VLSI Implementation for Interpolation-based Matrix Inversion for MIMO-OFDM Receivers

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Abstract: - Matrix inversion block is one of the computationally intensive blocks in communications system. To reduce the size of hardware implementation is a challenging problem. In MIMO-OFDM system, the receiver which uses zero-forcing (ZF) or minimum mean square error (MMSE) scheme should calculate the inverse matrices of all sub-carriers. This paper proposes VLSI architecture for interpolation-based matrix inversion. For the result, more than 70% area was reduced by adopting interpolation-based scheme in 4×4 matrix inversion processing of IEEE 802.11n receiver.

Key-Words: - Matrix Inversion, Interpolation, ZF, MMSE

1 Introduction

In the current decade Multi-Input Multi-Output (MIMO) systems have generated tremendous research interest as they offer high reliability and high data rate [1] Due to significant performance gains provides by MIMO, it is being widely adopted in most of the current and next generation wireless communication systems. [2]
In MIMO receiver, the preprocessing module which computes inverse matrix or QR decomposition is computationally very heavy block. For example, in 802.11n receiver which adopt MMSE detector, preprocessing block is over 30% size of total receiver. In this paper, matrix inversion architecture which adopt interpolation based scheme [3] is proposed to reduce the preprocessing block size.

2 MIMO Systems

Consider M transmitter and N receiver antennas for MIMO system. Lets $s=[s_1 \ldots s_M]$ denoted the M ×1 vector of transmit symbols, then the corresponding $x=[x_1 \ldots x_N]^T$ receive signal vector is given by

$$x = Hs + n \qquad (1)$$

when $n=[n_1 \ldots n_N]^T$ represents the white Gaussian noise of variance σ_n^2 observed at the N receiver antennas while the average transmit power of each antenna is normalized to one.
In MIMO communications system utilizing multi antennas there are mainly four sorts of detectors which are zero forcing (ZF) detector, minimum mean square error (MMSE) detector, Bell laboratories layered space time (BLAST) detector and maximum likelihood (ML) detector. In cases of ZF detector and MMSE detector, inverse matrix should be calculated. On the other hand, in cases of BLAST detector and ML detector needs QR factorization of channel matrix. In this section, ZF and MMSE detection scheme is shortly described because proposed architecture can be utilized for these detection schemes.

2.1 Zero Forcing (ZF)

In ZF scheme, the interference introduced from the matrix channel is null out by multiplying directly channel inversion matrix directly. The equation of ZF is described in (2).

$$\hat{x} = (H^*H)^{-1}H^* y \qquad (2)$$

2.2 Minimum Mean-Squared Error (MMSE)

In ZF scheme, the interference is null out without considering the noise. Thus the noise can be boost up. To solve this MMSE minimize the mean squared-error. The equation of MMSE detector is

$$\hat{x} = (H^*H + \sigma_n^2/\sigma_x^2)^{-1}H^* y \qquad (3)$$

where σ_n^2 and σ_x^2 are noise variance and signal variance respectively.

3 Interpolation based Matrix Inversion

The basic idea of interpolation based matrix inversion is presented in [3]. In this section the interpolation-based matrix inversion algorighm is simply described. When the MIMO channel is flat-fading channel, the transfer function of the matrix-valued channel impulse response is given by

$$H(e^{j2\pi\theta}) = \sum_{l=0}^{L-1} H_l e^{-j2\pi l\theta}, \quad 0 \le \theta \le 1 \qquad (4)$$

where H_l is the matrix-valued taps. The channel model (4) is derived from the assumption of L resolvable paths. This means the entries of channel matrix are polynomial equation of L-1 order.

When we calculate channel inversion matrix, the channel matrix of subcarriers cannot be directly interpolated from neighbors because the channel matrix entries are not polynomial. For example, in case of 2×2 matrix, if the channel is

$$H(s_k) = \begin{bmatrix} h_{11}(s_k) & h_{12}(s_k) \\ h_{21}(s_k) & h_{22}(s_k) \end{bmatrix} \qquad (5)$$

then, the inverse matrix H^{-1} is

$$H^{-1}(s_k) = \frac{1}{h_{11}(s_k)h_{22}(s_k) - h_{12}(s_k)h_{21}(s_k)} \times \begin{bmatrix} h_{11}(s_k) & h_{12}(s_k) \\ h_{21}(s_k) & h_{22}(s_k) \end{bmatrix} \qquad (6)$$

We can observe this H^{-1} matrix can be calculated with adjoint matrix and determinent.

For general cases, we can write

$$H^{-1}(s_k) = \frac{\operatorname{adj} H(s_k)}{\det H(s_k)} \qquad (7)$$

adjoint matrix of H can be calculated by finding

$$[\operatorname{adj}(H^{-1})]_{ji} = (-1)^{i+j} \det(H^{-1})_{ij} \qquad (8)$$

where H_{ij} is the i^{th} row and j^{th} column entry of matrix H. Also determinant can be calculated by following.

$$\det(H^{-1}) = \sum_{j=1}^{M} (-1)^{i+j} a_{ij} \det(H^{-1})_{ij} \qquad (9)$$

$$\textit{for an arbitrary } i^{th} \textit{ row}$$

It is now obvious that H^{-1} can be calculated interpolating adj(H^{-1}) and det(H^{-1}) distinctly because adj(H^{-1}) and det(H^{-1}) are all polynomial equation.

4 Implementation

4.1 Top Block

The interpolation based matrix inversion block was implemented for target system which is IEEE802.11n and the utilized transmitter and receiver antennas are assumed four. In case of 802.11n system, the number of sub-carriers is 64 which contain 48 data carrying tones.

Fig. 1 shows the diagram of matrix inversion for 64 matrices corresponding 64 sub-carriers. The channel matrix of each sub-carrier is 4×4 matrix because of the numbers of transmitter and receiver antennas. The forty-seven matrices are interpolated from seventeen matrices. This means we can save matrix inversion operation for forty-seven sub-carriers. Also, seventeen sub-carriers which used for interpolating other sub-carriers means this architecture can cover five delay tabs in flat fading channel. For the case of the number of delay tabs is five and the channel matrix is 4×4 matrix, the order of determinant is 16 and the order of entries of adjoint matrix is 12 and we needs at least seventeen matrix inversion operations for seventeen sub-carriers.

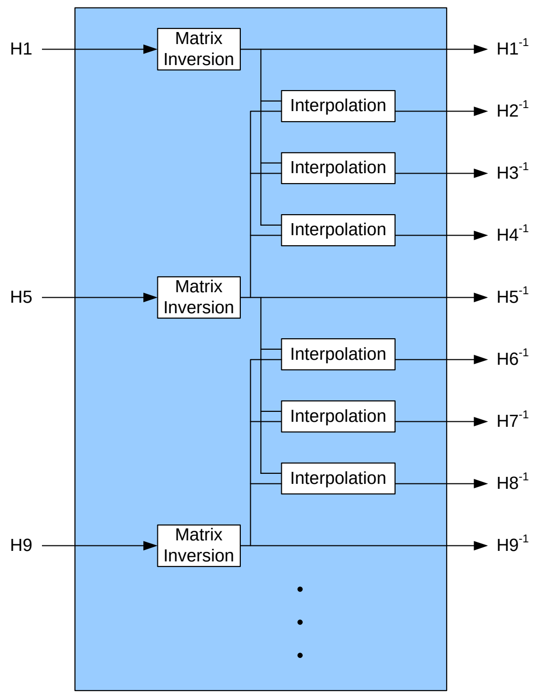

Fig. 1. Top block

1) *Interpolate* $H(s_k),\ k \in \mathrm{P}$, *to obtain* $H(s_k)$ *for* $k \in B_{M-1}$

2) *Compute* R_2 2*-minors for each* $k \in B_2$. *Set* $m \leftarrow 2$

3*a*) *If* $m = M - 1$, *goto step* 5). *Otherwise,*
interpolate the m*-minors to all tones* $k \in B_{m+1}$

b) *Compute* R_{m+1} $(m+1)$*-minors for each* $k \in B_{m+1}$
using Laplace expansion with the m*-minors obtained in 3a)*
and the $H(s_k)$ *obtained in 1). Set* $m \leftarrow m+1$ *and goto step 3a)*

4) *Interpolate* $\mathrm{adj}H(s_k),\ k \in B_{M-1}$, *computed in* 3), *to obtain* $\mathrm{adj}H(s_k)$ *for* $k \in D$

5) *Continue with steps* 2) *and* 3)

$$
\begin{cases}
\mathrm{P} : \mathrm{P} \subset \{0,1,\ldots,N-1\}, \text{ selected interpolation subcarrier} \\
\mathrm{D} : \mathrm{D} \subset \{0,1,\ldots,N-1\}, \text{ data carrying subcarrier} \\
M : \text{number of antennas} \\
N : \text{number of subcarriers} \\
B_m : B_m \subseteq D \text{ base point used for interpolation } m\text{-minor} \\
\quad (B_M = ML+1) \text{ where } L \text{ is the number of delay tab}
\end{cases}
$$

Fig. 2. Interpolation-based Matrix Inversion Algorithm

4.2 Matrix Inversion Block

Fig. 3 shows Matrix Inversion block. For 4×4 matrix inversion, we need to calculate determinants of twelve 2×2 sub-matrices, sixteen 3×3 sub-matrices and one 4×4 sub-matrix. The Matrix Inversion block is composed twelve ADJ3 blocks and sixteen ADJ4 blocks and one DET4 block. To calculate an adjoint matrix of 4×4 matrix, ADJ3 blocks and ADJ4 blocks are utilized and to calculate a determinant of 4×4 matrix, DET4 block is utilized. The determinant and adjoint matrix is obtained recursively from (8) and (9) and the algorithm is described in Fig. 2. One Matrix inversion block is composed of 76 multipliers and 47 adders.

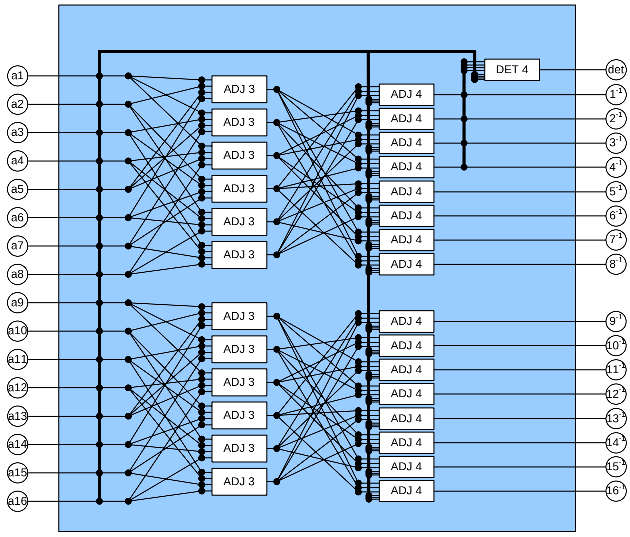

Fig. 3. Matrix Inversion Block

4.3 Interpolation Block

Interpolation block is for calculation of inverse matrix by interpolation. This block takes determinant and sixteen entries of adjoint matrix of neighbor sub-carriers. For general cases, to interpolate 4×4 matrix, interpolation block needs 34 multipliers and 17 adders. However, in special cases which interpolate one matrix from two matrices we need only seventeen adders and seventeen one-bit shift operations. Also, for the case of obtaining three matrices by interpolating two neighbor matrices, interpolation block needs fifty-one adders and seventeen two-bit shift operations for two 4×4 matrix interpolations and seventeen adders and seventeen one-bit shift operation for one 4×4 matrix interpolation. General case of interpolation block is showed in Fig 5. In Fig 5, first

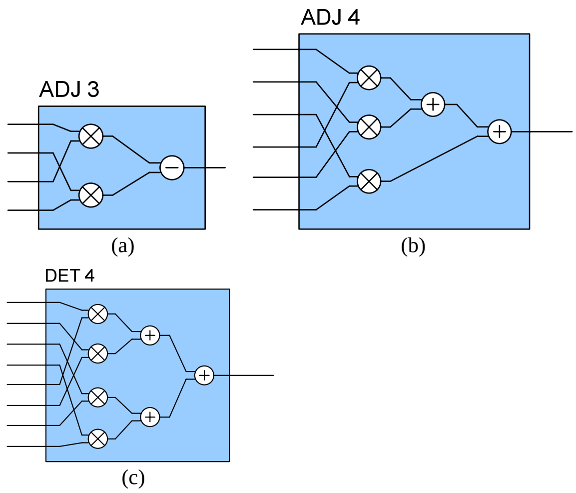

Fig. 4. (a) ADJ3 block (B) ADJ4 block (C) DET4 block

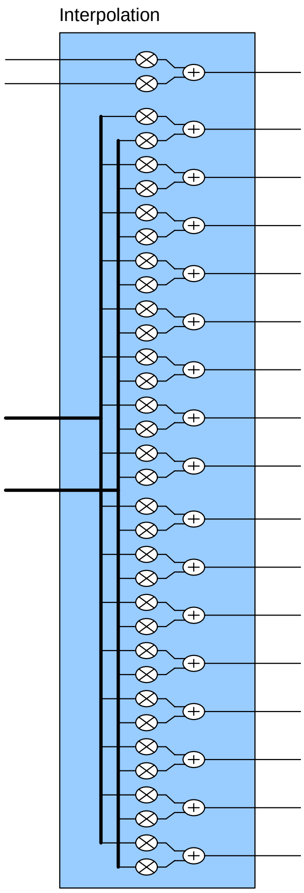

Fig.5. Interpolation Block for general cases

two multipliers and one adder are utilized to interpolate determinant and the others are used to interpolate sixteen entries of adjoint matrix. In this block, linear interpolation is used for its simplicity.

5 Result

In case of 4×4 matrix inversion of 64 sub-carriers MIMO-OFDM system. The complexity is can be reduced by adopting interpolation-based scheme and the result is in Table 1. The origin number of multipliers and adders was 4864 and 3008 respectively. When interpolation-based matrix inversion scheme was adopted necessary multipliers and adders reduced to 1292 and 906. This shows by adopting interpolation-based scheme, we can save 73.4% multiplier and 69.9% adder.

Table 1. Complexity of Preprocessing Block

	Multiplier	Adder
Brute-Force Matrix Inversion	4864	3008
Interpolation-based Matrix Inversion	1292	906

6 Conclusion

In digital communications system, preprocessing block is one of the most computationally intensive blocks. By adopting interpolation-based matrix inversion scheme, we reduced the size of preprocessing block more than 70%.

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