Particle Swarm Optimization Based on Model Space Theory and Its Application on Transmission Network Planning

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Abstract: - A model space and space compression theory is developed. The theory defined an N-1 security network as a model, and then defined a model space according a given model. Using this theory in Particle Swarm Optimization can prevent particles form searching in fund areas and improve their search efficiency. Put forth a model structure mutation method, which can make particles jump out of local optima. And compared the performance of basic PSO and model based PSO (MPSO). Numerical simulation results demonstrate this theory is correct and efficient.

Key-Words: - Transmission network expansion planning, Particle swarm optimization, Model, Model space, Model structure mutation

1. Introduction

Particle Swarm Optimization's [1~5] (PSO) strongpoint is fast convergence velocity [6], however, when the size of power network is large, its calculation velocity is also slow. This is because PSO doesn't store some useful information, that particles may search the same space more than once. Furthermore PSO is tending to converge to local optima, because the global-best particle or local-best particles have great effects on every particle. To overcome these two flaws, this paper developed a model space and space compression theory according to transmission network planning (TNP) characters. Appling this theory to PSO can increase its calculation velocity. Moreover, for the purpose of find areas hadn't found before by particles, a model structure mutation method is brought forward. This method use model information to produce new solutions, which ensure particles find new areas, and then insure the algorithm's overall convergence performance.

2. Model Space and Space Compression

The task of TNP is to find a network expansion scheme, which can satisfy economic and secure demands at the same time.

TNP problem can be formulated as follows.

$$F = \sum_{i=1}^{m} A_i x_i \tag{1}$$

$s.t.$

$$\sum p_{ij} = P_{Gi} - P_{Di} \quad j\omega i \quad i \in B \tag{2}$$

$$\sum p_{ij}^{(N-1)} = P_{Gi}^{(N-1)} - P_{Di}^{(N-1)} \quad j\omega i \quad i \in B \tag{3}$$

$$-p_i^{\min} \le p_i \le p_i^{\max} \quad i \in m \tag{4}$$

$$-p_i^{\min} \le p_i^{N-1} \le p_i^{\max} \quad i \in m \tag{5}$$

Fitness value F is construction costs of new lines. A_i is the cost of line i; Restriction (2) and (3) are power equivalent law for buses of N security and N-1 security network respectively. Restriction (4) and (5) are lines' load restriction of N security and N-1 security network respectively. m is the total number of right-of-way. B is total bus number.

A local optimum in TNP context is a safe but not the most economic network. That is to say, the local optimum satisfied the constrictions from (2) to (5), but can't insure the value of function (1) minimal. Taking an N-1 security network for consideration, it consists of two parts: exiting lines and expansion lines. Expansion lines scheme can be represented by array $\boldsymbol{p}$

$$\boldsymbol{p} = (x_1, x_2, \ldots x_m)$$

x_i is integer from 0 to upper bound constraint of circuit number can be added in right-of-way i. m is the total number of right-of-way.

Definition 1: If $\boldsymbol{p}$ *constitutes an N-1 security network, then define* $\boldsymbol{p}$ *as a model.*

It can deduce from definition 1 that any N-1 security expansion network $\boldsymbol{p}$ is a model; however its lines are redundant.

There still needs model sequence and the smallest model definitions before model space can be defined.

Definition 2: There have two models $\boldsymbol{p_i} = (x_{i1}, x_{i2}, \ldots x_{im})$ *and* $\boldsymbol{p_j} = (x_{j1}, x_{j2}, \ldots x_{jm})$, *if* $x_{il} \le x_{jl}$ $(l=1,2, \ldots, m)$, *then* $\boldsymbol{p_i} \le \boldsymbol{p_j}$.

Definition 3: decreasing the line number of any dimension of model $\boldsymbol{p_i}$ *won't make expansion network become a model, then* $\boldsymbol{p_i}$ *is a local model optimum.*

The meaning of *Definition 3* is that if cut any line in $\boldsymbol{p_i}$ will not make the network satisfy N-1 security, and then $\boldsymbol{p_i}$ is *a local model optimum.* There many have many models of different structures in problem space. Now we can define model space based on the former three definitions.

Definition 4: If $\boldsymbol{p_i}$ *is a local model optimum, then all the models which are bigger than* $\boldsymbol{p_i}$ *and model* $\boldsymbol{p_i}$ *constitute a model space with regard to* $\boldsymbol{p_i}$.

These four definitions are not only mathematic definitions, but also in the context of TNP. Their mathematic definitions would lose their meaning without their TNP meaning.

Now let's analyze the rationality of their TNP meanings.

The meaning of definition 1 is very straightforward. The key definition is definition 2 in the model theory. From the practice condition of TNP, if $\boldsymbol{p_i}$ is an N-1 security network, then add any line to this network will increase its safety and redundancy simultaneously; however its investment is also increased. From the N-1 security and economy viewpoints, this increase is diseconomy and unnecessary. So, in definition 2, any model $\boldsymbol{p_j}$ formed by increase line number in any dimension of model $\boldsymbol{p_i}$ is bigger than $\boldsymbol{p_i}$, that is to say $\boldsymbol{p_j}$ is a diseconomy model. The meaning of Definition 3 is all appearance on the basis of definition 2. When cut any line of $\boldsymbol{p_i}$ will not make $\boldsymbol{p_i}$ satisfy N-1 security rule, then $\boldsymbol{p_i}$ is a local model optimum, namely a local optimum expansion scheme. Definition 4 indicates that a model space is a combination of networks represented by its local model optimum. There is no expansion schemes can exceed their local model optimum in the model space as far as economy and safety concerned.

Now we can easily get space compression conception after former analysis. In optimal process, when we get a local model optimum, we can define a model space with regard to the local model optimum, thus divide the problem space into a model space and an unknown space, and record the model space, which is represented by a local optimum point. In this conception, the problem space is divided and compressed.

3. Model Structure Mutation

From the former definition we can educe that we can't get a more excellent solution by increase a line of any dimension of a local model optimum $\boldsymbol{p_i}$, and also can't get an N-1 security network by cut any line of any dimension of $\boldsymbol{p_i}$. For the purpose of make the algorithm find a new space hasn't searched before, this paper devolved a model structure mutation method. From the practice condition of TNP, when cut a line from a local model optimum, a feasible expansion scheme can only be anticipated by add one or more lines to other right-of-way, this may constitute a new model out of model spaces in existence. If this mutation solution is still in model spaces in existence, continue mutation until a new model space is found or stop iteration in specified iteration times.

For the model structure mutation, there define an incompatible model concept.

Definition 5: There have two models $\boldsymbol{p_i}$ *and* $\boldsymbol{p_j}$, *if* $\boldsymbol{p_{jl}} < \boldsymbol{p_{il}}$ *and* $\boldsymbol{p_{jk}} > \boldsymbol{p_{ik}}$ *then define* $\boldsymbol{p_i}$ *and* $\boldsymbol{p_j}$

incompatible model.

It's apparent that incompatible models are mutation structure reciprocally from definition 5.

Two following methods can be used for model structure mutation

1) Subtract-Add line method

From the incompatible model definition, a simple method can be found for structure mutation: subtract a line from l dimension while add a line to k dimension from a local model optimum $\boldsymbol{p_i}$. After this change, a new model alien to $\boldsymbol{p_i}$ come into being, but it's not always a model alien to all other local model optimum in existence. If it isn't alien to all the searched models, then keep mutation till an entire new model is found, or stop calculation after given iteration times.

2) Reproduction method

Randomly select two incompatible models $\boldsymbol{p_i}$ and $\boldsymbol{p_j}$. According to definition 5 "$\boldsymbol{p_{jl}}<\boldsymbol{p_{il}}$ and $\boldsymbol{p_{jk}}>\boldsymbol{p_{ik}}$", we can safely suppose that $l<k$, and then the following reproduction method can be adopted:

$$\left.\begin{matrix} x_{i1},\cdots x_{il},\cdots x_{ik},\cdots x_{im} \\ x_{j1},\cdots x_{jl},\cdots x_{jk},\cdots x_{jm} \end{matrix}\right\} -> \qquad (6)$$
$$\begin{cases} x_{i1},\cdots x_{il},\cdots x_{jk},\cdots x_{jm} \\ x_{j1},\cdots x_{jl},\cdots x_{ik},\cdots x_{im} \end{cases}$$

The same as to method 1, the two new particles produced by (6) are not always models alien to all other models in existence. If they are not alien to other models, then keep mutation till an entire new model is found, or stop calculation after given iteration times. This method derived from Genetic Algorithm's (GA) reproduction idea. By reproduction, new particles can inherit good information found by their parents.

4. Model Space based PSO (MPSO) used in TNP

In MPSO calculation, there needn't to assure that a model is a local model optimum, because it will spend much calculation time especially in large network planning. In this paper, an iterative approach mechanism is adopted. Compare new particles to smaller models in existence, as soon as new particles are formed by velocity and position update. If particle bigger than smaller models in existence, no power flow calculation, N-1 check and fitness value calculation are needed for the particle. If particle is smaller than or incompatible to smaller models in existence, then do power flow calculation, N-1 check and fitness value calculation. If it is an N-1 security network after check, compare it with all of the smaller models, and delete all those models bigger than it. By this process, problem space can be compressed step by step, while avoids much unnecessary calculation. If do power flow calculation, N-1 check and fitness value calculation for every particle without model compare, its calculation quantity is very large. At the same time, some simple methods such as heuristic method can be use to initialize N-1 security network as basic models. These models have prefect characters that may diminish problem space notability, and then accelerate the total calculation process.

In MPSO calculation, the start bus and end bus of right-of-way are stored, while they don't needed in MPSO calculation, because they don't change in iteration. Particle can represent by array $\boldsymbol{p_i}$ directly, and velocity also can be represented by isomorphic array $\boldsymbol{v_i}$. Particle position and velocity is updated by equations (7) and (8).

$$v_{id}^{k+1} = Fix(c_1 r_1 v_{id}^k + c_2 r_2 (p_{id} - x_{id}^k) + c_3 r_3 (p_{gd} - x_{id}^k)) \qquad (7)$$

$$x_{id}^{k+1} = x_{id}^k + v_{id}^{k+1} \qquad (8)$$

$$(i = 1,2,\cdots n \quad d = 1,2,\cdots m)$$

Operation *Fix* is getting the integer part of variant. c_1 is inertial weight. c_2 and c_3 are learning factors. Usually c_1 is 0.9, c_2 and c_3 are 2, r_1, r_2 and $r_3$are random number between 0-1, superscript is iterative times, n is particle population size, m is dimension of particle. Particle dimension equal to number of right-of-way that can add new lines.

MPSO planning flow chart is shown in fig.1.

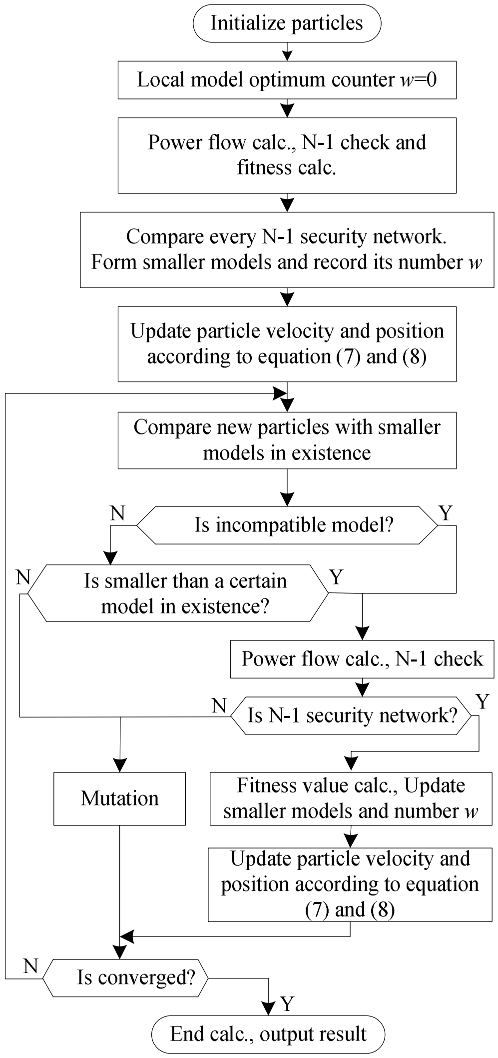

Fig.1 MPSO planning flow chart

From the flow chart we can see that all the power flow calculation, N-1 check and fitness value calculation are done after model compare except the initial particles. This insures the algorithm's efficient. Furthermore, those invalid particles mutate immediately. This configuration can avoid invalid particles engross the calculation time and computer memory, decrease the population size demand, and make particles have motive to search the entire problem space. In convergence judgments, use that fitness value didn't change in a given iterations or mutation can't find new model in a give iterations (in 30) as convergence judgment. Because the judgment that there is no new model found by model mutation in a given iterations can more easily confirm its convergence than that of fitness value didn't change in a given iterations. This convergence judgment insures the overall convergence performance of MPSO.

5. Numeric Simulation

5.1 Example 1

This example use garver's 6-bus system [7], its optimal result is as follow. (Bold lines are lines in existence; thin lines are new lines added)

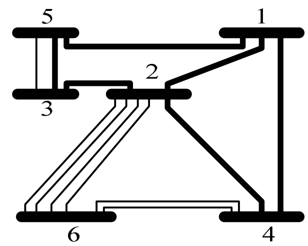

Fig.2. Optimal result for 6-buses network

MPSO and basic PSO compare is show in table 1. From table 1, we can find out that MPSO's population size demand is smaller than that of basic PSO's remarkably. Basic PSO converged 30 times only when population size is 80. When the size decreased to 20, it convergence times also decreased and only 7 times occurred. While MPSO only needs 10 particles to insure all 30 times calculation converged. As far as the calculation time and convergence performance concerned, MPSO is better than basic PSO observably.

Table 1 MPSO and Basic PSO compare

	Basic PSO	Basic PSO	MPSO
Right-of-ways can add lines	15	15	15
Population size	80	20	10
Simulation times	30	30	30
Convergence times	30	7	30
Calculation time (s)	54~79	39~47	13~28

GBest particle's fitness value curves in MPSO and basic PSO are shown in figure 3. From figure 3, we can conclude that single particle's convergence performance has no much difference in MPSO and basic PSO, although particle in MPSO is a little faster than basic PSO. From the entire performance of these two algorithms, we can find out that MPSO's fast performance attribute to its pertinent calculation strategy. MPSO doesn't calculate invalid particle's power flow and fitness value. And mutation these invalid particle immediately, therefore decrease the population demand. Fewer particles make MPSO more quickly and efficient.

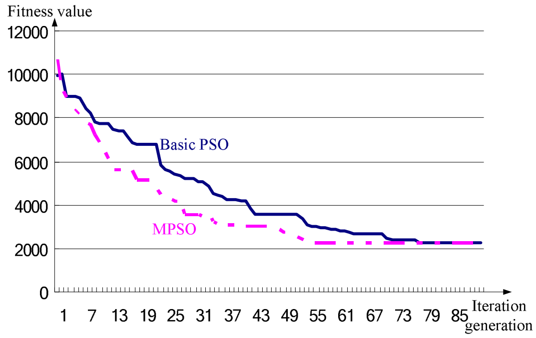

Fig.3. Particle's Fitness value curves

5.2 Example 2

This example use 46-bus system [8]. Optimal result is shown in table 2, compare result is shown in table 3. GBest particle's fitness value curves in MPSO and basic PSO are shown in figure 4.

As can be seen from table 3, MPSO's advantages are more noticeable when the particle dimension is increased. Basic PSO's population size is 5 times than that of MPSO's. And because particle dimension is larger in large power system, this decrease in population size makes the MPSO more quickly in large-scale network.

Table 2 optimal result for 46-bus system

Serial number	Right-of-way	Circuit number
1	8--13	1
2	14--15	1
3	46--10	1
4	5--11	1
5	28--31	1
6	28—30	1
7	24--34	2
8	26--29	1
9	24--33	1
10	46--11	1
11	24--25	1
12	40--41	1
13	40—42	3
14	2--3	1
15	5--6	1

Table 3 MPSO and Basic PSO compare for 46-bus system

	PSO	MPSO
Right-of-ways can add lines	79	79
Population size	250	50
Simulation times	30	30
Convergence times	29	30
Calculation time (s)	319~583	75~134

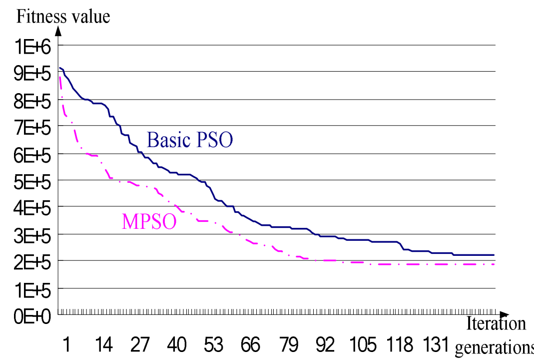

Fig.4. Particle's Fitness value curves

6. Conclusion and Prospect

This paper first developed a model space and space compression theory according to the characters of transmission network expansion planning. The following two characters represent its merits:

1) All the power flow calculation, N-1 check and fitness value calculation are done after model compare except the initial particles. This reduces much unnecessary calculation time, and insures the algorithm's efficient
2) Model structure mutation provides a mechanism for particles to jump out of local optima, avoids invalid particles occupying computer memory and reduces the population size needed for MPSO. Fewer particles make MPSO more quickly and efficient.

The former two examples demonstrate MPSO is efficient optimal theory.

The five definitions of model space theory provide an optimal theory foundation for transmission network expansion planning.

Model space theory not only can combine with PSO but also can combine with many other algorithms such as Genetic Algorithm (GA), Ant Colony Optimization (ACO). Model space theory can provide abundant information to direct search process of these algorithms.

Model space theory can also provide optimal theory basis for other engineering optimal problems besides TNP. The key definition in model space theory is definition 2 (sequence character). If it can constitute a sequence character for a specifically engineering optimal problem, model space theory can be used in its optimization

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