# The Lateral Restraint Network Model on the Processing of Image

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*Abstract:* -- Based on the ocular neural system, the Lateral Restraint Network Model on the processing of the image is presented in paper. The mathematic model of the network is also presented. In the network model, in addition to the negative feedback of the ocular neural cell, an ocular neural cell is only excited by the corresponding light spot in the field of vision and inhibited by the other light spot. The network has its advantages on the processing of image. It can detect the edge, sharp-angle, end-point, flexure, crotch and junction in the image. The learning algorithm of the network model is discussed and the convergence of the algorithm is proven too. In the end of the paper, some examples are introduced about the application of the network.

*Key- Words:* -- Biophysics, Optical Networking, Network Model, Image Processing, Intelligent Systems

## 1 Introduction

The human ocular system has its advantages on the processing of image. The biophysics study discovers that an ocular neural cell is only excited by the corresponding light spot in the field of vision; meanwhile it is inhibited by the other light spot. The inhibit function is also called lateral restraint [1][2]. Based on the human ocular system model discussed above, the lateral restraint network model on the processing of image (Fig1) is presented in the paper.

Let $V_{ij}$ represent the (i, j) cell in retinal. The $X_{ij}$ is its power. The image matrix $M(n,m)$ excites the network in such a way that the (i, j) light spot $I_{ij}$ excites $V_{ij}$ by mass action at a constant rate $B_{ij}$ meanwhile $V_{ij}$ spontaneously decay at a constant rate $A_{ij}$. In order to solve the saturation problem, lateral input $I_{kl}$ inhibit $V_{ij}$ at a rate $D_{klij}$ in a feed forward competitive anatomy (illustrated in Fig 1).

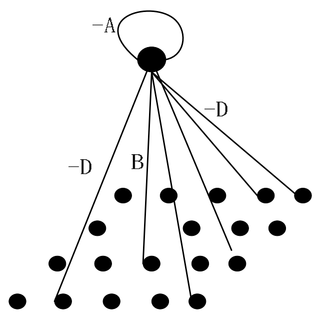


Fig1: The Lateral Restraint Network Model

The equation (1) is the mathematic mode of the network :

$$\frac{dx_{ij}}{dt} = -A_{ij}x_{ij} + B_{ij}I_{ij} - \sum_{kl \neq ij} I_{kl}D_{klij} \tag{1}$$

Where

$$A_{ij} > 0$$
$$1 > B_{ij} > 0$$

$D_{klij}$ represents the intensity which the lateral input $I_{kl}$ inhibits the $V_{ij}$. The biophysics study discovers that the inhibition action only occurs among

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the near cells. The inhibit action becomes weak by the exponential function when the length becomes long between the input light spot and the cell [3][4].

Based on the equation (1) we can get:

$$X_{ij}(t)=\frac{B_{ij}I_{ij}-\sum_{j\neq i} I_{kl}D_{klij}}{A_{ij}}e^{-A_{ij}t}+\frac{B_{ij}I_{ij}-\sum_{kl\neq ij} I_{kl}D_{klij}}{A_{ij}} \qquad (2)$$

If t=0 then

$$X_{ij}(0)=0 \qquad (3)$$

If $t\to\infty$ then

$$\lim_{t\to\infty} X_{ij}(t)=\frac{B_{ij}I_{ij}-\sum_{kl\neq ij} I_{kl}D_{klij}}{A_{ij}} \qquad (4)$$

When the M=0, i.e. there is no input, the formulate (1) becomes:

$$\frac{dx_{ij}}{dt}=-A_{ij}x_{ij} \qquad (5)$$

So

$$X_{ij}(t)=-\frac{1}{A_{ij}}e^{-A_{ij}t} \qquad (6)$$

According to the equation (2) to (6), the curve of function $X_{ij}(t)$ is shown in Fig 2.

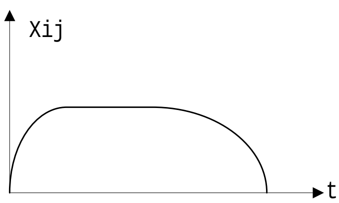


Fig2: The curve line of $X_{ij}(t)$

The curve in Figure 2 accords with the human ocular activity. The ocular neural cell becomes saturated gradually when the light excites the cell. The ascended period is very short. The descend line means the duration of vision [4].

It is on the basis of the saturated value for the human to recognize the image [4], so we mainly discuss the saturated value $y_{ij}$ of the cell $V_{ij}$ in the following section:

$$y_{ij}=X_{ij}(\infty)=\frac{B_{ij}I_{ij}-\sum_{kl\neq ij} I_{kl}D_{klij}}{A_{ij}} \qquad (7)$$

## 2 Learning Algorithm

### 2.1 Learning Algorithm

The output of the network $y_{ij}$ is not the copy of the input image. It expresses the character of an image. In order to detect the important information of an image by the network, we must regulate the parameter $A_{ij}, B_{ij}, D_{klij}$. In this section, we mainly discuss how to regulate the parameters $A_{ij}, B_{ij}, D_{klij}$. The total error of the network is defined as:

$$E=\frac{1}{2}\sum_{i=1;j=1}^{n,m}(y'_{ij}-y_{ij})^2 \qquad (8)$$

The $y'_{ij}$ is the expected output and the $y_{ij}$ is really output of the (i, j) cell.

The parameters are corrected according to the total error. The regulation of the parameters should be in direct ratio to the negative differential coefficient of the E with respect to the parameters, i.e.:

$$\Delta A_{ij}\propto-\frac{\partial E}{\partial A_{ij}} \qquad (9)$$

$$\Delta B_{ij}\propto-\frac{\partial E}{\partial B_{ij}} \qquad (10)$$

$$\Delta D_{klij}\propto-\frac{\partial E}{\partial D_{klij}} \qquad (11)$$

So

$$\Delta A_{ij}=-\theta_{1ij}*\frac{\partial E}{\partial A_{ij}}=-\theta_{1ij}*\frac{\partial E}{\partial y_{ij}}*\frac{\partial y_{ij}}{\partial A_{ij}} \qquad (12)$$

$$\Delta B_{ij}=-\theta_{2ij}*\frac{\partial E}{\partial B_{ij}}=-\theta_{2ij}*\frac{\partial E}{\partial y_{ij}}*\frac{\partial y_{ij}}{\partial B_{ij}} \qquad (13)$$

$$\Delta D_{klij}=-\theta_{3ij}*\frac{\partial E}{\partial D_{klij}}=-\theta_{3ij}*\frac{\partial E}{\partial y_{ij}}*\frac{\partial y_{ij}}{\partial D_{klij}} \qquad (14)$$

Where $\theta_{1ij},\theta_{2ij},\theta_{3ij}$ are the learn coefficients:

$$0 < \theta_{1ij}, \theta_{2ij}, \theta_{3ij} < 1$$

By equation (8) we know:

$$\frac{\partial E}{\partial y_{ij}} = -(y_{ij}^{'} - y_{ij}) \quad (15)$$

$$\frac{\partial y_{ij}}{\partial A_{ij}} = \frac{B_{ij} I_{ij} - \sum_{kl \neq ij} I_{kl} D_{klij}}{A_{ij}^2} = \frac{A_{ij} * y_{ij}}{A_{ij}^2} = \frac{y_{ij}}{A_{ij}} \quad (16)$$

$$\frac{\partial y_{ij}}{\partial B_{ij}} = \frac{I_{ij}}{A_{ij}} \quad (17)$$

$$\frac{\partial y_{ij}}{\partial D_{klij}} = -\frac{I_{kl}}{A_{ij}} \quad (18)$$

So:

$$A_{ij}(t+1) = A_{ij}(t) + \Delta A$$
$$= A_{ij}(t) - \theta_{1ij} * (y_{ij}^{'} - y_{ij}) * \frac{y_{ij}}{A_{ij}(t)} \quad (19)$$

$$B_{ij}(t+1) = B_{ij}(t) + \Delta B$$
$$= B_{ij}(t) - (y_{ij}^{'} - y_{ij}) * \frac{I_{ij}}{A_{ij}(t)} \quad (20)$$

$$D_{klij}(t+1) = D_{klij}(t) + \Delta D$$
$$= D_{klij}(t) + (y_{ij}^{'} - y_{ij}) * \frac{I_{kl}}{A_{ij}} \quad (21)$$

In order to accelerate the learning process, the network can be also modified to a local linked frame, where the (i,j ) input only links to it's near cells in the out layer.

## 2.2 Convergence of the Learn Algorithm

As to the equation (19) (20) (21), the following formulation (22) is a general format to regulate the parameter J.

$$J_{k+1} = J_k - \eta \frac{dE}{dJ_k} \quad (22)$$

The learn iterative equation denoted by the Algorithm:

$$J_{k+1} \in S(J_k, -\frac{dE}{dJ_k})$$

$S(J_k, -\frac{dE}{dJ_k})$ presents the start point and direction of linear search.

The S is defined as:

$$S(G,d) = \{J_{k+1} : J_{k+1} = J_k + \alpha \frac{dE}{dJ_k}$$
$$E(J_{k+1}) = \min_{0<\alpha<1} E(J_k + \alpha \frac{dE}{dJ_k})\}$$

If $\frac{dE}{dJ_k} \neq 0$

Then S is closed and G is continued.

The solution is defined as

$$\{J_k : \frac{dE}{dJ_k} = 0\}$$

For $\frac{dE}{dJ_k} \neq 0$

So $E(J_k)$ is a descend function.

By the global convergence theory [5], if the $\{J_k\}$ is bounded then it will have limited point and every limit is a solution.

So the algorithm is convergent.

# 3 Analysis and Application

The network has different function on the processing of image if parameters have different character.

Suppose the lateral restraint intensity is symmetrical and

$$v_{00} = \frac{B_{00}}{A_{00}} \quad (23)$$

$$v_{10} = -\frac{D_{10}}{A_{00}} = -\frac{D_{01}}{A_{00}} = -\frac{D_{-1,0}}{A_{00}} = -\frac{D_{0,-1}}{A_{00}} \quad (24)$$

$$v_{11} = -\frac{D_{11}}{A_{00}} = -\frac{D_{-1,1}}{A_{00}} = -\frac{D_{-1,1}}{A_{00}} = -\frac{D_{1,-1}}{A_{00}} \quad (25)$$

According to equation (7):

$$
\begin{aligned}
y_{00} &= v_{00} I(0,0) \\
&+v_{10}[I(\delta,0)+I(-\delta,0)+I(0,-\delta) \\
&\quad +I(0,\delta)] \\
&+v_{11}[I(\delta,\delta)+I(-\delta,\delta)+I(\delta,-\delta) \\
&\quad +I(-\delta,-\delta)] \\
&+v_{20}[I(2\delta,0)+I(-2\delta,0)+I(0,-2\delta) \\
&\quad +I(0,2\delta)] \\
&+v_{21}[I(2\delta,\delta)+I(-2\delta,\delta) \\
&\quad +I(\delta,-2\delta)+I(-\delta,-2\delta) \\
&\quad +I(-2\delta,-\delta)+I(2\delta,-\delta) \\
&\quad +I(\delta,2\delta)+I(-\delta,2\delta)] \\
&+\ldots
\end{aligned} \tag{26}
$$

The $\delta$ is the length between the adjacent points in the input image.

The Taylor Expanding Equation of $I(\xi,\pi)$ is:

$$I(\pm\delta,0)=I(0,0)\pm I'_{\xi}\delta+\frac{1}{2}I''_{\xi\xi}\delta^2+\ldots \tag{27}$$

$$I(0,\pm\delta)=I(0,0)\pm I'_{\pi}\delta+\frac{1}{2}I''_{\pi\pi}\delta^2+\ldots \tag{28}$$

$$
\begin{aligned}
I(\pm\delta,\delta) &= I(0,0)+(\pm I'_{\xi}+I'_{\pi})\delta \\
&+\frac{1}{2}(I''_{\xi\xi}\pm 2I''_{\xi\pi}+I''_{\pi\pi})\delta^2+\ldots
\end{aligned} \tag{29}
$$

$$
\begin{aligned}
I(\delta,\pm\delta) &= I(0,0)+(I'_{\xi}\pm I'_{\pi})\delta \\
&+\frac{1}{2}(I''_{\xi\xi}\pm 2I''_{\xi\pi}+I''_{\pi\pi})\delta^2+\ldots
\end{aligned} \tag{30}
$$

Substitute the equation (27) – (30) into equation (26):

$$
\begin{aligned}
y(0,0) &= I(0,0)[v_{00}+4v_{10}+4v_{11} \\
&+4v_{20}+8v_{21}+\ldots+n_{ij}v_{ij}] \\
&+I''_{\xi\xi}[v_{10}+2v_{11}+2v_{20}+\ldots + \\
&+\tfrac{1}{2}n_{ij}v_{ij}(i^2+j^2)]\delta^2 \\
&+I''_{\pi\pi}[v_{10}+2v_{11}+2v_{20}+\ldots \\
&+\tfrac{1}{2}n_{ij}v_{ij}(i^2+j^2)]\delta^2 \\
&+\ldots
\end{aligned} \tag{31}
$$

Where $n_{ij}$ is the number of the cells which are equidistant away from the $V_{00}$. The $(i,j)$ is the coordinates of the cell.

Because the lateral restraint intensity is symmetrical, there is not the item which is proportional to the odd order derivative in the equation (31). The network model can not detect the gray scale which transform uniformly.

Suppose

$$A=v_{00}+4v_{10}+4v_{11}+4v_{20}+8v_{21}+\ldots+n_{ij}v$$

$$B=v_{10}+2v_{11}+2v_{20}+\ldots+\tfrac{1}{2}n_{ij}v_{ij}(i^2+j^2)$$

If A=0, approximately the X(0,0) is

$$X(0,0)=B(I''_{\xi\xi}+I''_{\pi\pi})\delta^2$$

The network model can detect the gray scale which transform un-uniformly, etc. the network model can detect edges in the image.

If A≠0, the network can detect the average grey of the input image.

In the case of the cells in the out layer have the thresholds. The weights may be positive or negative, i.e the lateral cells may restrain or excite the center cell. If the thresholds and weights are adjusted suitability then the network can detect the sharp-angle, end-point, flexure, crotch and junction, which is called “Info Point”. The “Info Point” holds the important information of an image [6]. The network is an appropriate way on the processing of image.

The function of the network on the processing of the image is illustrated in the Fig 3.

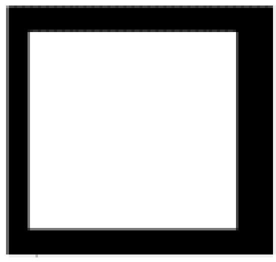

Fig3-a: The original image

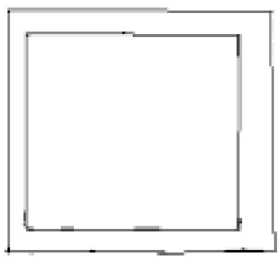

Fig3-b: The network can detect the edge

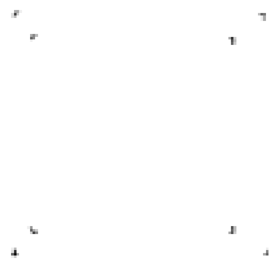

Fig3-c: The network can detect the cross

The network model can be used in the recognizing of human face too. In order to recognize the human face, It is important to detect cantus, corners of the mouth, bridge of a nose [7] [8]. They are all sharp-angle, end-point, flexure, crotch and junction. The traditional image processing methods are simply to detect the edge based on the several operators, such as Sobel operator, Laplacian operator etc.. A character of the face is picked meanwhile the other characters are exterminated too if we only apply one algorithm. In order to improve the performance, sometimes several algorithms is applied in the project, so the method become complex and is easy disturbed by the noise. To apply the network model discussed above in project, the method become simply. The network can detect the different charter by different parameters. The network can learn self so we can find the character what is needed without to search the comply algorithm. The experiment in table illustrates that the advantage of the introduced network.

We match the 100 human faces in the 1000 photos in the database. The result is illustrated in table 1.

Table1: The experiment result

| | Without noise | |
|---|---|---|
| | Time (second) | Mismatch Times |
| Traditional method | 3.1 | 15 |
| The network model | 2 | 10 |
| | Noise | |
| Traditional method | 3.1 | 25 |
| The network model | 2 | 15 |

In the fluorescence chromatogram detector project, the network is used to detect the peak, front-shoulder peak, back-shoulder peak, tail peak and overlap-peak [9]. The peak points indicate the characters of the material which is tested [10]. The example is illustrated in Fig4.

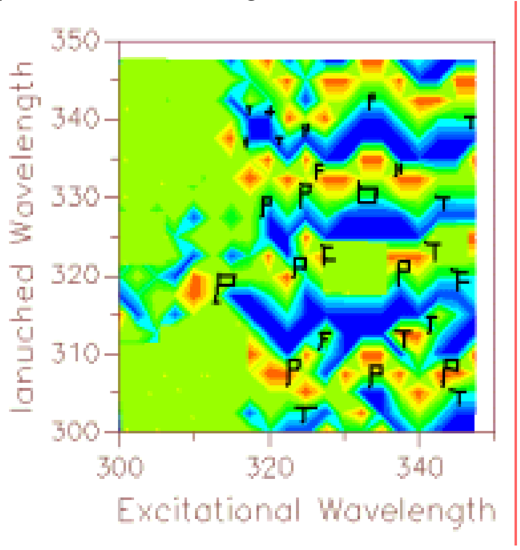


Fig4: The fluorescence chromatogram (P: peak, T: tail peak, F: front shoulder peak, H: back-shoulder peak, O: Over-Lap Peak)

## 4 Conclusion

The Lateral Restraint Network is modeled on the ocular neural system. It has some advantages on the processing of image. It can detect the sharp-angle, end-point, flexure, crotch, junction, the “Info Point” in an image. The network model has been used in the human face recognizing and fluorescence chromatogram detecting projects.

However, there are many problems in the network model is open. It is difficult to detect the several kind of “Info Point”. For example the net can not detect the edge and cross meanwhile.

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