Observer-based control for Time-delayed systems

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Abstract: In this paper, an observer-based control problem for time-delayed systems is considered. Based on the separation principle, we can design the controller and the observer gains independently. Since the resultant criterion is not formulated as *Linear Matrix Inequalities* (LMIs), we propose a relaxation technique that leads to an LMI-like iteration algorithm. Numerical examples demonstrate the effectiveness of the designed method.

Key–Words: Observer-based control, Delay system, Separation principle, Matrix inequalities

1 Introduction

The phenomenon of time-delay occurs in various industrial systems, such as distributed networks, manual control, long transmission lines in pneumatic systems, and neural networks. The time-delay causes instability or loss of performance. Therefore, the problem of stability analysis and controller synthesis for time-delayed systems have attracted considerable attention over the past years [2]–[4]. According to their dependence on the size of the delays, there are two categories of the stability and stabilization criteria, delay-independent criteria [5]–[6], and delay-dependent ones [7]–[9]. Delay-independent criteria guarantee the asymptotic stability of the system irrespective of the size of the delay. However, when the time-delay is small, these results are often more conservative than delay-dependent criteria.

In many practical systems, the states of system are not usually measured. In this case, state feedback control will not guarantee the stabilizability. The observer-based controls are often applied to the stabilizing problem. In the observer-based control, dynamic output-feedback control will be considered and the system states are estimated from the control process [10].

In this paper, we consider a stabilization problem for time-delayed systems based on Lyapunov-Krasovskii functional. A Luenberger-type observer is applied to estimate the states of system. We derive a simple stabilization criterion using the separation principle [11]. Unfortunately, the criterion is not LMI. By applying a kind of matrix inequality relationship, the original conditions can be relaxed, giving two phase-based iterative algorithms.

2 Separation Principle in Delayed Systems

Let us consider the following delayed system:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + A_h x(t-h) + Bu(t),\ t \geq 0, \\ y(t) &= Cx(t), \\ x(t) &= \phi(t),\ -h \leq t \leq 0, \end{aligned} \tag{1}$$

where h is a constant time-delay, and $\phi(t)$ is a given continuous vector valued initial condition. We shall consider observer-based controller of the form:

$$\begin{aligned} \dot{\hat{x}}(t) &= A\hat{x}(t) + A_h\hat{x}(t-h) + Bu(t) + L[y(t) - \\ &\quad C\hat{x}(t)] + L_h[y(t-h) - C\hat{x}(t-h)], \\ u(t) &= K\hat{x}(t) + K_h\hat{x}(t-h), \end{aligned} \tag{2}$$

where $\hat{x} \in \mathbb{R}^n$ is the estimation of the state x.
Let us define the state estimation error as

$$e(t) = x(t) - \hat{x}(t).$$

Then, the closed-loop system can be written as

$$\begin{bmatrix} \dot{x}(t) \\ \dot{e}(t) \end{bmatrix} = \begin{bmatrix} A+BK & -BK \\ 0 & A-LC \end{bmatrix} \begin{bmatrix} x(t) \\ e(t) \end{bmatrix} + \begin{bmatrix} A_h+BK_h & -BK_h \\ 0 & A_h-L_hC \end{bmatrix} \begin{bmatrix} x(t-h) \\ e(t-h) \end{bmatrix}, \tag{3}$$

which, by defining

$$z(t) \triangleq \begin{bmatrix} x(t) \\ e(t) \end{bmatrix}, \bar{A} \triangleq \begin{bmatrix} A+BK & -BK \\ 0 & A-LC \end{bmatrix}, \bar{A}_h \triangleq \begin{bmatrix} A_h+BK_h & -BK_h \\ 0 & A_h-L_hC \end{bmatrix}, \bar{C} \triangleq \begin{bmatrix} C & 0 \end{bmatrix}, \tag{4}$$

can be rewritten as

$$\begin{aligned}\dot{z}(t) &= \bar{A}z(t) + \bar{A}_h z(t-h),\\ y(t) &= \bar{C}z(t). \qquad (5)\end{aligned}$$

In view of the structures of $\bar{A}$ and $\bar{A}_h$, the characteristic equation of the system (5) can be written as

$$\begin{aligned}&\det[sI - \bar{A} - e^{-hs}\bar{A}_h]\\ =\ &\det[sI-(A+BK)-e^{-hs}(A_h+BK_h)]\times \quad (6)\\ &\det[sI-(A-LC)-e^{-hs}(A_h-L_hC)] \quad (7)\\ =\ &0.\end{aligned}$$

This shows that the separation principle [11] also holds for the time-delayed system and we can design the controller and the observer gains independently.

3 Main Result

Let us define $\chi(t)$ as $\chi(t) \triangleq [x^T(t)\ x^T(t-h)\ \dot{x}^T(t)]^T$ and the corresponding block entry matrices $e_i,\ i \in \{1,2,3\}$. Then, the system (1) with $u(t) = 0$ can be written as

$$0 = (Ae_1^T + A_h e_2^T - e_3^T)\chi(t). \qquad (8)$$

We shall choose the Lyapunov-Krasovskii functional as follows:

$$\begin{aligned}V(t) &= V_1(t) + V_2(t) + V_3(t),\\ V_1(t) &= x^T(t)Px(t),\ P > 0,\\ V_2(t) &= \int_{t-h}^{t} x^T(\alpha)Qx(\alpha)d\alpha,\ Q > 0,\\ V_3(t) &= \int_{-h}^{0}\int_{t+\alpha}^{t} \dot{x}^T(\beta)S\dot{x}(\beta)d\beta d\alpha,\ S > 0.\end{aligned}$$

Then, the time-derivative of $V(t)$ becomes as

$$\begin{aligned}\dot{V}_1(t) &= 2\dot{x}^T(t)Px(t) = 2\chi^T(t)e_3Pe_1^T\chi(t),\\ \dot{V}_2(t) &= x^T(t)Qx(t) - x^T(t-h)Qx(t-h)\\ &= \chi^T(t)\{e_1Qe_1^T - e_2Qe_2^T\}\chi(t),\\ \dot{V}_3(t) &= h\dot{x}^T(t)S\dot{x}(t) - \int_{t-h}^{t}\dot{x}^T(\alpha)S\dot{x}(\alpha)d\alpha\\ &= h\chi^T(t)e_3Se_3^T\chi(t) - \int_{t-h}^{t}\dot{x}^T(\alpha)S\dot{x}(\alpha)d\alpha.\end{aligned}$$

Furthermore, using the lemma [12]–[13], for

$$\begin{bmatrix} Y_{11} & Y_{12}\\ Y_{12}^T & Y_{22}\end{bmatrix} > 0,$$

we have

$$\begin{aligned}0 &\le \int_{t-h}^{t}\begin{bmatrix}\chi(t)\\ \dot{x}(\alpha)\end{bmatrix}^T\begin{bmatrix} Y_{11} & Y_{12}\\ Y_{12}^T & Y_{22}\end{bmatrix}\begin{bmatrix}\chi(t)\\ \dot{x}(\alpha)\end{bmatrix}d\alpha\\ &= \chi^T(t)\{hY_{11} + Y_{12}(e_1-e_2)^T +\\ &\quad (e_1-e_2)Y_{12}^T\}\chi(t) + \int_{t-h}^{t}\dot{x}^T(\alpha)Y_{22}\dot{x}(\alpha)d\alpha\end{aligned}$$

such that $\dot{V}(t)$ can be upper-bounded by the following quantities:

$$\begin{aligned}\dot{V}(t) \le \chi^T(t)\{&hY_{11}+Y_{12}(e_1-e_2)^T+(e_1-e_2)Y_{12}^T+\\ &e_3Pe_1^T+e_1Pe_3^T+e_1Qe_1^T-e_2Qe_2^T+\\ &he_3Se_3^T\}\chi(t) - \int_{t-h}^{t}\dot{x}^T(\alpha)(S-Y_{22})\dot{x}(\alpha)d\alpha.\end{aligned}$$

Finally, we shall remove the constraints of the model dynamics itself in (8) by introducing free variables Σ such that

$$0 \equiv \chi^T(t)\Sigma(Ae_1^T + A_he_2^T - e_3^T)\chi(t), \qquad (9)$$

which concludes the following lemma.

Lemma 1 *The delayed system (1) with $u(t) = 0$ is asymptotically stable if there exist matrices $P,\ Q,\ S,\ Y_{11},\ Y_{12},\ Y_{22}$ and Σ such that the following conditions hold:*

$$\begin{aligned}0 > \ &\Sigma(Ae_1^T+A_he_2^T-e_3^T)+(Ae_1^T+A_he_2^T-e_3^T)^T\Sigma^T+\\ &e_3Pe_1^T+e_1Pe_3^T+e_1Qe_1^T-e_2Qe_2^T+he_3Se_3^T+\\ &hY_{11}+Y_{12}(e_1-e_2)^T+(e_1-e_2)Y_{12}^T, \qquad (10)\\ &\begin{bmatrix} Y_{11} & Y_{12}\\ Y_{12}^T & Y_{22}\end{bmatrix} > 0,\ P>0,\ Q>0, S-Y_{22}\ge 0. \quad (11)\end{aligned}$$

Now, we shall employ a simplified stability criterion for Lemma 1 by assigning $S = Y_{22}$.

For the controller design procedure, we need to convert the $(\cdot)A + A^T(\cdot)^T$ term in (8) to its dual counterpart: $(\cdot)A^T + A(\cdot)^T$. Let us divide Σ into three parts:

$$\Sigma \triangleq \begin{bmatrix}\Sigma_1^T & \Sigma_2^T & \Sigma_3^T\end{bmatrix}^T.$$

Then, judging from the $(3,3)$-entry in (10), we can claim the following condition:

$$\Sigma_3 + \Sigma_3^T > 0,$$

which guarantees the invertibility of the matrix Σ_3. Also, from the following development:

$$\begin{aligned}&e_1Pe_3^T + \Sigma(Ae_1^T + A_he_2^T - e_3^T)\\ =\ &\begin{bmatrix} P & 0 & \Sigma_1\\ 0 & P & \Sigma_2\\ 0 & 0 & \Sigma_3\end{bmatrix}\begin{bmatrix} 0 & 0 & I\\ 0 & 0 & 0\\ A & A_h & -I\end{bmatrix},\end{aligned}$$

an attractive following-up is to utilize the invertible block matrix

$$\begin{bmatrix} P & 0 & \Sigma_1 \\ 0 & P & \Sigma_2 \\ 0 & 0 & \Sigma_3 \end{bmatrix}$$

whose inverse can be found and will be defined as

$$\begin{bmatrix} P^{-1} & 0 & -P^{-1}\Sigma_1\Sigma_3^{-1} \\ 0 & P^{-1} & -P^{-1}\Sigma_2\Sigma_3^{-1} \\ 0 & 0 & \Sigma_3^{-1} \end{bmatrix} \triangleq \begin{bmatrix} \bar{P} & 0 & \bar{\Sigma}_1 \\ 0 & \bar{P} & \bar{\Sigma}_2 \\ 0 & 0 & \bar{\Sigma}_3 \end{bmatrix} \triangleq \bar{\mathcal{P}}, \quad (12)$$

with $\bar{\Sigma} \triangleq \begin{bmatrix} \bar{\Sigma}_1^T & \bar{\Sigma}_2^T & \bar{\Sigma}_3^T \end{bmatrix}^T$.

For further development, we shall define some matrices as follow:

$$\bar{Q} \triangleq \bar{P}Q\bar{P}, \bar{Y}_{22} \triangleq Y_{22}^{-1},$$

$$\begin{bmatrix} \bar{Y}_{11} & \bar{Y}_{12} \\ \bar{Y}_{12}^T & \bar{P}\bar{Y}_{22}^{-1}\bar{P} \end{bmatrix} \triangleq \begin{bmatrix} \bar{\mathcal{P}} & 0 \\ 0 & \bar{P} \end{bmatrix} \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{12}^T & Y_{22} \end{bmatrix} \begin{bmatrix} \bar{\mathcal{P}} & 0 \\ 0 & \bar{P} \end{bmatrix}^T.$$

Multiplying on the left side of (10) by (12) and on the right side by its transpose, we can obtain an equivalent stability criterion that will be derived below in a step-by-step manner.

- For $e_1Pe_3^T + \Sigma(Ae_1^T + A_he_2^T - e_3^T)$:

$$\begin{aligned} &\bar{\mathcal{P}}\{e_1Pe_3^T + \Sigma(Ae_1^T + A_he_2^T - e_3^T)\}\bar{\mathcal{P}}^T \\ &= \bar{\mathcal{P}} \begin{bmatrix} P & 0 & \Sigma_1 \\ 0 & P & \Sigma_2 \\ 0 & 0 & \Sigma_3 \end{bmatrix} \begin{bmatrix} 0 & 0 & I \\ 0 & 0 & 0 \\ A & A_h & -I \end{bmatrix} \begin{bmatrix} \bar{P} & 0 & 0 \\ 0 & \bar{P} & 0 \\ \bar{\Sigma}_1^T & \bar{\Sigma}_2^T & \bar{\Sigma}_3^T \end{bmatrix} \\ &= \begin{bmatrix} \bar{\Sigma}_1^T & \bar{\Sigma}_2^T & \bar{\Sigma}_3^T \\ 0 & 0 & 0 \\ A\bar{P} - \bar{\Sigma}_1^T & A_h\bar{P} - \bar{\Sigma}_2^T & -\bar{\Sigma}_3^T \end{bmatrix} \\ &= e_1\bar{\Sigma}^T + e_3A\bar{P}e_1^T + e_3A_h\bar{P}e_2^T - e_3\bar{\Sigma}^T. \end{aligned}$$

- For $hY_{11} + Y_{12}(e_1 - e_2)^T$:

$$\begin{aligned} &\bar{\mathcal{P}}\{hY_{11} + Y_{12}(e_1 - e_2)^T\}\bar{\mathcal{P}}^T \\ &= h\bar{Y}_{11} + \bar{Y}_{12}(e_1 - e_2)^T. \end{aligned}$$

- For $e_1Qe_1^T - e_2Qe_2^T$:

$$\bar{\mathcal{P}}\{e_1Qe_1^T - e_2Qe_2^T\}\bar{\mathcal{P}}^T = e_1\bar{Q}e_1^T - e_2\bar{Q}e_2^T.$$

- For $he_3Y_{22}e_3^T$:

$$\bar{\mathcal{P}}(he_3Y_{22}e_3^T)\bar{\mathcal{P}}^T = h\bar{\Sigma}Y_{22}\bar{\Sigma}^T,$$

Applying the Schur complement technique [1] to the results derived, we can verify the stability of the closed loop system obtained by applying a control law

$$u(t) = Kx(t) + K_hx(t-h) \quad (13)$$

to the system if we replace the matrices (A, A_h) in the result by $(A + BK, A_h + BK_h)$ and verify the resulting inequalities are feasible. This leads to the following theorem.

Theorem 2 *The control law (13) asymptotically stablilizes the delayed system (1) if there exist matrices $\bar{P}$, $\bar{Q}$, $\bar{Y}_{11}$, $\bar{Y}_{12}$, $\bar{Y}_{22}$, $\bar{\Sigma}$, $\bar{K}$ and $\bar{K}_h$ such that the following conditions hold:*

$$\bar{P} > 0,\ \bar{Q} > 0,\ 0 > \begin{bmatrix} \bar{\Pi} & \bar{\Sigma} \\ \bar{\Sigma}^T & -h^{-1}\bar{Y}_{22} \end{bmatrix}, \quad (14)$$

$$\begin{bmatrix} \bar{Y}_{11} & \bar{Y}_{12} \\ \bar{Y}_{12}^T & \bar{P}\bar{Y}_{22}^{-1}\bar{P} \end{bmatrix} > 0, \quad (15)$$

where

$$\begin{aligned} \bar{\Pi} \triangleq\ & h\bar{Y}_{11} + \bar{Y}_{12}(e_1 - e_2)^T + (e_1 - e_2)\bar{Y}_{12}^T + \\ & e_3(A\bar{P}e_1^T + A_h\bar{P}e_2^T) + (A\bar{P}e_1^T + A_h\bar{P}e_2^T)^Te_3^T + \\ & e_1\bar{Q}e_1^T - e_2\bar{Q}e_2^T + (e_1 - e_3)\bar{\Sigma}^T + \bar{\Sigma}(e_1 - e_3)^T + \\ & e_3B\bar{K}e_1^T + e_1\bar{K}^TB^Te_3^T + e_3B\bar{K}_he_2^T + e_2\bar{K}_h^TB^Te_3^T. \end{aligned}$$

Since, the characteristic equation of this temporal state-feedbacked system is equivalent to (6), the obtained controller gains should stabilize the original system (1) asymptotically, assuming that the observer is well-designed.

To handle the nonlinear condition (15), the following matrix inequality will be applied.

Lemma 3 *For any matrices $Q > 0$, M and N with compatible dimension, it holds that*

$$NQ^{-1}N^T \geq NM + M^TN^T - M^TQM. \quad (16)$$

Proof: See the following matrix inequality:

$$(N - M^TQ)Q^{-1}(N^T - QM) \geq 0,$$

which immediately concludes (16). □

Then, for any non-positive marrix Λ, we have the following matrix inequality:

$$\begin{aligned} \bar{P}\bar{Y}_{22}^{-1}\bar{P} &\geq \bar{P}M + M^T\bar{P} - M^T\bar{Y}_{22}M \\ &\geq \bar{P}M + M^T\bar{P} - M^T\bar{Y}_{22}M + \Lambda, \end{aligned}$$

which, using the Schur complement technique [1], gives the following two-phase-based algorithm.

Algorithm 1

1. Set initial $M = I$.

2. Solve the following convex optimization problem:

$$minimize\ \alpha\ subject\ to\ (14)\ and$$

$$\alpha I \geq \Lambda, \begin{bmatrix} \bar{Y}_{11} & \bar{Y}_{12} & 0 \\ \bar{Y}_{12}^T & \bar{P}M + M^T\bar{P} + \Lambda & M^T\bar{Y}_{22} \\ 0 & \bar{Y}_{22}M & \bar{Y}_{22} \end{bmatrix} > 0.$$

3. Save the feasible matrix variables $(\bar{P}, \bar{Y}_{22})$ in the resultant solution set.

4. If $\alpha \leq 0$, stop the iteration (feasible solutions for the Theorem 2 are found!). Else, solve the convex optimization problem in step 2. In this step, $(\bar{P}, \bar{Y}_{22})$ will be fixed and M would be free variable.

5. Save the feasible matrix variable M in the resultant solution set.

6. If $\alpha \leq 0$, stop the iteration (feasible solutions for the Theorem 2 are found!). Else, solve the convex optimization problem in step 2. In this step, M will be fixed and $(\bar{P}, \bar{Y}_{22})$ would be free variables.

7. Stop the loop if no progress is expected.

Remark 4 In this case, we can obtain the state-feedback gains through $K = \bar{K}\bar{P}^{-1},\ K_h = \bar{K}_h\bar{P}^{-1}$.

Since, the sign of the real part of the solution for (7), which concerns the behavior of the observer directly, is invariant under transpose operation, we can design observer gains that estimate the state of the original system (1) asymptotically by replacing (A, $\bar{K}$, $\bar{K}_h$, B) in Theorem 2 with (A^T, $\bar{L}$, $\bar{L}_h$, $-C^T$). In this case, we can obtain the observer gains through $L = (\bar{L}\bar{P}^{-1})^T,\ L_h = (\bar{L}_h\bar{P}^{-1})^T$.

4 Example

Example 5 Consider the delayed system (1). The following parameters are used.

$$A = \begin{bmatrix} 0.2 & 0.0 \\ 0.1 & -0.1 \end{bmatrix}, A_h = \begin{bmatrix} 0.0 & 0.1 \\ -0.1 & 0.1 \end{bmatrix},$$
$$B = \begin{bmatrix} 0.0 \\ 1.0 \end{bmatrix}, C = \begin{bmatrix} -0.5 & 1.0 \end{bmatrix}.$$

Table 1 shows maximum allowable delay upper-bounds for various controller structure.

Table 1: maximum delay upper-bound

method	maximal h	gains
using K, K_h	3.4990	$K = \begin{bmatrix} -27.7691 & -9.9007 \end{bmatrix}$, $K_h = \begin{bmatrix} 0.1093 & 0.0961 \end{bmatrix}$, $L = \begin{bmatrix} -0.9579 \\ -0.2797 \end{bmatrix}, L_h = \begin{bmatrix} -0.2821 \\ 0.2337 \end{bmatrix}$
using K	2.9992	$K = \begin{bmatrix} -104.4998 & -32.7384 \end{bmatrix}$, $L = \begin{bmatrix} -1.0443 \\ -0.2861 \end{bmatrix}, L_h = \begin{bmatrix} -0.2762 \\ 0.2416 \end{bmatrix}$
using K_h	1.9727	$K_h = \begin{bmatrix} -2.7357 & -1.0175 \end{bmatrix}$, $L = \begin{bmatrix} -1.2994 \\ -0.2506 \end{bmatrix}, L_h = \begin{bmatrix} -0.1830 \\ 0.1810 \end{bmatrix}$

Example 6 Consider the delayed system (1) for the following parameters:

$$A = \begin{bmatrix} -2.0 & 0.0 \\ 0.0 & -0.9 \end{bmatrix}, A_h = \begin{bmatrix} \epsilon & 0.0 \\ -1.0 & -1.0 \end{bmatrix},$$
$$B = \begin{bmatrix} 0.0 \\ 1.0 \end{bmatrix}, C = \begin{bmatrix} 0.0 & 1.0 \end{bmatrix}.$$

Table 2 shows stable range of ϵ in A_h for various delay bounds. Comparison our results with those of the delay-independent criterion [14] shows that our method outperforms over the existing criterion.

Table 2: stable range of ϵ

	delay	lower ϵ	upper ϵ
delay-independent [14]	·	-1.99999	2
delay-dependent	0.5	-3.73703	2
	0.3	-5.75688	2
	0.1	-15.0199	2

5 Conclusion

In this paper, we considered an observer-based control problem for time-delayed systems. Based on the separation principle, we could design the controller and the observer gains independently. Since the resultant criterion was not formulated as LMIs, we proposed a relaxation technique that leads to an LMI-like iteration algorithm. Numerical examples demonstrated the effectiveness of the designed method.

As a future work, we plan to extend this work in several directions; output-feedback stabilization, $\mathcal{H}_2/\mathcal{H}_\infty$ control problems and so on.

Acknowledgements: This work was supported by the center for Broadband OFDM Mobile Access

(BrOMA) at POSTECH through the ITRC program of the Korean MIC, supervised by IITA. (IITA-2006-C1090-0603-0037)

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