Study on the WaitingTimes in Chinese Stock Markets

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Abstract: - In this paper, we investigate the statistical properties of the waiting times between two successive price changes above a fixed threshold (or a fixed point) for the indices of Shanghai Stock Exchange and Shenzhen Stock Exchange. The database is from the indices of Shanghai and Shenzhen in the 10-year period from January 1997 to December 2006, and the empirical research shows that the distribution of the waiting times for price changes follows power-law decay. Further, for different values of the threshold, we discuss the conditional distributions of returns for the indices of Shanghai and Shenzhen.

Key-Words: - waiting times; power-law distribution; price changes; returns

1 Introduction

In this paper, we study the statistical properties of the waiting times between two successive price changes above a fixed point or above a fixed threshold in Chinese stock markets. China has two stock markets, Shanghai Stock Exchange and Shenzhen Stock Exchange, and the indices studied in the present paper are Shanghai Composite Index and Shenzhen Composite Index. These two indices play an important role in Chinese stock markets. The database is from Shanghai Stock Exchange and Shenzhen Stock Exchange, see www.sse.com.cn and www.sse.org.cn. We select the data for the closing prices of each trading day in the 10-year period from January 1997 to December 2006, the total number of observed data is about 2400 for Shanghai Composite Index and Shenzhen Composite Index respectively. Through analyzing this data, and also by the plots in the present paper, we try to discuss the statistical properties of the waiting times and the conditional returns (price changes above a fixed point).

In recent years, the empirical research in financial market fluctuations has been made. Many statistical properties for market fluctuations uncovered by the high frequency financial time series, such as fat tails distribution of price changes, the power-law of logarithmic returns and volume, volatility clustering which is described as on-off intermittency in literature of nonlinear dynamics, and multifractality of volatility, etc.. In this paper, we study another random variable of the market fluctuations, the waiting time distribution of price changes. Above the threshold, we show that the waiting time distribution follows a power-law decay distribution, and the property of power-law exponents and the conditional distribution of returns are also discussed in the following of the present paper.

There is a long tradition of studies on power-law scaling in financial markets. Scaling of market prices was first reported by Mandelbrot in 1963 in his work on cotton prices. He presented empirical evidence that the distribution of daily fluctuations in cotton prices followed a power-law asymptotic behavior, characterized by an exponent $\alpha = 1.7$. He also proposed stable Levy distributions as candidates for the probability density function of price changes in financial assets. Fama analyzed quantitatively the daily data on the share prices of 30 companies over a five-year period in 1965, and showed a power-law distribution of the price changes. Resent studies have confirmed the presence of scaled distributions of price changes in various financial markets: exchange rate markets, German stocks, the stock price index, and individual stocks, see [1-4][6-9][11]. These results suggest the possibility of the existence of universal properties of price changes.

Although the waiting time of price changes is, like price, an important quantity that characterizes the activities of financial markets, only a few attempts have so far been made to understand the statistical properties of the waiting times. In this paper we study the statistical properties of fluctuations in waiting times of price changes. By analyzing the indices of Shanghai and Shenzhen in the 10-year period from January 1997 to December 2006, we found that the probability distributions of waiting time can be well described by a power-law decay and characterized by an exponent within the stable Levy domain $0 < \alpha < 2$. Further more, we found that the distribution of the logarithm returns as the price changes above a fixed threshold also obeys a power-law decay and characterized by an exponent out of the stable Levy domain.

2 The Waiting Time Distribution of Price Changes

In this section, through analyzing the data of indices for Shanghai Stock Exchange and Shenzhen Stock Exchange in the 10-year period 1997-2006, we study the waiting time distributions of price changes. Let the price time series $P(t)$ denote the daily closing price at time $t(t = 0,1,2,\cdots,T)$, and let price changes $\Delta P(t)$ be the increment of the price $\Delta P(t) = P(t) - P(t-1)$. The normalized price change is defined as

$$s(t) = (\Delta P(t) - \overline{\Delta P(t)}) / S$$

where $\overline{\Delta P(t)} = (\sum_{t=0}^{T} \Delta P(t)) / T$, and S is the standard deviation and $\overline{\Delta P(t)}$ is a time average. Then, the total number of normalized price changes $s(t)$ for the index of Shanghai in the 10-year period is 2379. In order to study the waiting time intervals, we introduce a threshold θ for the absolute normalized price changes $|s(t)|$. A waiting time interval X is defined as the time interval, which is the time length from one day that the absolute normalized price change is above the threshold to the next day that it exceeds the same threshold, more specifically, let $X_0 = 0$, and for $i = 1,2,3\cdots$, then the waiting time interval X defined as following,

$$X_i = \min\{t : | s(t + \sum_{k=0}^{i-1} X_k - 1) | > \theta, t \in \mathrm{N}\}$$

where θ is the threshold value, so we have random waiting time series $\{X_i, i = 1,2,3\cdots\}$. According to the data from the index of Shanghai, we plot the waiting time intervals $\{X_i\}$ of the normalized price changes $|s(t)|$ in following Figure 1, and show the statistical properties of the normalized price changes for Index of Shanghai Stock Exchange (ISHSE) and Index of Shenzhen Stock Exchange (ISZSE) in the Table 1, see the reference [12].

Fig.1 illustrates of the waiting time intervals X_i of the price changes for Shanghai, where the threshold at $\theta = 1.7$. In Table 1, the kurtosis are 17.8711 and 14.46 respectively for the two Chinese stock markets, much bigger than 3, this implies that the fluctuations of the price changes are greater than that of the corresponding Gaussian distribution.

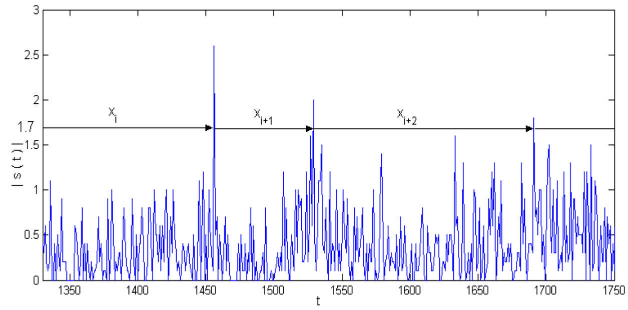


Fig.1. The time series of the waiting time interval of price changes. The vertical axis indicates the absolute normalized price change $|s(t)|$, and the horizontal axis indicates the corresponding trading date. The arrows on the figure illustrate the waiting time intervals, and the threshold $\theta = 1.7$.

Table 1. Properties of the normalized price change

Index	mean	Std.	skewness
ISZSE ISHSE	0.6696 0.6905	0.7426 0.7232	3.0967 2.6636
Index	kurtosis	min	max
ISZSE ISHSE	17.8711 14.46	0.000244 0.000297	7.6227 6.7283

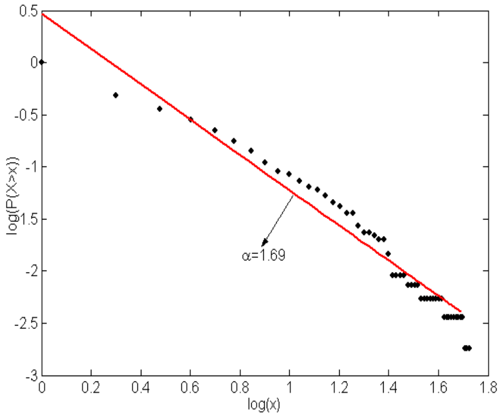
Next we show that the log-log plots of the probability distribution of the waiting time intervals of price changes with the fixed threshold at $\theta = 0.9$. In Figure(2a), the dots represent the probability distributions of the waiting times for the index of Shenzhen, where the solid line represents the power-law distribution, which is expressed by

$$P(X_{Shen} > x) \sim x^{-\alpha}$$

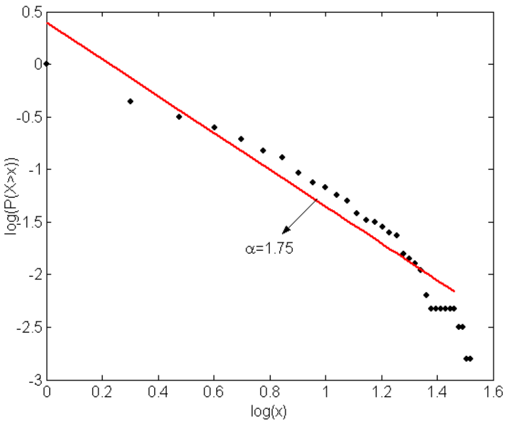
and the linear regression is applied to analyze the observed data, yields $\alpha = 1.69 \pm 0.14$ with the coefficient of determination $R^2 = 0.99$. Similarly in Figure(2b), We analyze the price series data for the Shanghai Stock Exchange Index in the period from January 1997 to December 2006, the dots represent the observed probability distribution

$$P(X_{Shang} > x) \sim x^{-\alpha}$$

with $\alpha = 1.75 \pm 0.19$ and $R^2 = 0.99$.



Figure(2a)



Figure(2b)

Fig.2. The log-log plots of the probability distribution of the waiting times for the Shenzhen Stock Exchange Index and Shanghai Stock Exchange Index in the period from January 1997 to December 2006 with the threshold at $\theta = 0.9$. The dots represent the observed probability distributions, where the solid lines represent the power-law distributions $P(X > x) \sim x^{-\alpha}$. (a) The log-log plot for Shenzhen, $\alpha = 1.69$. (b) The log-log plot for Shanghai, $\alpha = 1.75$.

3 Comparison of the Power-Law Exponents on the Thresholds

In this section, we discuss the relation of power-law exponents α with the different thresholds θ . In Figure 3, we only study the data from Shenzhen Stock Exchange Index, Fig.3 presents the plot of the probability distributions of the waiting times for the Index of SZSE at θ ranging from 0.5 to 1.7. Table2 shows the relation of the threshold θ and the corresponding power-law exponent α for Index of SZSE and Index of SHSE. From Fig.3 and Table 2, one can see that, in two stock markets, the power-law exponent α monotonically decreases with respect to the threshold θ. From Table 2, we also know that the relation of the exponent α and the threshold θ is nonlinear. And in this paper, when the threshold θ ranges from 0.5 to 1.7, we have a good description for the relation between the exponent α and the threshold θ. In fact, we are also interested in θ ranging at other area, and discuss the relation between α and θ, here we omit this part of work.

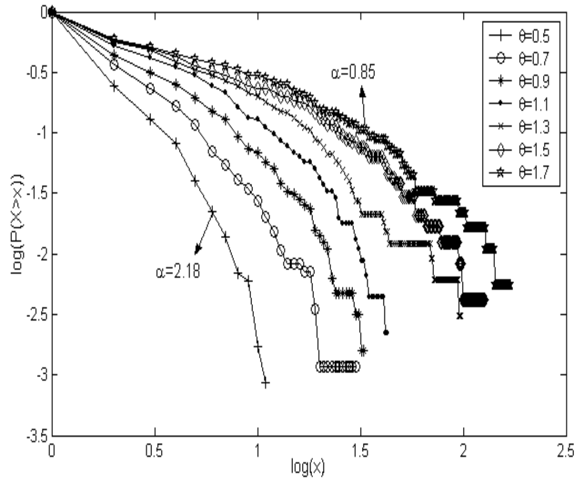


Fig.3. The plot of the probability distributions of the waiting times for the SZSE Index at threshold θ ranging from 0.5 to 1.7.

Table 2. Relation of the power-law exponent α and the threshold θ

ISZSE		ISHSE	
θ	α	θ	α
0.5	2.18	0.5	2.53
0.7	1.79	0.7	1.82
0.9	1.69	0.9	1.75
1.1	1.45	1.1	1.47
1.3	1.24	1.3	1.24
1.5	1.01	1.5	1.06
1.7	0.85	1.7	0.92
1.9	0.63	1.9	0.66
2.1	0.59	2.1	0.60

4 The Power-Law of Returns above the Threshold θ

In this section, we estimate the probability distribution of the logarithm returns for the indices of SZSE and SHSE with the threshold at $\theta = 0.9$. For a price series $P(t)$, the returns $R(t)$ over a time scale Δt is defined as the forward change in the logarithm of Δt, i.e. (see [5][10][13])

$$R(t) = \ln(P(t+\Delta t)) - \ln(P(t)).$$

In recent years, the probability distribution of fluctuations for returns has been studied, the empirical research results show that the distribution of large returns follow a power law distribution with exponent 3, that is,

$$P(R(t) > r) \sim r^{-\alpha}$$

where $\alpha \approx 3$. In this paper, we select the returns of SZSE and SHSE with the condition the threshold at $\theta = 0.9$, then study the probability distribution of the logarithm returns, and show the conditional probability distribution follows the power-law behavior

$$P\left(|R(t)| > r \;||\; s(t)| > \theta\;\right) \sim \frac{1}{r^{\beta}}.$$

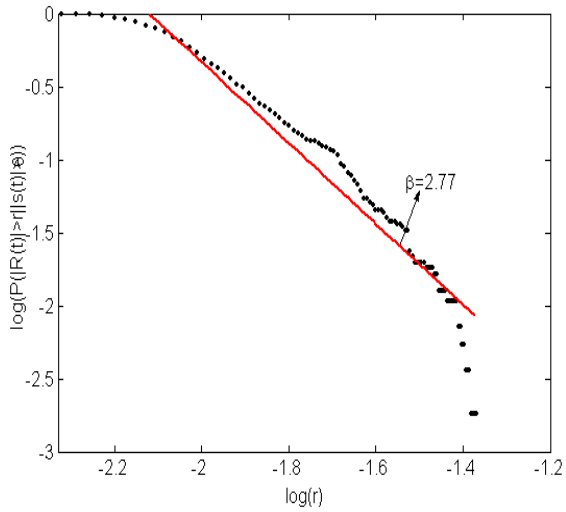
This means that, in this paper, the data of returns with the threshold at $\theta = 0.9$ is the part of data for returns, we only study the statistical properties of returns with the threshold at $\theta = 0.9$. From this data, we have the following Figure 4. Fig.4 shows the log-log plots of the corresponding returns for indices of SZSE and SHSE with the threshold $\theta = 0.9$, the dots represent the observed probability distribution, and the solid lines are of the power-law distributions, and then we have

$$\beta_{\text{shen}} = 2.77 \pm 0.09, \;\; R^2 = 0.99$$

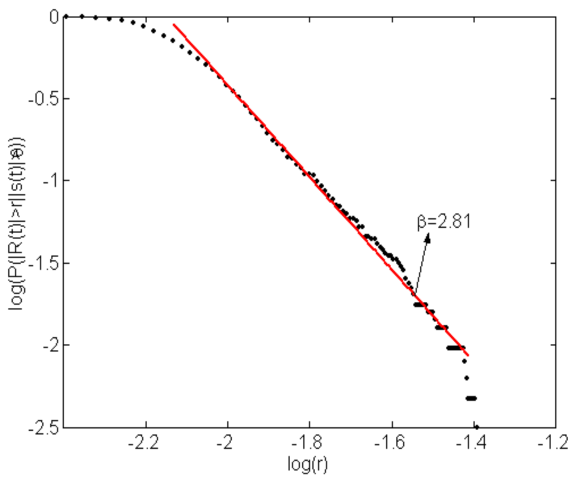
and

$$\beta_{\text{shang}} = 2.81 \pm 0.11, \;\; R^2 = 0.99.$$

Next we investigate the relation of the power-law exponent β and the threshold θ. We plot the probability distributions of the returns for SHSE Index at θ ranging from 0.5 to 1.7 in the following Figure 5. Here, we use the similar methods as we do in Section 3. Fig.5 shows the relation of the threshold θ and the corresponding power-law exponent β, one can see that the power-law exponent β monotonically increases with respect to the threshold θ. We also can obtain the similar Table as Table 2, which can give a good description for the relation between the exponent α and the threshold θ, here we omit the Table.



Figure(4a)



Figure(4b)

Fig.4. The log-log plots of the probability distributions of the logarithm returns for SZSE and SHSE Indices from January 1997 to December 2006 with the threshold at $\theta = 0.9$. The dots represent the observed probability distributions, the solid lines represent the power-law distributions, $P(|R(t)| > r \;||\; s(t)| > \theta\;) \sim \frac{1}{r^{\beta}}$. (a) For SZSE Index, β=2.77. (b) For SHSE Index, β=2.81.

5 Conclusion

In conclusion, the power-law statistics and other statistical methods are applied to analyze the data from Shanghai and Shenzhen Stock Exchanges in the 10-year period 1997-2006. Since the history of Chinese stock market is less than twenty years, we select the data by considering the financial policy of Chinese stock market during past two decades, for

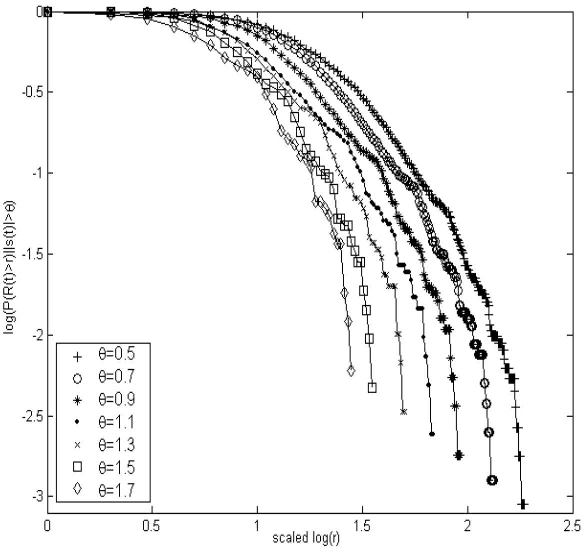


Fig.5. The plot of the probability conditional distributions of returns for Shanghai Stock Exchange Index at threshold θ ranging from 0.5 to 1.7.

example, the daily price limit (now the limit is 10%), the trading rules of SHSE and SZSE, etc.. According to the analysis, tables and figures in the present paper, we have shown the statistical properties of the waiting time intervals between large fluctuations of daily stock prices in two Chinese stock markets, and we also discuss the conditional probability distributions of returns for the indices of Shanghai Stock Exchange and Shenzhen Stock Exchange with the threshold at $\theta = 0.9$. We also have shown the power-scaling behaviors for fluctuations in waiting time of price changes, and found that the waiting time distribution of price changes obeys a power-law decay, the distribution of the logarithm returns as the price changes above a fixed threshold also obeys a power-law decay. The other main result of this paper is that Section 3 shows that, in two Chinese stock markets, the power-law exponent α monotonically decreases with respect to the threshold θ, and Section 4 shows that the power-law exponent β monotonically increases with respect to the threshold θ. The power-law statistics show here, not only an interesting theoretical finding, but presumably also a practical tool for measuring the risk of security investments.

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