An Efficient Rearrangement of Wavelet Packet Coefficients for Embedded Quad-Tree Image Coding

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Abstract: -In this paper, we propose an efficient method for rearranging the wavelet packet coefficients of an image to form hierarchical trees, by which the well known SPIHT algorithm can be applied. For images with textures, the high frequency wavelet coefficients are likely to become significant after several code passes of SPIHT, which degrades substantially the coding performance. As a result, the high frequency wavelet coefficients representing most of the high detail content of images need to be decomposed into wavelet packet coefficients for a further exploitation. The proposed rearrangement scheme has been applied to the highest frequency wavelet packet coefficients of images. Experimental results show that the performance of SPIHT can be improved, especially for fingerprint images.

Key-Words: -Wavelet, Wavelet packet, Embedded coding, SPIHT, Fingerprint.

1 Introduction

With the rapid growth of modern communications and computer technologies, image compression continues to be in great demand. The discrete cosine transform (DCT) based Joint Photographic Experts Group (JPEG) coder shows good compression results at moderate to high rates of bits per pixel [1]. Wavelet transform, which has drawn a lot of attention to the image compression applications, provides many desirable properties and therefore has been adopted by JPEG2000 [2].By wavelet transform, an image can be decomposed into subbands with orientation selectivity. Shapiro first introduced the so-called self similarity of wavelet transform and developed an efficient embedded zerotree wavelet (EZW) coding algorithm [3]. Its improved version known as set partitioning in hierarchical trees (SPIHT) proposed by Said and Pearlman has been considered a benchmark [4]. Mukherjee and Mitra developed a vector extension of SPIHT called VSPIHT, in which wavelet coefficients are first grouped into small vectors and then coded by SPIHT [5]. Both EZW and SPIHT are quad-tree algorithms, in which the spatially related wavelet coefficients taken from all the subbands of the same orientation are arranged to form hierarchical trees. Whereas quad-tree coders are based on the self similarity of wavelet transform, there are some coders based on block coding in wavelet domain. The basic idea behind the block coding strategy is to exploit the energy clustering of wavelet coefficients within each subband. The AGP algorithm divides regions of high energy coefficients into small blocks by quad-tree partitioning [6]. The SWEET algorithm takes octave-band partitioning to exploit the pyramid structure of wavelet transform [7]. Pearlman et al. developed the set partitioning embedded block (SPECK) algorithm [8], in which, both quad-tree partitioning and octave-band partitioning are utilized. The embedded block coding with optimized truncation (EBCOT) algorithm proposed by Taubman [9] has been adopted by JPEG2000, which is preferable to JPEG. However, the complexity of JPEG2000 is much more intense than JPEG [10].

For images with high detail textures, lots of wavelet coefficients are found significant in the high frequency subbands. Thus, the detailed information of an image represented by the significant wavelet coefficients in the high frequency subbands demands a further decomposition. Whereas wavelet transform only decomposes the low frequency subband in the iterative manner, wavelet packet transform decomposes both the low and high frequency subbands and therefore provides a much larger family of subband decompositions [11]. In this paper, an efficient method for rearranging the wavelet packet coefficients of an image is proposed to construct hierarchical wavelet packet trees, based on which, images can be coded by using an efficient quad-tree based algorithm, e.g. SPIHT.

The remainder of this paper proceeds as follows. In Section 2, wavelet transform, wavelet packet transform and SPIHT are reviewed briefly. Section 3 describes the proposed scheme to construct the wavelet packet trees of an image. Experimental results are presented in Section 4. Conclusion is given in Section 5.

2 Review of Wavelet Based SPIHT

2.1 Wavelet Transform

The wavelet transform (WT) of $S_\ell(n)$, which is a 1-D signal at resolution ℓ, is given by

$$\begin{aligned} S_{\ell+1}(n) &= \sum_k S_\ell(k)\, h(2n-k) \\ D_{\ell+1}(n) &= \sum_k S_\ell(k)\, g(2n-k) \end{aligned} \tag{1}$$

where $S_{\ell+1}(n)$ denotes the approximation at the next coarser resolution $\ell+1$, $D_{\ell+1}(n)$ denotes the detail between resolution ℓ and resolution $\ell+1$, $h(n) = <\phi, \phi_{-1,-n}>$, $g(n) = <\psi, \phi_{-1,-n}>$, $<\cdot,\cdot>$ is an inner product operator, ψ is a valid (mother) wavelet, ϕ is the corresponding scaling function, and $\phi_{-1,-n}(x) = 2^{-1/2}\phi(2^{-1}x-n)$. $S_\ell(n)$ can be exactly reconstructed by using the following inverse wavelet transform.

$$S_\ell(n) = \sum_k S_{\ell+1}(k)\, \tilde{h}(n-2k) + \sum_k D_{\ell+1}(k)\, \tilde{g}(n-2k) \tag{2}$$

where $\tilde{h}(n) = h(-n)$ and $\tilde{g}(n) = g(-n)$.

It is through the tensor product of 1-D WT that 2-D WT can be obtained. Figure 1(a) shows a 3-level 2-D WT. Where HL_ℓ, LH_ℓ and HH_ℓ denote the subbands composed of wavelet coefficients $D_\ell^1(m,n)$, $D_\ell^2(m,n)$ and $D_\ell^3(m,n)$ representing the detail information at resolution ℓ in the horizontal, vertical and diagonal orientations, respectively. LL_3 denotes the approximation at the coarsest resolution 3.

2.2 Wavelet Packet Transform

Wavelet transform is focused on the low frequency decomposition, i.e. only the scaling coefficients are successively decomposed. As a result, wavelet transform may not be suitable for images with large regions of textures. However, both the scaling coefficients and wavelet coefficients can be decomposed, which leads to the so-called wavelet packet transform (WPT). Similarly, 2-D WPT can be obtained by the tensor product of 1-D WPT. Figure 2(a) shows a 3-level 2-D WPT, where the wavelet coefficients: $D_1^3(m,n)$ and $D_2^3(m,n)$ are decomposed further into wavelet packets.

2.3 The SPIHT Algorithm

In wavelet domain, the related wavelet coefficients taken from all the subbands of the same orientation can be arranged to form hierarchical wavelet trees. A wavelet coefficient called parent node at a resolution level has four related wavelet coefficients called children nodes at the next finer resolution level. The root node is at the coarsest resolution level and the leaf nodes are at the finest resolution level. Figure 1(b) shows a wavelet tree in the diagonal orientation. Images are usually composed of homogeneous regions, textures and a small portion of edges, which are typically the low, middle and high frequency components, respectively. Most of the significant wavelet coefficients of homogeneous regions are at the coarser resolution levels, i.e. in the lower frequency subbands, whereas the significant wavelet coefficients representing the noticeable textures and edges are at the finer resolution levels, i.e. in the higher frequency subbands.

The wavelet based SPIHT algorithm has received a lot of attention since its introduction in 1996. In SPIHT, two code passes, namely sorting pass and refinement pass are involved, which can be combined to form a single scan pass. Three code symbols, namely zero tree (ZT), insignificant pixel (IP) and significant pixel (SP) are utilized to code the wavelet trees of an image. These symbols are stored in their respective lists, i.e. list of insignificant sets (LIS), list of insignificant pixels (LIP) and list of significant pixels (LSP). In sorting pass, tree nodes stored in LIS and LIP are evaluated as follows. For a node with magnitude greater than the current threshold, it becomes significant and then is stored in LSP. For an insignificant node with respect to the current threshold, if all the descendants are also insignificant, it is stored in LIS; otherwise, it is stored in LIP. In refinement pass, the significant nodes are refined with one bit per node to update their respective magnitudes. The basic idea behind SPIHT is as follows. If a parent node is insignificant, all the descendants are likely to be insignificant and therefore they can be coded efficiently by a single code symbol: ZT.

3 Proposed Wavelet Packet Trees

In the framework of SPIHT with wavelet trees, the spatially related wavelet coefficients are organized into hierarchical trees. Typically, most of images' energy is concentrated in the lower frequency subbands, i.e. at the upper tree levels, and decreases in magnitude from top to bottom. However, for images with high detail textures, many significant wavelet coefficients are distributed in the high frequency subbands. Thus, instead of wavelet functions, other basis functions might be more suitable for coding the high frequency components of images.

3.1 Image Coding with Both Wavelets and Wavelet Packets

High quality image compression at low bit rates can be achieved by representing the low frequency and high frequency components of images using distinct basis functions. The low frequency components can be well represented by wavelet transform. However, the significant wavelet coefficients of the high frequency components are likely to be in the high frequency subbands. As a result, the high frequency wavelet coefficients of images with textures demand a further exploitation to improve the coding performance.

As a much larger family of basis functions can be obtained by wavelet packet transform, an image might be represented preferably by using wavelet packets in the adaptive manner. Wavelet packet transform offers a great diversity of representations with more basis functions that can be adapted to the content of an image. However, image coding with wavelet packets induces intense complexity. A tree structure was proposed to code images with an arbitrary decomposition through wavelet packet transform [12]. In which, a scheme composed of two steps with four rules was used to solve the so-called parenting conflict problem: if the spatial resolution of a parent node, which is a wavelet packet coefficient in a low frequency subband, is finer than the four children nodes, which are the related wavelet packet coefficients in the next higher frequency subband. In that case, all the children nodes will be moved up in the tree structure and an appropriate ancestor node at the same or coarser resolution level is taken as the newly parent node. This reorganization scheme for wavelet packet coefficients is complicated and moreover the resulting trees may not be quad trees any more, which puts the image compression task to the expense of increasing complexity.

3.2 Proposed Wavelet Packet Trees

In wavelet domain, the construction of hierarchical trees with a parent-child relationship is straightforward due to the pyramid structure of 2-D wavelet transform. To take the contributions of image textures into account, the high frequency wavelet coefficients are deliberately decomposed further into wavelet packets. As the construction of wavelet trees is based on the spatial positions of wavelet coefficients between subbands, a spatial position based quad-tree structure is thus proposed to organize the low frequency wavelet coefficients and the high frequency wavelet packet coefficients into hierarchical trees. Figure 3(a) depicts block diagram of the proposed scheme composed of 1-level wavelet decomposition followed by a rearrangement procedure. In which, a sequence of high frequency wavelet coefficients is first decomposed into two sub-sequences of wavelet packet coefficients through 1-D WT, and these sub-sequences are then multiplexed into a single sequence by interleaving one with the other. Note that pairs of related wavelet packet coefficients corresponding to the same spatial position are grouped together and rearranged in ascending order of spatial frequency.

To code an image with both wavelet and wavelet packet functions, a separable 2-D rearrangement can be obtained by using the tensor product of 1-D rearrangements. More specifically, the high frequency wavelet coefficients are first decomposed into wavelet packet coefficients and then rearranged by using the proposed 1-D rearrangement horizontally followed by vertically, or vice versa. Figure 2 shows an example of 3-level 2-D WPT with a hierarchical wavelet packet tree containing the lowest frequency wavelet coefficient in the diagonal orientation.

3.3 Image Coding with Wavelet Packet Trees

It is through the proposed rearrangement scheme that the low frequency wavelet coefficients and the high frequency wavelet packet coefficients of an image can be organized into hierarchical trees, which are called wavelet packet trees. Thereafter an efficient quad-tree based algorithm can be applied. For example, an efficient image coder can be obtained by applying SPIHT to the wavelet packet trees of an image, which is presented in steps as follows.

<u>Initialization</u>: After wavelet transform, the high frequency wavelet coefficients are decomposed further into wavelet packet coefficients. Determine

the initial threshold T_b $(b=1)$ such that all the low frequency wavelet coefficients and high frequency wavelet packet coefficients are in the range of $[-2T_1, 2T_1]$.

Wavelet packet tree construction: Rearrange wavelet packet coefficients using the proposed 1-D rearrangement scheme (Figure 3(a)) horizontally followed by vertically, or vice versa. These rearranged wavelet packet coefficients together with the low frequency wavelet coefficients can be grouped to form the wavelet packet trees of an image.

Sorting pass: Identify the significant wavelet and wavelet packet coefficients by comparing their respective magnitudes to the current threshold T_b. If a coefficient becomes significant, output the sign bit.

Refinement pass: Output the refinement of the significant coefficients by 1 bit per coefficient.

The sorting pass followed by refinement pass with a sequence of successively smaller thresholds, which can be obtained by $T_{b+1} = T_b / 2$, is performed repeatedly until the bit budget is exhausted. Note that the constructed wavelet packet trees are still quad trees and therefore the proposed rearrangement scheme is efficient as well as suitable for quad-tree image coding. To decompress an image, the embedded code stream is first decoded into wavelet coefficients and wavelet packet coefficients. The high frequency wavelet packet coefficients are then rearranged by using the inverse 1-D rearrangement (shown in Figure 3(b)) vertically followed by horizontally, or vice versa. Finally, the decoded image can be obtained through inverse wavelet packet transform.

4 Experimental Results

In the proposed quad-tree image coding with both wavelets and wavelet packets, the highest frequency wavelet coefficients are presumably decomposed twice to generate wavelet packet coefficients. These highest frequency wavelet packet coefficients are then rearranged by using the proposed rearrangement scheme, and together with the low frequency wavelet coefficients, the wavelet packet trees of an image can be constructed. The coding results of images Sailboat on lake, Fingerprint, Earth and San Francisco are presented in this paper. The aforesaid images are shown in Figure 4. The coding performance of the proposed method is compared to SPIHT with wavelet trees. The compression rate is measured in bits per pixel (bpp). The distortion is measured by the peak signal to noise ratio (PSNR). The computed bit rates and PSNR values are collected to generate the rate distortion curves. Daubechies orthogonal wavelet with 4-tap filters is used. The number of decomposition levels is 5.

For images Sailboat on lake and Fingerprint, Table 1 shows the numbers of significant wavelet and wavelet packet coefficients in the highest frequency subbands, bit-plane by bit-plane from most to least significant. The number of significant coefficients is reduced starting from the 7th and 6th bit-plane, respectively. As one might expect, the coding performance of SPIHT can be improved significantly by rearranging the highest frequency wavelet packet coefficients through the proposed scheme. Fig. 5 and Fig. 6 show the rate distortion curves, where the horizontal and vertical axes are the bit rates and PSNR values, respectively.

For a satellite image of Earth and an aerial image of San Francisco, Table 2 shows the numbers of significant wavelet and wavelet packet coefficients in the highest frequency subbands. The number of significant coefficients is reduced significantly starting from the 9th and 7th bit-plane, respectively. As there is a large portion of high detail textures in these images, many wavelet coefficients in the highest frequency subbands are likely to become significant after several code passes of SPIHT, and therefore the proposed method is suitable for coding such images, as shown in Fig. 7 and Fig. 8.

5 Conclusion

Wavelet transform offers an efficient multi-resolution analysis with many desirable properties. The wavelet tree based SPIHT algorithm is based on the so-called self similarity of wavelet transform: if a node in a wavelet tree is insignificant with respective to a given threshold, all the descendants are likely to be insignificant with respective to the same threshold. For images with textures, however, many coefficients in the high frequency subbands are likely to become significant after several code passes of SPIHT. To improve the coding performance, the high frequency wavelet coefficients need to be decomposed further into wavelet packet coefficients. To incorporate with the framework of SPIHT, an efficient scheme has been proposed to rearrange the high frequency wavelet packet coefficients, together with the low frequency wavelet coefficients, the called wavelet packet trees of images are thus constructed. As the proposed hierarchical wavelet packet tree is still a quad tree, the coding performance can be improved at no extra cost of coding complexity. Experimental results show that the proposed method is suitable for images with textures, especially for fingerprint images, which is one of the most demanding tasks.

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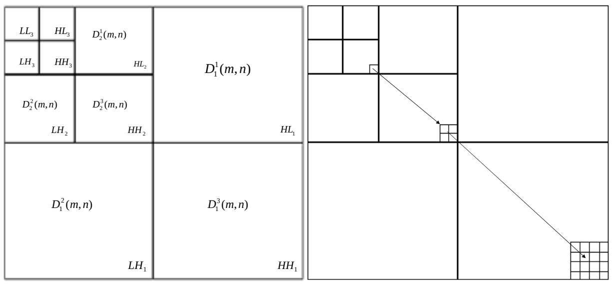
Table 1: Numbers of significant wavelet and wavelet packet coefficients of images: Sailboat on lake and Fingerprint, in the highest frequency subbands, bit-plane by bit-plane from most to least significant.

	Sailboat on lake image			Fingerprint image		
bit	W.C.	W.P.C.	Diff.	W.C.	W.P.C.	Diff.
1	0	0	0	0	0	0
2	0	0	0	0	0	0
3	0	0	0	0	5	-5
4	0	0	0	54	209	-155
5	0	7	-7	1633	2248	-615
6	130	238	-108	11202	7489	3713
7	1695	1403	292	21985	17023	4962
8	5388	4565	823	32478	27741	4737
9	10934	10225	709			
10	17923	17883	40			

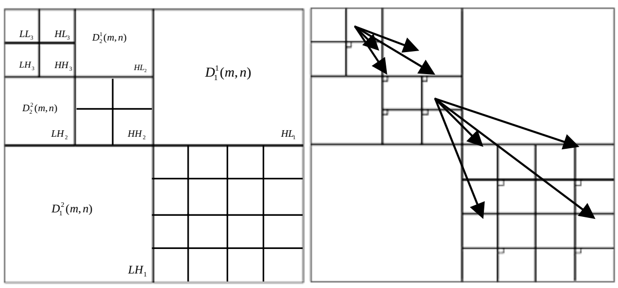
Table 2: Numbers of significant wavelet and wavelet packet coefficients of images: Earth and San Francisco, in the highest frequency subbands, bit-plane by bit-plane from most to least significant.

	Earth image			San Francisco image		
bit	W.C.	W.P.C.	Diff.	W.C.	W.P.C.	Diff.
1	0	0	0	0	0	0
2	0	0	0	0	0	0
3	0	0	0	0	0	0
4	0	0	0	0	0	0
5	0	0	0	11	85	-74
6	46	54	-8	2637	3858	-1221
7	249	247	2	26052	23223	2829
8	1380	1857	-477	71695	61790	9905
9	12370	10759	1611	118979	107863	11116
10	42947	36897	6050			



(a) (b)

Fig. 1 An example of 2-D WT; (a) 3-level 2-D WT; (b) a wavelet tree in the diagonal orientation.



(a) (b)

Fig. 2 An Example of 2-D WPT; (a) a 3-level 2-D WPT; (b) a hierarchical wavelet packet tree in the diagonal orientation.

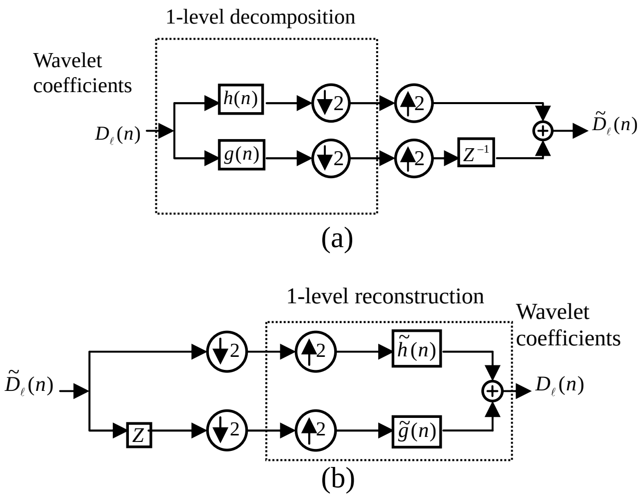


Fig. 3 Proposed scheme to rearrange 1-D wavelet packet coefficients to form hierarchical trees; (a) block diagram of forward rearrangement; (b) block diagram of inverse rearrangement.

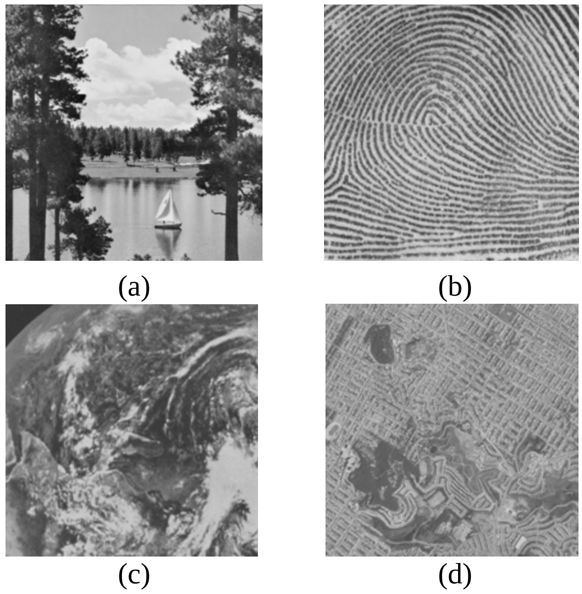


Fig. 4 Test images; (a) Sailboat on lake of size 256×256; (b) Fingerprint of size 256×256; (c) Earth of size 512×512; (d) San Francisco of size 512×512.

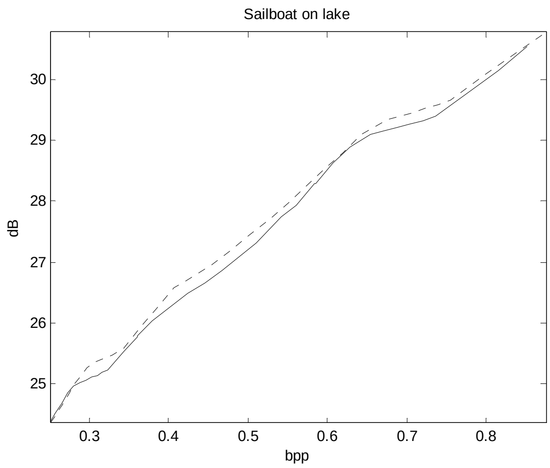


Fig. 5 Rate distortion curves of Sailboat image (dotted line: the proposed method; solid line: SPIHT).

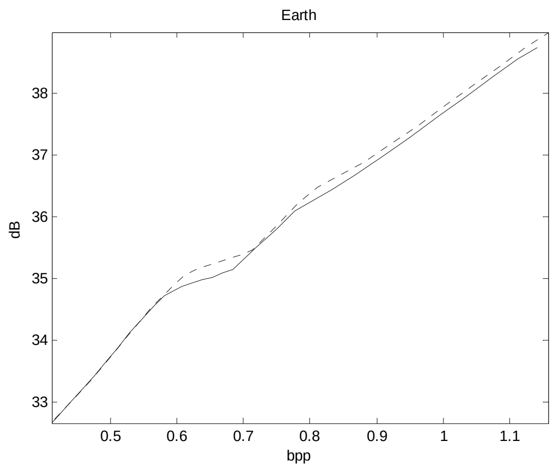


Fig. 7 Rate distortion curves of Earth image (dotted line: the proposed method; solid line: SPIHT).

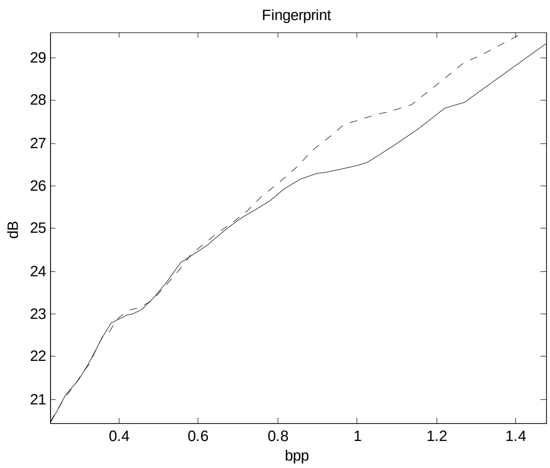


Fig. 6 Rate distortion curves of Fingerprint image (dotted line: the proposed method; solid line: SPIHT).

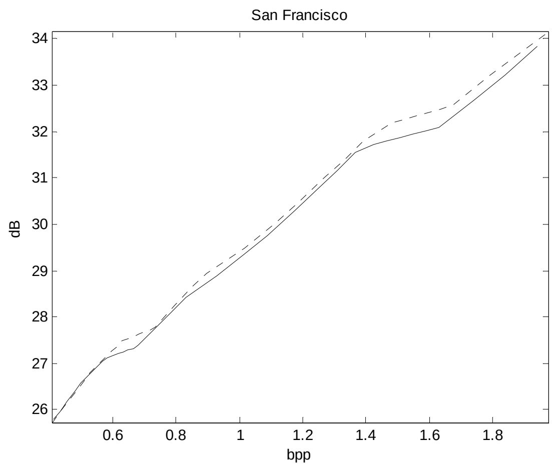


Fig. 8 Rate distortion curves of San Francisco image (dotted line: the proposed method; solid line: SPIHT).