Proposal of Search Method Compressed the One-Way Branch based on the Double-Array Structure

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Abstract: Digital search is expressed by the digital search tree which considers a part of keys as transitions. Therefore, it is frequently used for various applications, such as natural language dictionaries because of being dependent on the key length for search processing. There is the compressed digital search tree which compressed the one-way branches after the leaf and reduces nodes on digital search tree. However the one-way branches has still existed on the compressed digital search tree. In this paper, we present the search method which used the digital search tree compressed all one-way branches used the double-array structure. The simulation results confirmed that the proposed method is more efficient than the original method.

Key–Words: Information Retrieval, Digital Search, the Double-Array, Patricia, Data Structure, Trie

1 Introduction

Search processings such as hash based on comparison between keys are dependent on the number of keys. On the other hand, digital search comparisons between a part of keys[2]. Therefore, it is dependent on the key length and frequently used for various applications, such as natural language dictionaries, database systems and compilers.

Digital search is expressed by the digital search tree (DS-tree) which considers a part of keys as transitions. DS-trees are divides into three types, which are full DS-tree (FDS-tree), compressed DS-tree (CDS-tree) and Patricia. FDS-tree has all parts of keys. On the other hand, CDS-tree and Patricia have some parts of keys. CDS-tree expresses the common prefix. Patricia does not have nodes which have only one child (one-way branch) [1, 2]. DS-tree often expressed as the binary tree, actually Patricia is the binary tree. Because of the binary tree is easier to realize than the multi-way tree and its space efficiency are better than the multi-way tree.

However, the binary tree has many nodes and its search speed is slower than multi-way tree. There is the double-array structure as an effective data structure which realizes multi-way tree. It has quick search speed and high space efficiency. At the present, a variety of the high speed updating techniques are proposed to the double-array[3, 4].

The present work is intended to reduce the one-way branches of multi-way tree used the double-array. Therefore, the search technique and the updating techniques of having introduced the node which stored transition informations are proposed.

The rest of the paper is organized as follows. In section 2, we give overviews of the double-array structure. In section 3, we propose a search method compressed the one-way branches used the double-array structure. In section 4, we evaluation our proposed methods. Finally, we conclude the paper and describe future works in section 5.

2 The Double-Array Structure

The double-array is a data structure which realized FDS-tree and CDS-tree. In this section, we explain the double-array structure taking the case of key set K. Where numerical values of ‘a’, ‘b’, …, ‘o’ are 1, 2, …, 15 respectively and they are translated into binary numbers of 4 bits.

$$K = \{\text{academe},\ \text{academic},\ \text{cable},\ \text{cache},\ \text{call}\}$$

The double-array uses one-dimensional arrays BASE and CHECK[5]. Their index numbers correspond to node numbers in DS-tree. The BASE value of the node on DS-tree is the basis value which is the index number of the child. The CHECK value of the node on DS-tree is the index number of the parent.

The concept of the double-array is to represent transition by giving Eq.1 and Eq.2. Where $B[x]$ and $C[x]$ are the values of element x of BASE and

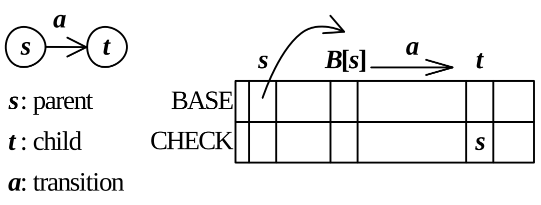
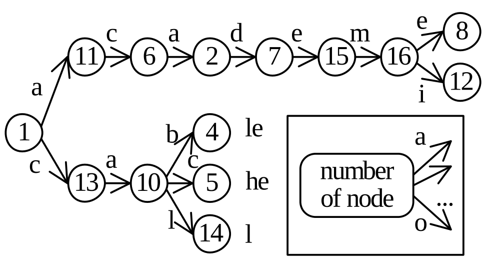


Fig. 1: Transition of DS-tree by the double-array.



	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
BASE	10	3		-4	-7	1	10	-1		2	3	-2	9	-10	3	3
CHECK		6		10	10	11	2	16		13	1	16	1	10	7	15

	1	2	4	7	10
TAIL		c	le	he	l

Fig. 2: Examples of CDS-tree and the double-array for K.

CHECK respectively, s is the original node, t is the child of s and a is the numerical value of transition. The transition by the double-array is shown in Fig.1.

$$B[s] + a = t \tag{1}$$

$$C[t] = s \tag{2}$$

For example, CDS-tree and the double-array for K are shown in Fig.2. As to a transition from original node 10 by ‘b’, a child of 10 is $B[10] + b = 2 + 2 = 4$ used Eq.1. After that, $C[4] = 10$ is compared with an original node 10 used Eq.2, in this case both are equal. Thus, a transition from 10 to 4 by ‘b’ is success.

Moreover, a doubly list called E-Link is created with the elements of the double-array called the empty element[5, 3, 4]. The empty element is not used as a node of DS-tree. E-Link is given by the BASE and CHECK values of the empty element. In the example of the double-array, for simplicity, the values of the empty elements are made blank.

A one-dimensional array TAIL is used to realize CDS-tree[5]. The double-array expresses the common prefix, TAIL expresses the one-way branches after the common prefix called the suffix. Where $T[x]$ is the value of element x of TAIL and $T + x$ is the suffix which begin from $T[x]$.

The value of BASE as to the leaf on CDS-tree is

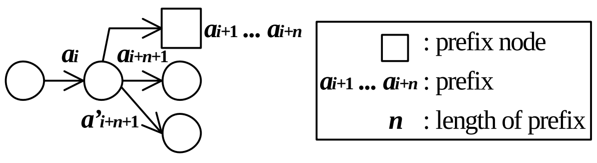


Fig. 3: Definition of the prefix node.

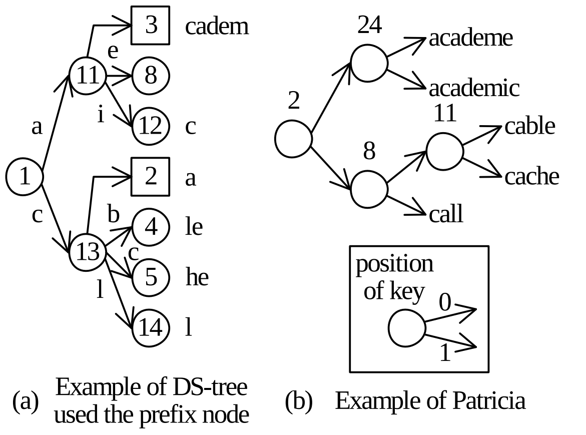


Fig. 4: Examples of DS-tree used the prefix node and Patricia for K.

negative value, and considered as the link of the suffix on TAIL by giving Eq.3. Where $leaf$ is the node number of leaf on CDS-tree and loc is index number on TAIL of the suffix. For example, as to a suffix of a leaf 4 is $T + (-B[4]) = T + 4 =$“le” used Eq.3.

$$B[leaf] = -loc \tag{3}$$

3 An Approach Compressed the One-Way Branches

In this section, we propose the search methods focused on the one-way branches which exist on CDS-tree. It is important to mention here that the node which stored transitions information is defined.

3.1 Data Structure

As to the one-way branches $a_{i+1},\ a_{i+2},\ \ldots,\ a_{i+n}$ of length n, the parent of a_{i+1}-transition to the child of a_{i+n}-transition are unique identifiable. To reduce the node on FDS-tree, the suffixes on the double-array are replaced on TAIL.

However, CDS-tree has the one-way branches called the prefix. Therefore, the prefixes on the double-array are also replaced on TAIL. The node called the prefix node is defined to manage the prefix. Then, the definition of the prefix node is shown in

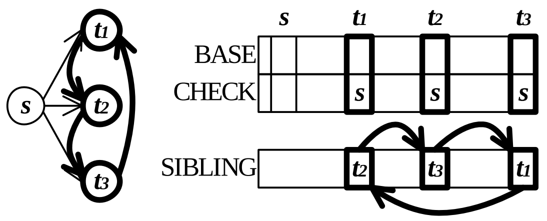


Fig. 5: Illustration of SIBLING.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14
BASE	10	-9	-1	-10	-13			-6			3	-7	2	16
CHECK		-1	-5	13	13			11			1	11	1	13
SIBLING	1	4	8	5	14			12			-13	3	-11	2

	1	6	7	9	10	13	16
TAIL	cadem		c	a	le	he	l

Fig. 6: Example of the double-array used the prefix node for K.

Fig.3. And, the DS-tree used the prefix node for K is shown in Fig.4(a).

The BASE value of the prefix node is the link of prefix on TAIL, and the CEHCK value is the prefix length by giving Eq.4.

$$B[p] = -loc, C[p] = -n \tag{4}$$

The transition to the prefix node is the constant PCODE. If two or more parents with an equal BASE value exist, each parents do not have the prefix node or one of them has the prefix node. In the case of the later, it can not judge which the prefix node has the prefix node, because of the CHECK value is not the index number of its parent.

Therefore, a one-dimensional array SIBLING is defined. SIBLING manages the siblings which are the other children of its parent, in order to fast computation of the children searching in the updating processing. The SIBLING value of the child is other child of its parent, and create the circular list which is shown in Fig.5. In this paper, this list called S-Link.

Then, the SIBLING value is given by Eq.5. Where $S[x]$ is the values of element x of SIBLING , the default value of $S[x]$ is x, b is the number of the children of the arbitrary parent s and $t_1, t_2, \ldots, t_b$ are index numbers of the children of s.

$$S[t_i] = t_{i+1} (1 \le i \le b-1), \quad S[t_b] = t_1 \tag{5}$$

In addition, SIBLING manages whether the parent has the prefix node. The SIBLING value of the parent which has the prefix node is negative value. Where Abs(v) is a function which return to the absolute value of v.

Function: Search(key)

Step(S-1): Initialization

Store 0 in pos which is the position of the key and store 1 in s which is the parent.

Step(S-2): Existence of the prefix node

If $S[s]$ is the negative value, store $B[s]$ + PCODE in p which is the prefix node, enqueue $\{pos, p\}$ and advance pos by $-C[p]$.

Step(S-3): Transition

Store $B[s]+key[pos]$ in t which is child. If s and $C[t]$ are equal, store t in s and advance pos by 1. Otherwise this process is fail. If $B[s]$ is the negative value, proceed to (S-4), otherwise proceed to (S-2).

Step(S-4): Comparison about the prefixes

Dequeue $\{pos_q, p_q\}$ and compare $key + pos_q$ and $T + (-B[p_q])$ of length $-C[p_q]$. If both are equal as to all element of queue, proceed to (S-5), otherwise this process is fail.

Step(S-5): Comparison about the suffix

Compare $key[pos]$ and $T+(-B[t])$. If both are equal, this process is success, otherwise this process is fail.

Fig. 7: Algorithm of Search

For example, the double-array used the prefix node and SIBLING for K is shown in Fig.6. Where numerical values of PCODE is 0. A next sibling of a node 11 is Abs(S[11])= 13, and a next sibling is Abs(S[13])= 11. In addition, a node 11 has a prefix node because of $S[11]$ is the negative value. A prefix node of 11 is $B[11]$ + PCODE $= 3$. A prefix node 3 shows a prefix $T + (-B[3]) =$ "cadem" of length $-C[3] = 5$.

Patricia for K is shown in Fig.4(b). Patricia is the binary tree and two nodes are needed to distinguish "cable", "cache" and "call". However, DS-tree used the prefix node needs only one node.

3.2 A Searching Algorithm

A searching algorithm is presented in Fig.7. There are three conditions in which search succeed. First, transition from a root to a leaf on DS-tree by satisfied Eq.1 and Eq.2 in (S-2) and (S-3). Unlike the double-array, the transitions about the prefixes are not checked. Second, all prefixes which prefix nodes shows are equal key in (S-4). It is the point that this also differs from the double-array. Last, the suffix which leaf shows is equal key in (S-5).

For example, searching "academic" in Fig.6 is explained. In (S-1), store 0 in pos and the root 1 in s. In (S-2), $S[1] = 1$ is not the negative and a node 1 does not have a prefix node. Then in (S-3), transit from $s = 1$ to $t = B[1] + key[0] = 11$ by $key[0] =$ 'a' and store 11 in s, $0 + 1$ in pos.

In (S-2), 11 has a prefix node 3 which shows a prefix of length $-C[3] = 5$. Then, enqueue $\{1, 3\}$ and store $1 + 5 = 6$ in pos. In (S-3), transit from $s = 11$ to $t = B[11] + key[6] = 12$ by $key[6] =$ 'i' and store 12 in s, $6+1 = 7$ in pos. After that, proceed to (S-4) because of $B[12] = -7$ is the negative value.

In (S-4), as to $\{1, 3\}$, compare $key+1 =$ "cadem" and $T + (-B[3]) =$ "cadem". In this case both are equal and queue is empty, then proceed to (S-5). In (S-5), compare $key + 7 =$ "c" and $T + (-B[12]) =$ "c", in this case both are equal. Thus, searching "academic" is success.

3.3 Insertion Algorithms

The insert processing is performed when the search processing is fail.

A case of fail on DS-tree in (S-3), call InsLeaf(s, t, key, pos) to create a leaf for key[3]. This function, if a new leaf t which is a child of s by $key[pos]$ is an empty element, create a leaf on t. Otherwise, update $B[s]$ or $B[s']$ and create a leaf on $B[s] + key[pos]$[5].

A case of fail on the TAIL in (S-4) and (S-5), call InsPrefixNode(t, key, pos, $T + (-B[t])$, n) to update the prefix or the suffix. Where n is the minimum comparison position where $key + pos$ differs from $T + (-B[t])$.

The insert processing utilizes following variables and functions.

$tailPos$**:** A global variable which indicates the minimum index of available entries of TAIL.

L**:** The set of transitions.

NewBase(L**):** Return to a BASE value to which all element of L can transit used E-Link.

InsNode(x**):** x is deleted from E-Link in order to s is made usable on DS-tree. And, update S-Link.

DelNode(x**):** x is inserted to E-Link in order to s is deleted from DS-tree. And, update S-Link.

CreateLeaf(s, t, $suffix$**):** Call InsNode(t) and store s in $C[t]$. Store $-tailPos$ in $B[t]$ and a suffix $suffix$ is stored from $T[tailPos]$.

RenewalCheck(new, old**):** As to the children of new except a prefix node, the CHECK value old is updated to new used S-Link.

InsPrefixNode is presented in Fig.8. This function is called the case of updating prefix or suffix.

In the first case, determine the BASE value to which $\{key[pos + n]$, $tail[n]$, PCODE$\}$ can transit, and create a prefix node p which shows prefix $tail$ of length n. Then create a leaf t of s by $key[pos+n]$ and a child t' of s by $tail[n]$. And update a prefix pp of t' which shows prefix $tail + n + 1$.

```
Function: InsPrefixNode($s$, $key$, $pos$, $tail$, $n$)
Step(P-1): Initialization
    Store $B[s]$ in $base$ which is a old BASE value of $s$.
    Set $L$ to $\{key[pos + n], tail[n]\}$.
Step(P-2): Definition of a prefix node of $s$
    If $n$ is equal to 0, store NewBase($L$) in $B[s]$ and
    proceed (P-3). Otherwise, store NewBase($L \cup$
    {PCODE}) in $B[s]$ and $B[s]$ + PCODE in $p$. Call
    InsNode($p$) and $S[s]$ is set as a negative value. A
    prefix $tail$ of length $n$ is stored from $T[tailPos]$ and
    store $-tailPos$ in $B[p]$, $-n$ in $C[p]$.
Step(P-3): Definition of a node about $tail$
    Store $B[s] + tail[n]$ in $t'$. Call InsNode($t'$) and store
    $s$ in $C[t']$. If $base$ is the negative value, proceed to
    (P-4), otherwise proceed to (P-5).
Step(P-4): Updating of the $B[t']$($t'$ is a leaf)
    Store $base - n$ in $B[t']$. If $n$ is more than 0, store $base$
    in $B[p]$. Proceed to (P-6).
Step(P-5): Updating of the $B[t']$($t'$ is not a leaf)
    Store $base$ in $B[t']$ and call RenewalCheck($t'$, $s$).
    Store $base$ + PCODE in $pp$ which a prefix node of
    $t'$. If $n$ is not equal to 0, store $B[pp]$ in $B[p]$ and $-n$
    in $C[p]$. If $n+1$ is equal to $-C[pp]$, call DelNode($pp$)
    and $S[t']$ is set as a positive value, otherwise advance
    $B[pp]$ by $-(n + 1)$ and $C[pp]$ by $n + 1$.
Step(P-6): Definition of a leaf about $key$
    Store $B[s] + key[pos + n]$ in $t$ and call CreateLeaf($s$,
    $t$, $key + (pos + n + 1)$).
```

Fig. 8: Algorithm of InsPrefixNode

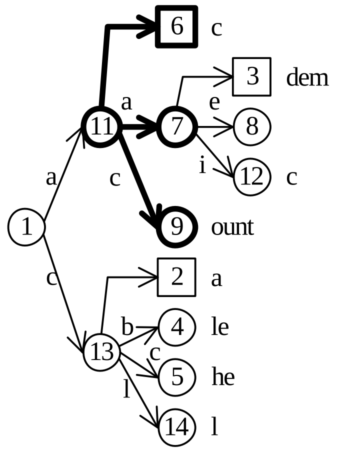
In the second case, if n is equal to 0, create a leaf t of s by $key[pos]$. As to $tail$, create a child t' of s by $tail[0]$ and update $B[t']$ to updating $tail$. If n is not equal to 0, need to create a prefix node p.

For example, insertion "account" in Fig.6 is explained. Searching "account" is fail on the TAIL and call InsPrefixNode(11, "account", 1, "cadem", 1). InsPrefixNode processes in the following order, (P-1), (P-2), (P-3), (P-5), (P-6). In (P-2), call NewBase({'c', 'a', PCODE}) and create a prefix node 6, and update a prefix node 3 in (P-5). In (P-6), create a leaf which shows "ount" on 9. DS-tree and the double-array after insertion processing are shown in Fig.9.

3.4 A Deletion Algorithm

The delete processing is performed when the search processing is success. Delete is presented in Fig.10. This function consists of deletion of key and maintaining a form of DS-tree. The first processing is (D-2), the second processing is (D-3) to (D-7). The deletion processing utilizes following functions.

leaf: A leaf which shows a delete key.



	1	2	3	4	5	6	7	8	9	10	11	12	13	14
BASE	10	-9	-3	-10	-13	-1	3	-6	-18		6	-7	2	-16
CHECK	14	-1	-3	10	10	-1	11	7	11		1	7	1	10
SIBLING	1	4	8	5	14	7	-9	12	6		-13	3	-11	2

	1	3	6	7	9	10	13	16	18
TAIL	c	dem		c	a	le	he	l	ount

Fig. 9: Examples of DS-tree and the double-array used the prefix node for $\{K$, "account"$\}$.

tail: A prefix or a suffix.

GetLabels(s): Return a set of transitions from s used S-Link.

TransNode(s, $newBase$, $oldBase$, L): In connection with the BASE value of s is updated $oldBase$ to $newBase$, all children of s are moved. If the child of s has children, call RenewalCheck.

For example, deletion "account" in Fig.9 is explained. Searching "account" is success on a leaf 9 and call Delete(9). In Delete, it processes in the following order, (D-1), (D-2), (D-3), (D-5), (D-6), (D-7). In (D-2), delete a leaf 9. In (D-3), set "ca" to $tail$ and delete a prefix node 6. In (D-5), $tail$ is updated "cadem", and update a prefix shown a prefix 3 to "cadem" in (D-6). In (D-7), delete a node 7. DS-tree and the double-array after deletion processing are return to Fig.6 and Fig.4(a) respectively.

4 Experimental Observations

We conducted simulation experiments to evaluate the proposed methods. Where DA, DA+T and DA+TS methods are the double-array and DA method does not use TAIL[5], DA+T and DA+TS methods used TAIL[5], DA+TS method used SIBLING. In addition, P method is Patricia[1]. In this simulation, we used two set of key which are randomly selected 200,000 English words called $S1_e$ and 5,000,000 URI called $S1_u$. The average lengths of key are 10.47 and 58.52 bytes respectively.

```
Function: Delete($leaf$)
Step(D-1): Initialization
    Store $C[leaf]$ in $s$ and $S[leaf]$ in $t$.
Step(D-2): Deletion of $key$
    Call DelNode($leaf$) and store Abs($S[t]$) in $sibling$
    which is a sibling of $t$. If $s$ is equal to root of DS-tree
    or there is no different child from $t$, this process is
    success, otherwise proceed to (D-3). The second case
    can be judged that, if $s$ has a prefix node, whether
    Abs($S[sibling]$) is equal to $t$, otherwise $sibling$ is
    equal to $t$.
Step(D-3): Updating $tail$
    If $s$ has a prefix node $pp$, set prefix $T + (-B[pp])$
    to $tail$, call DelNode($pp$) and $S[s]$ is set as a positive
    value. A transition of $s$ to $t$ is added to $tail$. If $t$ has
    not a prefix node, proceed to (D-4), otherwise proceed
    to (D-5).
Step(D-4): Addition suffix to $tail$
    A suffix $T + (-B[t])$ is added to $tail$ and store
    $-tailPos$ in $B[s]$. Proceed to (D-7).
Step(D-5): Addition prefix to $tail$
    If $t$ has a prefix node $p$, $T + (-B[p])$ is added to $tail$.
    Store $-tailPos$ in $B[p]$, a negative value which is
    length of $tail$ in $C[p]$ and $S[s]$ is set as a negative
    value. Store $B[t]$ in $B[s]$ and call RenewalCheck($s$,
    $t$). Otherwise proceed to (D-6).
Step(D-6): Definition of a prefix node
    Store $B[s]$ + PCODE in $p$. If $p$ is an empty element,
    store $B[t]$ in $B[s]$ and call RenewalCheck($s$, $t$). Oth-
    erwise, set to $L$ by GetLabels($t$), store NewBase($L \cup$
    {PCODE}) in $B[s]$ and call TransNode($s$, $B[s]$, $B[t]$,
    $L$). Store $B[s]$ + PCODE in $p$, $-tailPos$ in $B[p]$ and
    a negative value which is length of $tail$ in $C[p]$.
Step(D-7): Deletion of $t$
    Call DelNode($t$) and $tail$ is stored from $T[tailPos]$.
```

Fig. 10: Algorithm of Delete

Simulation results of the searching processing are shown in Tab.1. Tab.1 shows that the average of searching times on DS-tree and on TAIL. In addition, shows the average of search lengths on DS-tree and on TAIL.

From the simulation results, the searching speed of the proposed method is faster than the original methods for $S1_u$, about 1.16 to 1.32 times. On the other hand, later than DA+T method for $S1_e$, about 0.78 to 0.90 time. Notice that the averages of search length on DS-tree are reduced by the proposed method for both set. However, the searching speed on DS-tree of proposed method is later than DA and DA+T methods. This indicate that a processing of enqueue in (S-2) is taken time.

As to the updating processing, the initial states

Tab. 1: The simulation results of search processing.

	searching time(μs)			search length	
	On DS-tree	On TAIL	Total	On DS-tree	On TAIL
English					
DA	0.77	0.00	0.77	10.59	0.00
DA+T	0.54	0.06	0.60	8.08	2.57
P	1.29	0.15	1.44	24.61	10.47
Proposed	0.65	0.12	0.76	7.05	3.60
URI					
DA	4.03	0.00	4.03	58.48	0.00
DA+T	3.65	0.17	3.83	48.67	9.82
P	3.87	0.49	4.36	52.21	58.47
Proposed	2.44	0.87	3.31	15.97	42.52

Tab. 2: The simulation results of updating processing.

	insertion time(μs)	deletion time(μs)	search length of children	nodes (1,000)	tail (1,000)	used space (MB)
English						
DA	7.60	9.60	950.60	125.6	0.0	1.01
DA+T	4.48	5.68	284.40	35.2	90.3	0.37
DA+TS	4.46	4.11	33.42	35.2	90.3	0.51
P	4.86	4.30		20.2	1,279.3	1.70
Proposed	4.42	4.31	25.20	32.6	90.3	0.48
URI						
DA	13.421	26.73	2,229.59	9,519.3	0.0	76.16
DA+T	16.502	13.18	659.41	2,168.4	7,350.9	24.70
DA+TS	15.721	7.64	40.87	2,168.4	7,350.9	33.37
P	7.987	6.67		501.1	178,341.3	186.36
Proposed	6.234	6.60	7.84	955.4	7,350.9	18.82

are set to 20,000 words for $S1_e$ and 500,000 URI for $S1_u$ at random, and insertion and deletion processing used $S2_e$ and $S2_u$. Where $S2_e$ and $S2_u$ are randomly selected 200,000 words from $S1_e$ and 5,000,000 URI from $S1_u$ respectively. The number of insertion keys and deletion keys are almost equal and the number of keys on search methods are always almost equal during the updating processing.

This simulation results are shown in Tab.2. Tab.2 shows that the average of insertion and deletion times. In addition, shows the average of search length for the children searches which are GetLabels, RenewalCheck and the judgment whether parent has only one child.

Moreover, shows that the number of nodes on DS-tree, used elements on TAIL and the used space. In addition, as to the proposed method for $S2_e$ and $S2_u$, the number of the prefix nodes are 2,592 and 172,708, the average of prefix are 2.01 and 8.02 respectively. Where an used space of the P method is computed from a position of key, two information of child, an index on TAIL of key and TAIL.

From the simulation result, the insertion speed of the proposed method is faster than the original methods for $S2_u$, about 1.28 to 2.64 times. And for $S2_e$, equivalent to the insertion speeds of the original methods. This indicates that the inserted number of nodes on DS-tree is reduced for the prefix node, about 1.08 to 3.85 times for $S2_e$ and 2.27 to 9.96 times for $S2_u$. And, the average of search length for children search is reduced of SIBLING.

The deletion speed of the proposed method is as well or faster than the original methods. This indicates that the number of deleted nodes on DS-tree is reduced for the prefix node and the complexity of the judgment whether parent has only one child is reduced for SIBLING.

The used space of the proposed method is larger than the DA+T method for $S2_e$ about 1.30 times. This indicates is using SIBLING. However, the proposed method is well or smaller than other methods for $S2_e$, about 0.94 to 0.28 times. Especially for $S2_u$, the proposed method is smaller than the original method, about 0.10 to 0.76 times. This indicates that the number of nodes on DS-tree is reduced for the prefix node.

Simulation results prove that the proposed method is more efficient.

5 Conclusion

This paper proposed the search method which used the digital search tree compressed all one-way branches used the double-array structure, and the searching and updating algorithms are also shown. In addition, the space and time efficiency of the proposed method is shown from the simulation result.

As a future research work, we plan to implement the proposed method on an actual system to verify practicality and evaluate applicability.

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