STUDY OF ALGORITHMS ON MACHINES THROUGH GRAPHS II

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Abstract

In continuation of our work [5] in which we had floored an algorithm to investigate the Indeterminacy of a Plane Mechanism, here we try to give a proof of the aforesaid Algorithm. We have also given some more examples for a system of rigid bodies and their contacts to be the Plane Mechanism.

Key Words: Plane Mechanism, Graph- Theory, Algorithm, Adjacency Matrix of a Graph.

1. Introduction:

In our previous article [5], we have given an Algorithm which decides the Determination of a Plane Mechanism.
The theory of Mechanisms and Machines studies

1. Geometrical aspects of Motion.
2. Various forces involved in Motion.

The first is called Kinematics and the later is called Dynamics.

All the relevant definitions and their detailed expositions can be found in [5].

We shall be only confined to motions in two dimensions thus we are dealing with Plane Mechanisms only. Further work is under progress for Space Mechanisms.
The concept of structure diagram of plane mechanisms is helpful in identifying the unknown velocity poles. In structure diagrams the links are represented by points and each lower pair [5] is represented by a solid line joining the corresponding links. It is also possible to represent higher pairs [5] by broken lines. Thus the structure diagram of the four bar mechanism (Fig.1a) will be shown in (Fig. 1b).
The present Algorithm is Graph-Theoretic.
Many Algorithms in Graph Theory are known cf.[1,2,3],but our Algorithm is advantageous since it is easily comprehensible, and it obeys Polynomial Complexity[5].
A Graph G has a four cycle comprising of the vertices (u_1, u_2, u_3, u_4) if row and column sums of the corresponding (4 by 4) main minor is ≥ 2.
We say that the information flows in G, if by drawing the diagonals in a four cycle results in a new four cycle. Drawing diagonals is equivalent to augmenting the (4 by 4) corresponding main minor to $(K-I)_4$.
It can be easily seen that information flows in a graph if the main minor corresponding to the completed four cycle results in

another minor satisfying the above property.
A mechanism is determinate if the corresponding adjacency matrix A can be progressively augmented to become $(K-I)$, otherwise the mechanism is indeterminate.
Thus a mechanism is determinate if information keeps flowing in a graph.

Four bar mechanism

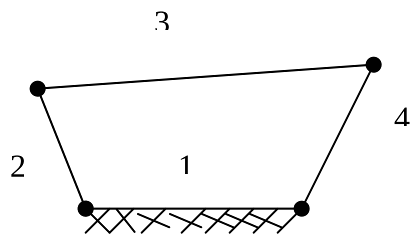

Fig. 1a

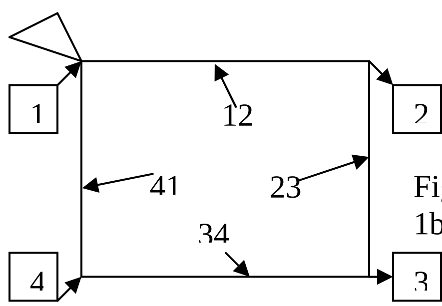

Fig. 1b

Structure Diagram

2. Problem Formulation:

In [5] we have explained that if there exists a 4- Cycle in the system then the System is Determinate otherwise it is Indeterminate. Presently we shall prove two Theorems.

Theorem 1: The number of four cycles C involving the vertex i are given by,

$$C_i = \sum_{j \neq i} a_{ij}^2 \left(a_{ij}^2 - 1\right)^+$$

Where a_{ij}^2 is the (i,j)th entry in A^2.

Theorem 2: The vertex v_j is associated with the vertex v_i in a 4- cycle if

$b_{ij} > 0$ where

$$b_{ij} = \left[a_{ij}^2 - a_{ii}^2 - a_{jj}^2 + 1\right]^+ (a_{ij})$$

Here
$$x^+ = x \quad x > 0$$
$$= 0 \quad otherwise$$

3. Problem Solution:

Before we prove the above two results we state a Lemma which is to be used in sequel.
Lemma 1:
The number of walks of length r in a Graph G joining $v_i \, to \, v_j$ is the entry in position (i,j) of the adjacency matrix A^r.

Proof: The proof of the above lemma can be found in [1].
Proof of Theorem1:
The number of paths of length 4 from the vertex $v_i \, to \, v_j$ is given by

$$a_{ii}^4 = \sum_{j=1}^{N} a_{ij}^2 a_{ji}^2 ,$$

Now for every path of length 2 from $v_i \, to \, v_i$, there exist a_{ii}^2 paths of length 4 from $v_i \, to \, v_j$ which is of the form $v_i \to v_j \to v_i \to v_k \to v_i$.
The number of such paths is $a_{ii}^2 \otimes a_{ii}^2$. Moreover for every path of length 2 from $v_i \to v_j$ of the form, $v_i \to v_k \to v_j$, there is a path of length 4 from $v_i \to v_i$ of the form $v_i \to v_k \to v_j \to v_k \to v_i$, which does not give a 4-cycle.
The number of such paths is a_{ij}^2.
Therefore the number of 4-cycles is
$$\sum_j a_{ij}^2 a_{ji}^2 - a_{ii}^2 a_{ii}^2 - \sum_{j \neq i} a_{ij}^2$$
$$= \sum_{j \neq i} \left(a_{ij}^2 - 1\right)^+ a_{ij}^2 .$$
thus proving the result.

Proof of Theorem 2:

Now a_{ij}^3 gives the number of paths of length 3 from $v_i \to v_j$. Out of these a_{ii}^2 is

The number of paths of length 2 which give rise to a path of length 3 of the type $v_i \to v_j \to v_i \to v_j$.

Similarly a_{jj}^2 gives the paths of length 2 each of which gives the paths of length 3 of the type $v_i \to v_j \to v_k \to v_j$. Of these one path is common that is, $v_i \to v_j \to v_i \to v_j$. So the effective paths of length 3 which do not retrace themselves are $\left[a_{ii}^3 - a_{ii}^2 - a_{jj}^2 + 1\right]^+$.

These will give rise to a 4- cycle if j is next to i, that is $a_{ji} = a_{ij} > 0$. Thus the 3- path from i to j give rise to a 4 cycle if

$\left(a_{ii}^3 - a_{ii}^2 - a_{jj}^2 + 1\right)^+ a_{ij} > 0$, which proves the result.

Examples of Plane Mechanisms:

1. G =(6,7).
2. G =(8,10).
3. G = (10,13).

CONCLUSION:

1. The present Algorithm is useful in exploring a Software for Watermarking, in which some advances has already being made cf.[6].The approach works with control/data flow graphs and uses ions, approximate k-partitions, and a random walk method. We are trying to use the random walk method to improve upon our algorithm.
2. We are trying to extend our present work to multiloop linkages, cf.[4].
3. Our Algorithm can also be used in Computer Network Architecture and design.
4. A Computer Program of our Algorithm is also under process.

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