7],[8],[10],[11]. Compare with the Eq.(19), we know the Eq.(24) is right, so we have finished the proof of Theorem 1.

In [12], it also presents a construction of cocyclic Jacket matrices based on q -ary first-order Reed Muller codes which have some similarity with the above approach. This can be applied to design fast decoding algorithm for $RM_p(1,m)$, where p is a prime number. We can make an example for the 9-by-9 cocyclic Jacket matrix. Let $p=3$, $n=1$, $m=2$, the finite field of $q=p^n=3$ elements $F_3=\{0,1,2\}$. $RM_3(1,2)$ is as following

$$RM_3(1,2)=\begin{pmatrix} 0&0&0&0&0&0&0&0&0\\ 0&1&2&0&1&2&0&1&2\\ 0&2&1&0&2&1&0&2&1\\ 0&0&0&1&1&1&2&2&2\\ 0&1&2&1&2&0&2&0&1\\ 0&2&1&1&0&2&2&1&0\\ 0&0&0&2&2&2&1&1&1\\ 0&1&2&2&0&1&1&2&0\\ 0&2&1&2&1&0&1&0&2 \end{pmatrix} . \tag{21}$$

Let ω be a primitive third root of unity. The cocyclic Jacket matrix obtained by

$$J_{3^2}=\begin{bmatrix} \omega^0&\omega^0&\omega^0&\omega^0&\omega^0&\omega^0&\omega^0&\omega^0&\omega^0\\ \omega^0&\omega^1&\omega^2&\omega^0&\omega^1&\omega^2&\omega^0&\omega^1&\omega^2\\ \omega^0&\omega^2&\omega^1&\omega^0&\omega^2&\omega^1&\omega^0&\omega^2&\omega^1\\ \omega^0&\omega^0&\omega^0&\omega^1&\omega^1&\omega^1&\omega^2&\omega^2&\omega^2\\ \omega^0&\omega^1&\omega^2&\omega^1&\omega^2&\omega^0&\omega^2&\omega^0&\omega^1\\ \omega^0&\omega^2&\omega^1&\omega^1&\omega^0&\omega^2&\omega^2&\omega^1&\omega^0\\ \omega^0&\omega^0&\omega^0&\omega^2&\omega^2&\omega^2&\omega^1&\omega^1&\omega^1\\ \omega^0&\omega^1&\omega^2&\omega^2&\omega^0&\omega^1&\omega^1&\omega^2&\omega^0\\ \omega^0&\omega^2&\omega^1&\omega^2&\omega^1&\omega^0&\omega^1&\omega^0&\omega^2 \end{bmatrix} .$$

(22)It's the same with the matrix presented in the Table 1.

From the formula (7) – (9), we can factorize this matrix, and the fast cocyclic Jacket transform can be written as

$$J_{3^2} = [J]_3 \otimes [J]_3 = A^1_{3^2} A^2_{3^2} \ . \quad (23)$$

where

$$A^1_{3^2} = J_3 \otimes I_3 \quad , \quad A^2_{3^2} = I_3 \otimes J_3 \quad . \quad (24)$$

thus we have

$$J_{3^2} = A^1_{3^2} A^2_{3^2} = \begin{bmatrix} \omega^0 & 0 & 0 & \omega^0 & 0 & 0 & \omega^0 & 0 & 0 \\ 0 & \omega^0 & 0 & 0 & \omega^0 & 0 & 0 & \omega^0 & 0 \\ 0 & 0 & \omega^0 & 0 & 0 & \omega^0 & 0 & 0 & \omega^0 \\ \omega^0 & 0 & 0 & \omega^1 & 0 & 0 & \omega^2 & 0 & 0 \\ 0 & \omega^0 & 0 & 0 & \omega^1 & 0 & 0 & \omega^2 & 0 \\ 0 & 0 & \omega^0 & 0 & 0 & \omega^1 & 0 & 0 & \omega^2 \\ \omega^0 & 0 & 0 & \omega^2 & 0 & 0 & \omega^1 & 0 & 0 \\ 0 & \omega^0 & 0 & 0 & \omega^2 & 0 & 0 & \omega^1 & 0 \\ 0 & 0 & \omega^0 & 0 & 0 & \omega^2 & 0 & 0 & \omega^1 \end{bmatrix} \cdot \begin{bmatrix} \omega^0 & \omega^0 & \omega^0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \omega^0 & \omega^1 & \omega^2 & 0 & 0 & 0 & 0 & 0 & 0 \\ \omega^0 & \omega^2 & \omega^1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \omega^0 & \omega^0 & \omega^0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \omega^0 & \omega^1 & \omega^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & \omega^0 & \omega^2 & \omega^1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \omega^0 & \omega^0 & \omega^0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \omega^0 & \omega^1 & \omega^2 \\ 0 & 0 & 0 & 0 & 0 & 0 & \omega^0 & \omega^2 & \omega^1 \end{bmatrix} . \quad (25)$$

From these sparse matrices, we can easily draw the signal flow graph as show in Fig .2(a).

As we know arbitrary number can be decomposed by prime number. We will show that the higher order cocyclic Jacket matrices can be constructed by the lower prime order cocyclic Jacket matrices. Theorem 1 just introduce a special case: $N = p^n$, now we will present a more general method which satisfied all the size.

Theorem 2. If the cocyclic Jacket matrices of order $N = p_1 p_2 \cdots p_n$, then

$$J_N = J_{p_1} \otimes J_{p_2} \otimes \cdots \otimes J_{p_n} \quad . \quad (26)$$

where $J_{p_m} = [\omega_m^{\vec{i} \circ \vec{j}}]$, $\omega_m = e^{i(\frac{2\pi}{p_m})}$, p_m is a prime number, and $1 \le m \le n$. Its factorization is expressed as

$$J_N = A^1_{p_1} A^2_{p_2} \cdots A^n_{p_n} \ . \quad (27)$$

Where

$$A^m_{p_m} = \underbrace{I_{p_1} \otimes I_{p_2} \otimes \cdots I_{p_{m-1}}}_{m-1} \otimes J_{p_m} \otimes \underbrace{I_{p_{m+1}} \otimes I_{p_{m+2}} \otimes \cdots \otimes I_{p_n}}_{n-m} \ . \quad (28)$$

Proof: We have already proved the J_{p_m} is cocyclic Jacket matrix in Theorem 1. Base on the Proposition 1, J_N is also cocyclic Jacket matrix. As for Eq.(32) and (33) the proof is similar with that of Eq. (8) and (9), we only need to change $I_{p_1} \otimes I_{p_2} \otimes \cdots I_{p_{m-1}}$ with $I_{p^{i-1}}$, and $I_{p_{m+1}} \otimes I_{p_{m+2}} \otimes \cdots \otimes I_{p_n}$ with $I_{p^{n-i}}$, then carry out proof by the same method. Specially, when $p_1 = p_2 = \cdots = p_n$, so $N = p^n$ and we can get

$$J_{p^n} = \prod_{i=1}^{n} (I_{p^{i-1}} \otimes J_p \otimes I_{p^{n-i}}) \ . \quad (29)$$

It's the special case described by Theorem 1.

Example 1. The fast cocyclic Jacket transform with the order $N = 12 = 2 \times 2 \times 3$, so $p_1 = p_2 = 2, p_3 = 3$. Using Theorem 2 it is easy to know

$$J_{12} = A^1_2 A^2_2 A^3_3 \ . \quad (30)$$

$$= (J_2 \otimes I_2 \otimes I_3)(I_2 \otimes J_2 \otimes I_3)(I_2 \otimes I_2 \otimes J_3)$$

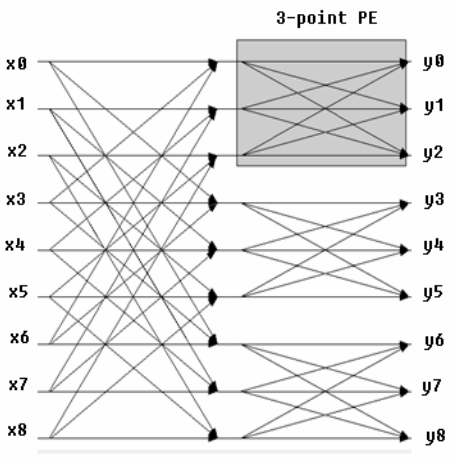
$$= \begin{bmatrix} A & 0 & A & 0 \\ 0 & A & 0 & A \\ A & 0 & A_1 & 0 \\ 0 & A & 0 & A_1 \end{bmatrix} \begin{bmatrix} A & A & 0 & 0 \\ A & A_1 & 0 & 0 \\ 0 & 0 & A & A \\ 0 & 0 & A & A_1 \end{bmatrix} \begin{bmatrix} B & 0 & 0 & 0 \\ 0 & B & 0 & 0 \\ 0 & 0 & B & 0 \\ 0 & 0 & 0 & B \end{bmatrix}$$

where

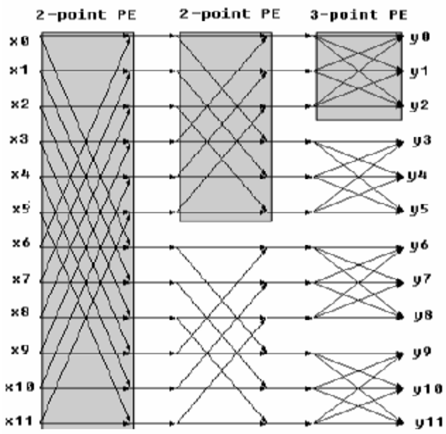
$$A = \begin{bmatrix} \alpha^0 & 0 & 0 \\ 0 & \alpha^0 & 0 \\ 0 & 0 & \alpha^0 \end{bmatrix}, A_1 = \begin{bmatrix} \alpha^1 & 0 & 0 \\ 0 & \alpha^1 & 0 \\ 0 & 0 & \alpha^1 \end{bmatrix}, B = \begin{bmatrix} \beta^0 & \beta^0 & \beta^0 \\ \beta^0 & \beta^1 & \beta^2 \\ \beta^0 & \beta^2 & \beta^1 \end{bmatrix}$$

.

The signal flow graph of fast algorithm is as shown in Fig.2(b)



(a) order-9



(b) order-12

Fig. 2 Fast cocyclic Jacket transform signal flow

For clearly, some construction approaches for cocyclic Jacket matrices j_i $(i=1,2,\cdots,20)$ are presented in the Table 2, in the table, the second column is the decomposition approaches for numbers, and the third column is the construction approaches for cocyclic Jacket matrices. Clearly, all this kind of matrices can be constructed using lower prime order matrices j_p, where p is a prime number.

Table 2. Decompositions of Numbers and The Cocyclic Jacket Matrices

	Decompositions	Cocyclic Jacket matrices
1	$1=1$	$J_1=1$
2	$2=2$	$J_2=J_2$
3	$3=3$	$J_3=J_3$
4	$4=2^2$	$J_4=J_2^{\otimes 2}$
5	$5=5$	$J_5=J_5$
6	$6=2\times 3$	$J_6=J_2\otimes J_3$
7	$7=7$	$J_7=J_7$
8	$8=2^3$	$J_8=J_2^{\otimes 3}$
9	$9=3^2$	$J_9=J_3^{\otimes 2}$
10	$10=2\times 5$	$J_{10}=J_2\otimes J_5$
11	$11=11$	$J_{11}=J_{11}$
12	$12=2^2\times 3$	$J_{12}=J_2^{\otimes 2}\otimes J_3$
13	$13=13$	$J_{13}=J_{13}$
14	$14=2\times 7$	$J_{14}=J_2\otimes J_7$
15	$15=3\times 5$	$J_{15}=J_3\otimes J_5$
16	$16=2^4$	$J_{16}=J_2^{\otimes 4}$
17	$17=17$	$J_{17}=J_{17}$
18	$18=2\times 3^2$	$J_{18}=J_3^{\otimes 2}\otimes J_2$
19	$19=19$	$J_{19}=J_{19}$
20	$20=2^2\times 5$	$J_{20}=J_2^{\otimes 2}\otimes J_5$

The Multi-Agent Systems in Figure.3 is one example of Systems which need different communication data rates and data lengths to transmit different kinds of information. As can be seen in Theorem 1,2 and Table.2 that Cocyclic Jacket Matrices with arbitrary size can be decomposed into smaller Jacket Matrices with size of prime numbers. That means we can provide arbitrary data rate and length for the communication in the Cooperative Multi-Agent Systems.

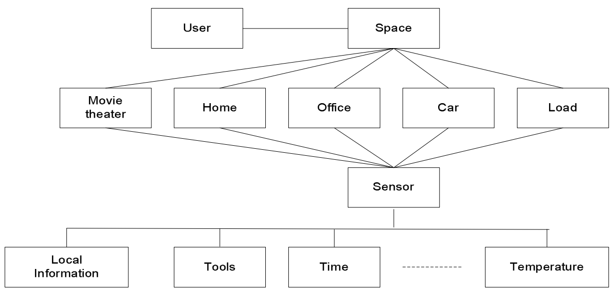
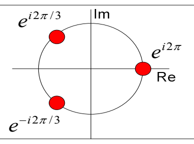
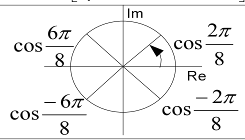
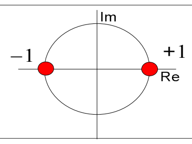
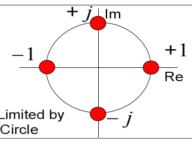


Fig. 3. Example of Multi-Agent Moble Communication System

Table 3: Compare with DFT, DCT, Hadamard and Jacket matrices

	DFT (1822) J. Fourier	DCT(1974) N. Ahmed, K.R. Rao,et.	Hadamard (1893) J. Hadamard	Jacket(1989)* Moon Ho Lee
Formula	$X(n)=\sum_{k=0}^{N-1}x(k)w^{nk}$ $n=0,1\ldots N-1, w=e^{-j2\pi/N}$	$[C_N]_{m,n}=\sqrt{\frac{2}{N}}k_m\cos\frac{m(n+\frac{1}{2})\pi}{N}$ $m,n=0,1,\ldots,N-1$	$[H]_2=\begin{bmatrix}1&1\\1&-1\end{bmatrix}$ $[H]_n=[H]_{n/2}\otimes[H]_2$	$[J]_4=\begin{bmatrix}1&1&1&1\\1&-j&j&-1\\1&j&-j&1\\1&-1&-1&1\end{bmatrix}$ $[J]_n=[J]_{n/2}\otimes[H]_2 \quad n>4$
Forward	$N=3$ $F_3=\begin{bmatrix}1&1&1\\1&w&w^2\\1&w^2&w\end{bmatrix}$ $w=e^{-i2\pi/3}$	$N=4$ $[C]_4=\begin{bmatrix}\frac{1}{\sqrt2}&\frac{1}{\sqrt2}&\frac{1}{\sqrt2}&\frac{1}{\sqrt2}\\C_8^1&C_8^3&C_8^5&C_8^7\\C_8^2&C_8^6&C_8^6&C_8^2\\C_8^3&C_8^7&C_8^1&C_8^5\end{bmatrix}$ $C_8^i=\cos\frac{i\pi}{8}$	$(-1)^{\oplus_{k=0}^{n-1}i_kj_k}$ $[H]_4=\begin{bmatrix}1&1&1&1\\1&-1&1&-1\\1&1&-1&-1\\1&-1&-1&1\end{bmatrix}$	$(-1)^{\oplus_{k=0}^{n-1}i_kj_k}w^{(i_{n-2}\oplus i_{n-1})(j_{n-2}\oplus j_{n-1})}$ $[J]_4=\begin{bmatrix}1&1&1&1\\1&-w&w&-1\\1&w&-w&-1\\1&-1&-1&1\end{bmatrix}$ w=1: Hadamard w=2: Center Weighted Hadamard
Inverse	Element-Wise Inverse $F_3^{-1}=\frac{1}{3}\begin{bmatrix}1&1&1\\1&w^{-1}&w^{-2}\\1&w^{-2}&w^{-1}\end{bmatrix}$	Non-Element-Wise Inverse $[C]_4^{-1}=\frac{1}{2}\begin{bmatrix}\frac{1}{\sqrt2}&C_8^1&C_8^2&C_8^3\\\frac{1}{\sqrt2}&C_8^3&C_8^6&C_8^7\\\frac{1}{\sqrt2}&C_8^5&C_8^6&C_8^1\\\frac{1}{\sqrt2}&C_8^7&C_8^2&C_8^5\end{bmatrix}$	Element-Wise Inverse $[H]_4^{-1}=\frac{1}{4}\begin{bmatrix}1&1&1&1\\1&-1&1&-1\\1&1&-1&-1\\1&-1&-1&1\end{bmatrix}$	Element-Wise Inverse or Block-Wise Inverse $[J]_4^{-1}=\frac{1}{4}\begin{bmatrix}1&1&1&1\\1&-1/w&1/w&-1\\1&1/w&-1/w&-1\\1&-1&-1&1\end{bmatrix}$
Circle				
Kronecker	$DFT\otimes DFT\neq DFT$	$DCT\otimes DCT\neq DCT$	$H\otimes H=Hadamard$	$J\otimes J=Jacket$
Size	$2^n, p=prime$	2^n	$2^n, 4n$	$Arbitrary$

It is shown in the Table.3 above that the size of the DFT/DCT are 2^p (p is a prime) and 2^n respectively can not be expanded by the kronecker product, the size of them is fixed. However, the Jacket matrices can have arbitrary matrix size with Kronecker product. In communication systems including the communication in the Multi-Agent cooperative systems, the data rate and data length depend on the size of DFT/DCT orthogonal transform, the proposed Jacket transform has a better flexibility and simpler inverse method than the DFT/DCT orthogonal transform.

5 Conclusion

The size of the DFT is 2^p (p is a prime) and the size of the DCT is 2^n respectively, and can not constructed by the Kronecker product. But the size of Jacket matrices can be arbitrary with Kronecker product of identity matrices and successively lower order Jacket Matrices and that is very useful in different data length for multi-agent network. The inverse of the Jacket matrix is from element wise inverse which can make the receiver of the agent low complexity. The contribution of this work lies in providing a new kind of Jacket matrices, we can provide flexibility compared with DFT and DCT which is important in the communicate channel in the Cooperative multi-agent network.

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