An Form Coalition Algorithm in Multi-Agent System

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Abstract

In multi-agent systems, autonomous agents may form coalition to increase the efficiency of problem solving. But the current coalition algorithm is very complex, and cannot satisfy the condition of optimality and stableness simultaneously. To solve the problem, an algorithm that uses the mechanism of distribution according to work for coalition formation is presented, which can achieve global optimal and stable solution in sub-additive task oriented domains. The validity of the algorithm is demonstrated by both experiments and theory

Key words: *multi-agent system (MAS); coalition; coalition utility*

1. Introduction

In multi-agent system(MAS), the autonomous agents may form coalition to increase the efficiency of problem solving, and divide the additional utility. But how to form a coalition that is both holistic optimal and stable is a critical issue. In this paper, an algorithm is presented to solve this problem.

2.Agent Coalition

In MAS, suitable rules must be specified to encourage agents to form coalition [1]. Usually the Shapley value is adopted[2], which prescribes the utility that the agent should achieve to be equal to the weighted average value of the increased utility that the agent contributes to the coalition in stochastic order and the order's probability is given by

$$u_i = \sum_{S \subset n, i \notin S} \frac{(n-|S|-1)!|S|!}{n!} \times v(S \bigcup \{i\} - v(S)). \quad (1)$$

Zlokin cited Rosenschein's theory, and provided several characteristics for agents to form coalition as validity, stability, simplicity, distribution and symmetry.

As for a stable, efficient and optimal coalition, the characteristic listed above is inadequate. The characteristic of symmetry is dispensable, because it damages the stability of agent interaction, which can be seen from the three postmen problem (TPP) as shown in Fig. 1.

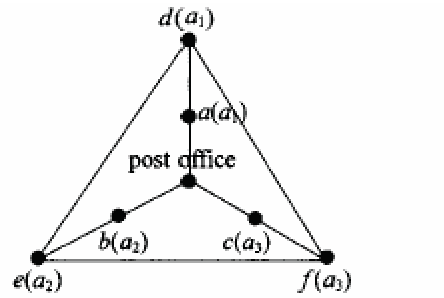

Fig. 1 Example of an unstable encounter

The post office is in the center. The length of the every arc is 1。The paths of three agents are $(T_1 = \{a,d\}, T_2 = \{b,e\}, T_3 = \{c, f\})$. The cost of every agent is 4.If any two agents cooperate, then the cost will be 5, and the total utility will be $(2\times 4) - 5 = 3$. If three agents cooperate, then the cost will be 8, and the total utility will be $(3\times 4) - 8 = 4$. So the payoff vector (u_1, u_2, u_3) will satisfy the following conditions:

① $\forall i \in \{1,2,3\}\ u_i \ge v(\{i\}) = 0;$

② $\forall i \ne j \in \{1,2,3\}\ u_i + u_j \ge v(\{i, j\}) = 3;$

③ $u_1 + u_2 + u_3 \ge v(\{1,2,3\}) = 4.$

Because α_1 can only achieve utility 4/3 in the holistic optimal solution $\{\alpha_1, \alpha_2, \alpha_3\}$ when it uses the Shapley value method, and is less than the utility 3/2 which it can achieve in the local optimal solution $\{\alpha_1, \alpha_2\}$, the coalition $\{\alpha_1, \alpha_2, \alpha_3\}$ is unstable. There are always two agents who would rather achieve higher utility in local sub-coalition.

Besides these, the mechanism didn't consider the velocity of the coalition formation. The agent might wait until the other agents join the coalition in order to achieve more utility. For example, three agents form a coalition, and the utility is:

$$v(\{\alpha_i = 1,2,3\}) = 0,$$

$v(\{\alpha_i, \alpha_j\} \mid i \neq j) = 2,$

$v(\{\alpha_1, \alpha_2, \alpha_3\}) = 4$

If the utility that an agent achieves equals the utility that the system increases: $v(\{\alpha_i\} \cup c) - v(c)$, then the agent who first joins the coalition will achieve utility 1, and the agent who joins later will get utility 2.So all the agents would wait until others join the coalition, therefore a deadlock is formed. An efficient coalition formation mechanism should avoid such a phenomenon, and the utility that agents have achieved should not decrease.

From the above, the mechanism of the coalition formation should satisfy the following characteristics: holistic optimality, stableness, simplicity, distribution, non-decrease, efficiency and asymmetry. From these points of view, the Shapley value method and Zlokin's mechanism are inadequate, which cannot ensure the holistic optimal; its computation is very complex; the extension of the coalition will decrease the utility of some agents, which might lead to an unstable coalition; the payoff that an agent gets in the coalition has no relationship with the order in which the agent joins the coalition, which cannot encourage the agent to form coalition timely.

3. Algorithm

In order to make the coalition not only holistic optimal but also stable, the non-decrease utility distribution must be used.

3.1 Agents Form a Coalition

When a lot of agents form a coalition, the utility distribution is "distribution according to work".

① Before the coalition forms, the total payoff is assumed to be C_b。 After the coalition is formed, the total payoff is assumed to be C_a. So the utility that the systems achieve is:

$$u = C_b - C_a \qquad (2)$$

②The utility u is divided into two parts pro rata $C_a : u$.

The inherent part is defined as

$$u_1 = uC_a / (C_a + u) \qquad (3)$$

The increased part is defined as:

$$u_w = uu / (C_a + u) \qquad (4)$$

③As for the inherent part, the distributive rule is share alike $u_i^1 = u_1 / n$

④As for the increased part, the distributive rule is distribution according to work, it is as follows: The utility that the agent i contributes to the coalition is Δu_i, then $u = \sum_{i=1}^{n} \Delta u_i$. The increased part that agent i should achieve is $u_i^w = u_w \Delta u_i / u$

⑤So the utility that agent i can achieve in the system is $u_i = u_i^1 + u_i^w$

3.2 Two Coalitions Form a New Coalition

When the two coalitions $\{\alpha_i \mid i \leq n\}$ and $\{b_j \mid j \leq m\}$ that exist already want to form a new coalition $\{\alpha_i\} \cup \{b_j\}$, the distribution rule is also "distribution according to work", the difference is that the former coalition can be regarded as an integer.

The single agent's strategy is to join the coalition, in which it can achieve the maximum utility.

The above contract, utility distribution rules and the strategy of the single agent compose the algorithm of coalition formation.

4.Experiments

In the postmen experiment, three algorithms are used to make a comparison among them.

①The Shapley algorithm with utility vector u^1;

②The algorithm presented by Luo and Shi [3] with utility vector u^2;

③ The algorithm presented in this paper with utility vector u^3

4.1 Experiment 1

When three different agents form the coalition simultaneously, and the tasks are finished by α_1, then three kinds of the utility distributions by three algorithms are show in Tab. 1.

Tab.1 Results of experiment 1

utility vector	u_1^i	u_2^i	u_3^i
u^1	4/3	4/3	4/3
u^2	4/3	4/3	4/3
u^3	20/9	8/9	8/9

Tab. 1 shows that the utility gained by system is distributed uniformly among three agents by using the Shapley algorithm and the algorithm in Ref. [4] ， which cannot reflect the difference of agents' contribution to the coalition. No one would do the work, because whether one does the work or not, the gain is the same (4/3). From this point of view, the efficiency of Shapley algorithm and the algorithm in Ref.[4] is rather low. By using the algorithm in this paper, since the contribution by α_1 is more than the other two α_2, α_3, it gets more utility (20/9) than α_2 (8/9) and α_3 (8/9). This adds efficiency to the coalition formation.

4.2 Experiment 2

When α_1, α_2 has already formed a coalition, in which α_1 does the work, and then α_3 wants to join. Three kinds of utility distributions obtained by the three algorithms are shown in Tab .2

Tab.2 Results of experiment 2

utility vector	u_1^i	u_2^i	u_3^i
u^1	4/3	4/3	4/3
u^2	11/6	11/6	1/3
u^3	1 027/432	533/432	8/27

Tab. 2 shows that by using Shapley algorithm the coalition is unstable, because there are always two agents who want to form a new coalition to get the utility (3/2), which is more than the utility (4/3) of three agents' coalition. The algorithm in Ref.[4] prescribes that the newly added utility is distributed uniformly, which is less efficient for agents to form coalition. Because α_1 does the work and α_2 doesn't do the work, but their gains are the same value (11/6).

By using the algorithm in this paper, after the coalition $c = \{\alpha_1, \alpha_2\}$ has formed, α_3 has three ways to choose: ①To join the coalition;②To choose one of α_1, α_2 to form coalition with himself; ③To keep independence. Obviously, the strategy ③ cannot increase the utility. Through②, α_3 cannot achieve positive utility. For example, if α_3 wants α_1 to form a new coalition c ' with him, then $u'(\alpha_1)$ must bigger than $u(\alpha_1) = 33/16$ and c ' needs to pay α_2 the utility 33/16. 6. So in the coalition $\{\alpha_1, \alpha_3\}$, α_3 can only achieve utility $u'(\alpha_3) = u'(\{\alpha_1, \alpha_3\}) - u'(\alpha_1) - u(\alpha_2)$ which is less than zero. Thus α_3 can only take the strategy ①Similarly, if α_3 wants α_2 to form a new coalition with him, the utility that α_3 achieves is also less than zero. After the coalition $\{\alpha_1, \alpha_2, \alpha_3\}$ has formed, if it wants to exist, then the compensatory cost 19/27 is higher than the utility 8/27 it can achieve in the coalition. So the optimal strategies of all the agents are to keep the coalition holistic optimal. From this point of view, the coalition by using the algorithm in this paper is stable and efficient.

5 Analyses of Algorithm's Performances

Zlotkin provided the theory of task oriented domains (TODs)[3], which indicates that if $v^*(T_1)$ and $v^*(T_2)$ are the minimal executive costs respectively for any two tasks T_1, T_2, then $T_1 \le T_2 \to v^*(T_1) \le v^*(T_2)$. Sub-additive TODs means $v^*(T_1 \cup T_2) \le v^*(T_1) + v^*(T_2)$ for any two tasks T_1, T_2. As for n tasks $\{T_i\}$, they have the minimal cost and maximal utility apparently. There are many examples of TODs, such as three Postmen problem, Brick problem [3], etc. In the following, the performances of the algorithm in this paper will be analyzed in sub-additive TODs.

5.1 Holistic Optimality

Lemma 1 By using the algorithm in this paper, suppose that $A = \{\alpha_1, \alpha_2, ... \alpha_n\}$, and the utility of coalition $c_i\{\alpha_{i1}, \alpha_{i2}, ... \alpha_{in}\}$ is $v(c_i)$, then the coalition will be optimal, namely

$$\sum_{i=1}^{n} v(c_i) = v(A) \quad (5)$$

Proof Assume that there is a coalition $\{c_i = \{\alpha_{i1}, \alpha_{i2}, ... \alpha_{in}\} \mid i = 1, 2, ..., l\}$. If $\sum_{i=1}^{n} v(c_i) < v(A)$, then there must be two coalitions c_1, c_2 which will form a new coalition with $v(c_1 \cup c_2) - v(c_1) - v(c_2) > 0$. If not, because $v(c_1 \cup c_2) - v(c_1) - v(c_2) \ge 0$ in TODs, then for any two coalitions, there will be $v(c_1 \cup c_2) - v(c_1) - v(c_2) = 0$. So $\sum_{i=1}^{n} v(c_i) = v(A)$, and the coalition is already optimal.

If *A* doesn't reach the optimality, then there must be *k* coalition, which would form a new coalition. Assume that the coalition is $B = \{c_1, c_{2,} ... c_k\}$ now. It can be proved that there must be two coalitions c_i, c_j; that would form a new coalition, if any one in *B* proposes to do that. For example:

If c_1 proposes to form a new coalition with c_2, then there will be two ways for c_2 to select: To form a new coalition with c_1, or form a new coalition with $B = \{c_1, c_{2,} ... c_k\}$ now. If it selects the former, then the lemma is proved; if it selects the later, and c_3 is assumed to be selected, then there are also two ways for α_3 to select: one way is to select c_2, the other is to select $B = \{c_1, c_{2,} ... c_k\}$. If it selects c_2, then the lemma is proved. If it selects $B = \{c_1, c_{2,} ... c_k\}$, then following this way, c_{k-1} proposes to form a new coalition with c_k .For c_k, there is only one way, which is to form a new coalition with c_{k-1}.

Similarly, if one coalition proposes to form a new coalition with multi coalition, then it also can be proved.

5.2 Stability

Lemma 2 In sub additive TODs, any coalition cannot be decomposed, which means that any individual cannot decompose from coalition and any sub coalition cannot decompose from coalition also.

Proof If a coalition will $c = \{\alpha_1, \alpha_2, ... \alpha_n\}$ will decompose, then assume that the new coalition is $c' = \{\alpha_1, \alpha_2, ... \alpha_m \mid m < n\}$. Assume that the utility vector of c is $(u_1, u_2, ... u_n)$, then $\sum_{i=1}^{m} u_i < \sum_{i-1}^{n} u'_i$. By using the algorithm in this paper, c' must pay c the losing utility $\sum_{i=m+1}^{n} u_i - u(\{\alpha_{m+1}, ... \alpha_n\})$ So the utility of c' is

$$u'(\{\alpha_1, \alpha_2, ... \alpha_m\}) =$$

$$\sum_{i=1}^{m} u'_i + \left(\sum_{i=m+1}^{n} u_i - u(\{\alpha_1, \alpha_2, ... \alpha_n\}) \right) >$$

$$u(c) - u(\{\alpha_{m+1}, ... \alpha_n\}). \qquad (6)$$

Then $u(c') + u(c - c') > u(d)$. This contradicts the theory of TODs. Thus the coalition c cannot decompose.

4.3 Other Attributes

①Simplicity; ②Distribution; ③Non-Decrease; ④High Efficiency; ⑤Asymmetry.

5 Conclusion

The algorithm doesn't need center control mechanism, and a compensative method is adopted to encourage agent to form coalition. Zlotkin provided a mechanism based on Shapley value, which has a very high complexity, and the coalition is stable only in some sub additive TODs occasions [4].Luo and Shi provided a share alike solution [3], which is less efficient. The experiments show that the algorithm in this paper can satisfy the characteristics of holistic optimality, stability, simplicity, distribution, non-decrease, high efficiency, and asymmetry and is better than Shapley value algorithm and the algorithm in Ref.[4].

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