Mathematical Modelling of a Living Room with Solar Radiation Source and Different Boundary Conditions

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Abstract: - Temperature and average turbulent airflows distributions in the 3D model of a living room with solar radiation modelling are developed using Ansys/CFX software. The distributions and heat balance are calculated depending on the solar radiation source through the window and on the pressure difference between opposite walls. The airflow velocities and indoor temperature with its gradients are also analysed as parameters of thermal comfort conditions. It is shown that the solar radiation source has an essential influence on the heat balance of the room and on comfort conditions.

Key-Words: - Mathematical modelling, living room, thermal radiation, solar source, heating, thermal comfort.

1 Introduction

Heat balance of a living room is very important from the energy consumption point of view and a possible solar radiation has the essential influence on it. For the first approximation this kind of heat transfer can be neglected and only heat conductivity in the boundary constructions and convection in the room with different ventilation condition can be observed. Corresponding numerical modelling for a living room with different boundary constructions, appropriate optimisation of an arrangement of the heater and ventilation system, corresponding heat consumption, temperature and velocity fields and its influence on heat balance and thermal comfort in the room is detailed analysed in [1, 2].

The radiation heat transfer from the heater's surface and through the window must be taken into account for more accurate numerical results and physical interpretation. In the sunny days due to the considerable solar heat transfer through the window required temperature in the room can be maintained with the less heating amount from the central heating system. The mathematical modelling enables to include the solar heat source and radiation heat transfer form the heater in the total heat consumption.

Other important factor, which is to be estimated for living rooms, is thermal comfort conditions. They are generally affected by many factors like velocity of airflows, humidity, absolute temperature and amplitude of the vertical temperature gradient in the room [3]. Hence it is necessary to analyse these factors in different models of living rooms.

Also an air exchange plays significant role for the rooms inhabited by humans in order to guarantee oxygen feeding, therefore airflows through openings and ventilation system are to be analysed in models with different pressure conditions. However, the greater air exchange rate means not only more fresh air, but also greater convective heat losses and increase of heating amount.

2 Problem Formulation

A living room with convective boundary conditions on border structures, heater's surface temperature and air openings is modelled. Radiation heat transfer model with an additional solar source and various angle of its attack is also included. Developed models help to predict the features of heat transfer process in the room and distributions of characteristic quantities. This approach characterises also influence of the radiation heat transfer and the conditions of the thermal comfort. *Ansys CFX* software [4] is used for development of 3D mathematical models and numerical calculations.

The calculations have been performed for the room with dimensions 2.75×4×6m shown in Fig.1, filled with an air. The window and the wall to the exterior air are modelled using different materials with heat transmittance for the window U_{window}=2.5 W/(m^2·K) and for the wall U_{wall}=0.35 W/(m^2·K). Such values are chosen similar to the room with an insulated outer wall and ordinary double-glazed window. Heat transmittance for other boundaries (walls to the adjoining rooms and to the corridor) U_{other} is set to 1 W/(m^2·K) and it

is included in heat transfer coefficient α from the surfaces using expression $\alpha = \left(1/\alpha_{st} + 1/U\right)^{-1}$ for all solid structures. Here α_{st} is standardised heat transfer coefficients from the surfaces (23.2 W/(m²·K) for outside and 8.1 W/(m²·K) for adjoining rooms).

It is assumed that the surrounding rooms (upstairs, downstairs and side rooms) have the temperature T of 20 °C, but the end wall is contiguous with a corridor or a staircase where the temperature is lower (T=15 °C). The outdoor temperature is chosen corresponding to the winter conditions (T=−10 °C). Temperature values are used for the setup of convective boundary conditions.

Solar radiation through the window is modelled as a heat source on the inner window's surface. Taking into account the measurements of a window solar transmittance [5], the solar heat flux density is set to 500 W/m² on the inner part of glass. Angle of attack for solar radiation varies from 30 to 60 degrees, which models different sun altitudes. For the radiation simulation the MonteCarlo model [4] with $2 \cdot 10^6$ histories is used. All objects expect transparent window is modelled as grey bodies with emission ε=0.9.

A small cranny between the window and the wall is created in some variants to model real gaps in old window-frames, however, in the opposite wall, there is created a ventilation opening. Opening boundary conditions with constant pressure and temperature of -10 °C and 15 °C are defined on the surfaces of crannies and ventilation opening. Pressure difference ΔP between opposite walls is set to constant 0 and 1 Pa underpressure or overpressure to model different windy conditions. Surface temperature of the heater is set to constant 50 °C. For all surfaces, except openings, non-slip boundary conditions are set.

Different features of all developed models are summarized in Table 1. The first model A represents the room with radiation modelling from the heater's surface, but without solar heat transfer through the window. The angle of solar radiation is varied in models B1-B3, what implies different powers on window surface and the additional heat source from the heater. Model C represents case with solar source through the window, but without additional heat transfer from the heater (adiabatic condition), which allows simulating a living room in winter conditions with switched off heating system. For the next tree models the room with an air infiltration is considered using different boundary conditions on the openings – model D1 without pressure difference (ΔP=0 Pa), model D2 with 1 Pa

overpressure in the room and model D3 with 1 Pa underpressure in the room.

The airflows in the room depend both on the convection created by the temperature difference and on the air exchange between the openings in the structures for the last three models. To describe the quasi-stationary behaviour of temperature and average turbulent flows, traditional differential equations are employed [6]:

- Reynolds averaged momentum equation;
- continuity equation;
- energy conservation equation.

As the velocities near the heater and in the openings are high, SST k-ω turbulence model [4, 7] is used.

The discretisation was performed with tetrahedral elements of varying size; boundary layers are discretised with smaller prismatic elements. The characteristic size of finite elements is from 20 cm in the middle of the room to 0.5 mm in the vicinity of the heating element and for the openings in the walls. Therefore, the total number of elements reaches $7 \cdot 10^5$ depending on geometry. The typical meshes in the middle cross-section of the room and near the heater are shown in Fig. 2.

The boundary conditions of the third type (convective) and the low viscosity of air essentially worsen the convergence of an iteration process. Therefore each modelling simulation has been performed in two steps – at first an steady state calculation, then transient state with constants properties and initial conditions from the steady state results. The total time required for one model calculations with a 3 GHz computer is about 2 days. The calculated heat imbalance between the heater power, the solar source and the losses from the outer surfaces and openings decreases below 3 %.
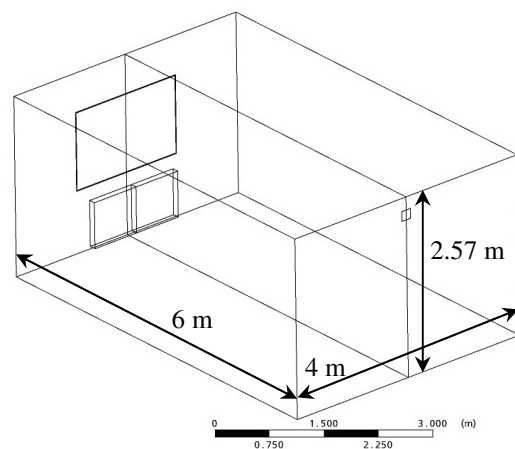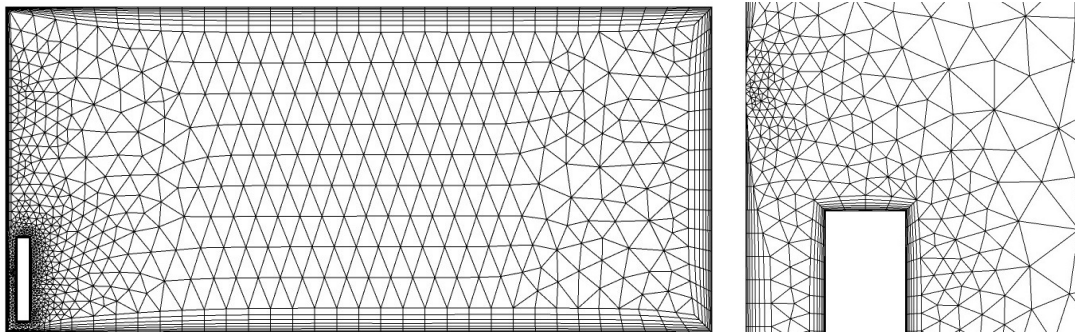

Fig. 1. A Layout of a modelled room.

Fig. 2. Typical meshes for the modelled room in the middle cross-section and near the heater.

Table 1. Properties for various considered models and main results of the modelling.

Properties/results	Model							
	A	B1	B2	B3	C	D1	D2	D3
Angle of attack (degrees)	-	**30**	**45**	**60**	45	45	45	45
Solar heat flux density (W/m^2)		500						
Boundary condition on heater		temperature, 50°C			**adiabatic**	temperature, 50°C		
Pressure difference ΔP (Pa)		-				**0**	**1**	**-1**
Heat amount for the heater (W)	225	173	173	178	0	254	232	315
Solar power W	0	411	327	228	333	326	325	325
Ventilation heat losses (W)		-				98	181	440
Air exchange rate (1/h)		-				0.3	1.4	1.4
Average velocity v (cm/s)	2	5	6	5	3	4	6	6
Average temperature T (°C)	24.2	32.5	30.8	28.4	26.0	23.1	26.1	22.4
Vertical temperature difference ΔT (°C)	2.1	1.7	1.7	2.5	1.5	1.5	1.2	2.4

3 Problem Solution

3.1 General results

Results for all nine developed models are summarized in Table 1 and visualized in Fig. 3. We can consider three result groups of models:

- the room without solar source (A);
- solar radiation with different angles of attack and constant heat flux density (B1, B2, B3);
- switched off heating system in winter conditions with solar source (C);
- inclusion of air infiltration process in the room with solar source (D1, D2, D3).

It is possible also to extract two significant parts of results – heat balance of the room (heating powers, ventilation heat losses and the air exchange) and thermal comfort conditions (average velocity and temperature, vertical temperature difference).

For the basic model without solar source an average temperature in the room is 24.2 °C and there exist a characteristic hot air flow at the top of the heater near the window (see Fig. 4a). Emission coefficient ε=0.9 set for the heater's surface in fact is greater than for the real heaters, therefore the temperature of the surface under real operation conditions may be less. However, the considered models in comparison with other developed models without radiation modelling [1, 2] show an essential influence of the radiation from the hot surfaces on the total heat balance and this kind of heat transfer can not be ignored for an accurate qualitative results.

Results of the models B1-B3 with solar source show that at the small angles of attack the solar radiation power is increased because the Sun is shining deeper into the room (Table 1, Figs. 4b-4d and Fig. 5). Thus, average temperature in model B1 with 60° angle of solar source is 28.4 °C, but in model B3 with 30° angle - 4 degrees higher. Heat amount from the radiator for models B1-B3 is nearly the same and high temperature level is determined only by solar radiation. Thermal comfort conditions in this situation are not suitable and the power of the heating system or installation of ventilation system is necessary to decrease the temperature in the room.

Results for model C with switched off heating system clearly show, that the power of solar radiation provides sufficient heat amount for the average temperature 26 °C (Table 1, Figs. 3) and an additional heating is not needed. This result is achieved for the room without air infiltration; convection heat transfer process is to be taken into account to model rooms more accurate.
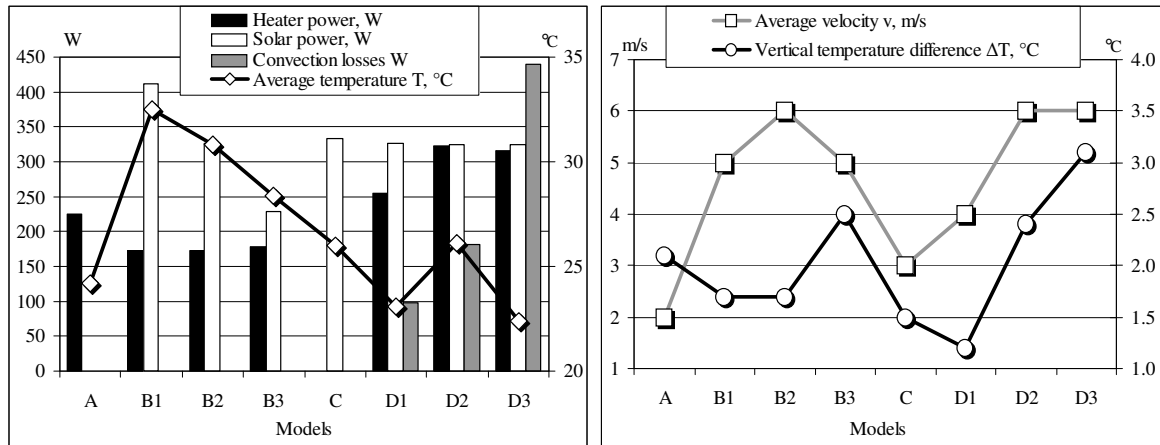
Fig. 3. Heating powers, convection losses, average temperature in the middle of the room, vertical temperature difference and airflow velocity in the middle of the room for different models.

Three different variants of air infiltration conditions in the room are considered in models D1-D3. In case when no pressure difference is set between the ventilation opening and the crannies in the window-frame, an insignificant air circulation due to thermoconvection exists in the room and the air exchange rate is 0.3 1/h. This value is minimal for an oxygen inflow needed for a human occupancy and does not produce great heat losses (Table 1).

In case of 1 Pa overpressure set on the ventilation opening (model D2), air masses with constant temperature of 15 °C are inflowing into the room. This creates intensive flow in the middle of the room (Fig, 4g) and average velocities in the room are high (Table 1 and Fig. 3). The model D3 describes the room with underpressure in the room due to 1 Pa pressure set on the crannies in window-frame, what means active cold exterior air with temperature of –10 °C inflow into the room (Fig, 4h). As a result of more intensive airflows heat transfer from the radiator remarkable increases – from 254 W without pressure difference to 315 in this model (Table 1 and Fig. 3). Cold masses form the outside decrease an average temperature in the room from 30.8 °C for model without air infiltration to 22.4 °C.

For both models with pressure difference 1 Pa (models D2 and D3) air exchange rate is 1.4 1/h; this causes great convection heat losses and is energetically disadvantageous.

3.2 Result analysis

Thermal conditions in the room are connected with the structure of an airflow velocities, pressure difference between opening in boundary constructions as well as with temperature distribution and thermal convection. Thus, an intensive upward airflow is formed at the top of the heater with maximum velocities up to 60 cm/s, at the same time an average air movement in the whole room does not exceed 5 cm/s. In case of solar radiation, hot region on the floor produces the notable convection and increase an average velocities in the room. However, the direction of this movement is horizontal in models with hot radiator because of existence of other big vortices in the room, but almost vertical in the model C with switched off heating system (Fig. 4). In general, the radiation source through the window increases airflow intensity in the middle part of the room (Table1 and Fig. 3).

A complicated multiple vortex air flow structure in the room with maximum velocity of 50 cm/s has been created in the case with an air infiltration opening without pressure difference (model D1, Fig. 4f). It is caused by the three main factors:

- inflowing air from the corridor with T=15 °C moves downwards near the wall;
- hot air from the alight region (Fig. 5) on the floor moves upwards and the flow splits;
- hot air flow form the heater moves upwards near the window and creates the intensive vortex.

The situation changes with an overpressure of 1 Pa in the room (model D2) and only one main vortex is formed in the room due to active air inflow from the corridor (Fig. 4g). As the air masses move through the relatively small ventilation opening, velocities there are very high – up to 1 m/s, but air movement in the other part of the room does not exceed 10 cm/s. Direction of the airflow vortex in the room changes with the pressure difference conditions change, but also in case of underpressure (model D3) air velocities near the hot region on the floor are about 30 cm/s.
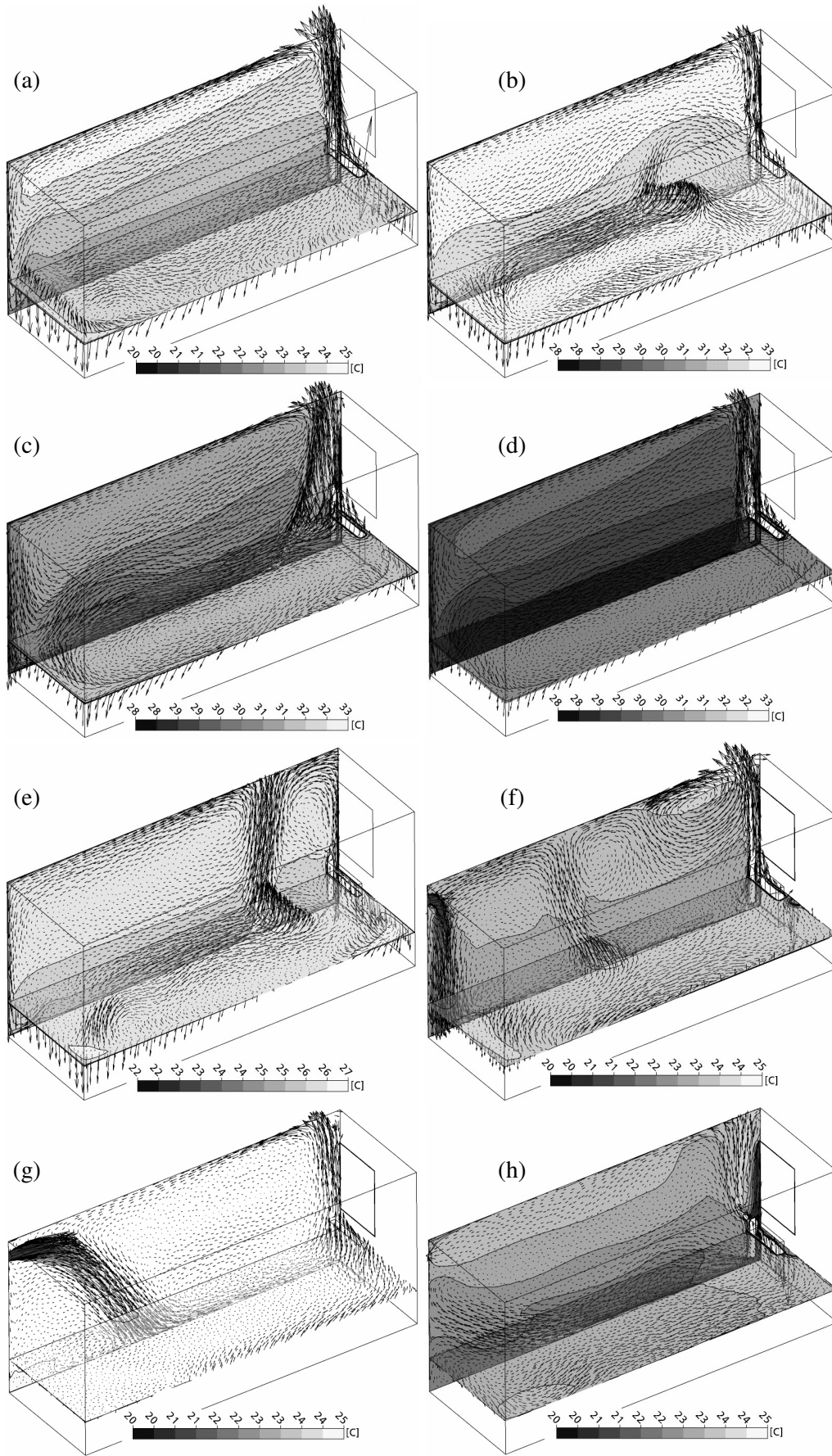
Fig. 4. Characteristic velocity vector field and temperature contours in middle vertical cross-section and 0.5m height horizontal cross-section for models A (a), B1-B3 (b-d), C (e) and D1-D3 (f-h).
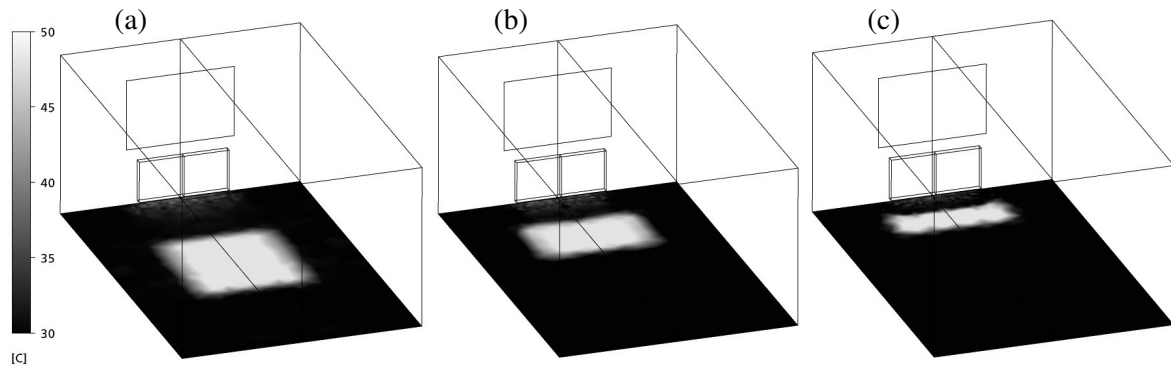
Fig. 5. Temperature distribution on the floor for models B1 (a), B3 (b) and B3(c).

Significant uprising airflows are observed from the hot radiator (models A, B1-B3 and D1-D3) and from the hot floor area (model C), but downwards movement exist in all models, what is the result of lower temperature of the neighbouring rooms (20 °C) and the corridor (15 °C). If it is assumed that heat exchange between the other rooms are excluded (adiabatic conditions are set), then the temperature distribution in the modelled room will noticeably increase, but an airflow velocity in the room – decrease.

Therefore we can assume that all considered models with solar radiation can be improved to reach better thermal comfort conditions. It is possible to forecast that the best conditions for the modelled room can be reached in following cases:

- the heater's power are being regulated in according with the solar radiation intensity and indoor temperature;
- a small overpressure is set in the room to exclude cold exterior air inflow (<0.3 Pa).
- the angle of solar radiation attack is >45°;
- temperature difference with the adjoining rooms is as small as possible.

By fulfil the above-mentioned conditions, intensive airflows will not be formed and an average temperature in the room will be acceptable for thermal comfort conditions.

4 Conclusion

A performed calculation for different models of a living room clearly demonstrates influence of solar radiation source and other factors on the heat balance and thermal comfort conditions in the room. The models allow estimating of temperature, airflow distributions and the tendencies of its changes, as well as a dependence on the heat loses trough the boundary structures under different solar radiation and ventilation conditions.

Obtained results show also an essential influence of some variables factors on the heat balance and the thermal comfort conditions. 3D modelling of a living room including of solar heat source is very important for the correct representation of a qualitative and quantitative heat transfer and convection processes in the living room.

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