Control performance evaluation of a switching controller using a laboratory scale plant with hybrid dynamics

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Abstract: This paper examines switching control from the practical and experimental perspective. Switched controllers belong to the class of hybrid systems and they can be used to compensate for the dynamics of certain classes of hybrid controlled systems and also as a better alternative to the traditional adaptive control. However, the evaluation of control performance achievable with these controllers available in the literature is usually not satisfactory. This topic is mostly treated at a purely theoretical level or in simulation at best. This paper goes a different way. It describes some extensions to the basic switching control structure, however the main focus is on practical evaluation of control performance of switching control using a laboratory scale plant. This laboratory scale plant was designed and built in such a way that it exhibits most of the hybrid phenomena typical of process control applications. As such it is used to test the control performance of switched control algorithms in this paper.

Keywords: Switched systems, Hybrid systems, Switched controller, Laboratory scale models

1. Introduction

The assumption that the controlled plant can be modelled with a continuous linear time invariant system is at least as common in control engineering literature as it is often wrong. Most real plants include non-linearities, their behaviour changes with time and quite often they have neither purely continuous nor purely discrete (or in a special case logical) dynamics but they include both continuous valued and discrete valued variables and elements (on/off switches or valves, speed selectors, etc.). Controller design may also complicated by modelling uncertainties and large unmeasurable disturbances.

The plants that combine continuous and discrete valued dynamics and components can be modelled with hybrid systems. The same holds for non-linear plants that exhibit considerably different dynamic behaviours in different operation modes or in different working points. Such plants can be modelled as a combination of several continuous models and discrete valued variables that determine which of these models is valid in the current operation mode or range. The plant model is thus again hybrid. In this way, the hybrid systems can be used as adequate models for a wide range of industrial plants and it is not surprising that the theory of hybrid systems has recently evolved in one of the most important research topics in the field of systems and control theory (see e.g. [9] for a survey).

If the controlled plant is modelled with a hybrid system, it is natural to expect that a suitable controller to compensate for the hybrid dynamics will also be of hybrid nature. Still there are just a few control approaches that were developed for hybrid systems. One approach that is best represented by research monograph [1] is based on an extension of model predictive control method. However, model predictive control of hybrid systems is marked by extensive and time consuming numerical computations based either on multi-parametric mixed integer programming or on dynamic programming.

A computationally considerably less demanding alternative is the use of switched controller approach. In this approach a set of candidate controllers is connected with a switching logic that selects the most appropriate controller for a given situation. This kind of controller has also the character of a hybrid system because continuous parts (individual candidate controllers) are associated with a discrete switching mechanism. An advantage of switched controllers is also the fact that they are not only able to cope with hybrid nature of the controlled system but unlike model predictive control they are able to cope with uncertainty in system parameters and notably with the changes of the controlled system behaviour with time. Thus they also represent an alternative to the classical methods of adaptive and robust control.

The use of switched controllers is a relatively new idea. Although its origins can be traced back to the eighties, most papers on switched controllers have been published during the last decade. A particularly important reference on this topic is [7]. In the research done by the authors the standard algorithm that makes the selection among different controllers was augmented with the bank of switched estimators that improve the response of the control system if the plant is subject to unmeasurable disturbances acting at various points of the controlled system. The results were published by the authors in [4] and [10].

However, both the results published in our previous papers as well as most other results from the field of switched controllers available in the literature have one weak point. Performance evaluation of the proposed control algorithms is either entirely neglected or it is done using just simple simulation examples. In fact, this problem is not just a problem of switched controllers but it appertains to the whole field of hybrid systems research.

This situation has motivated us to design and build a laboratory scale plant that is designed in such a way that it exhibit most of the hybrid phenomena usually encountered in process control applications. This plant can be used to test and evaluate various control strategies for hybrid systems in a setting that is much closer to real applications. A detailed description of this plant including proposals of research and educational experiments was given by the authors in [5] and [6]. In this paper, the plant will be used to evaluate the switched controller approach.

The paper will be organized as follows. First, a brief overview of the switched controllers will be given. Next section will introduce the experimental plant with hybrid behaviour and its mathematical model. After that, the practical evaluation of the proposed switched controller approach using this plant will be presented.

2. Switching control

Switching control is a potentially advantageous method for a variety of control problems such as control of highly uncertain systems with changing dynamics for which a single continuous control law cannot be found or control of systems where continuous control cannot be implemented due to sensor and/or actuator limitations. There are also certain classes of systems that are in principle not stabilizable by continuous feedback. The basic idea of switching control is simple. It is schematically shown in Fig. 1. A set of candidate controllers is associated with a decision logic (sometimes called supervisor) that makes the decision which of these controllers is currently the most appropriate one. However, the design of candidate controllers and even more the design of decision logic are quite difficult problems.

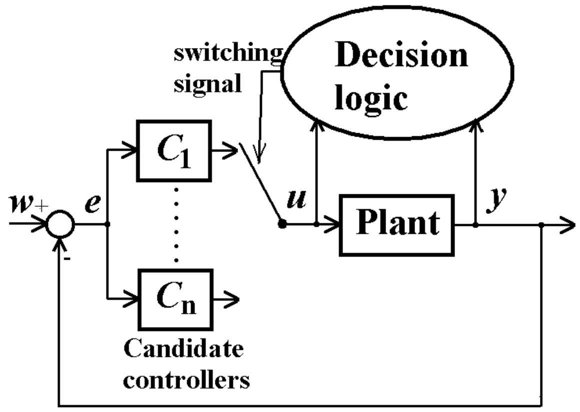

Fig. 1 Switching control

Algorithms proposed in the early papers used a simple switching mechanism based on a sequential or "pre-routed" blind search among a set of candidate controllers. This approach results in a very low control performance. Much better performance can be achieved with switching algorithms that evaluate online the potential performance of each candidate controller and use this to direct their search. These algorithms can further be divided into two main categories: switching algorithms based on process estimation (e.g. [7]). and algorithms based on a direct performance evaluation of each candidate controller (e.g. [8]).

Our approach as described e.g. in [4] basically follows the variant based on process estimation. However, the decision logic is augmented with another feature. If the controller uses state feedback, it is necessary not only to estimate which of the process models is the most appropriate one but it is also necessary to estimate the unmeasurable state variables. If conventional observer were used to estimate these unmeasurable state variables, the control performance would deteriorate because the estimation results can be corrupted by disturbances acting at various points of the controlled system (disturbance at system input or output, disturbance acting at some other point of the controlled system). To overcome this problem, the proposed architecture includes also a bank of switched estimators. Each of these estimators is tuned for a specific input point of a disturbance. Switching logic is extended so that it could select the most suitable estimator that yields the most reliable estimate in any particular control situation.

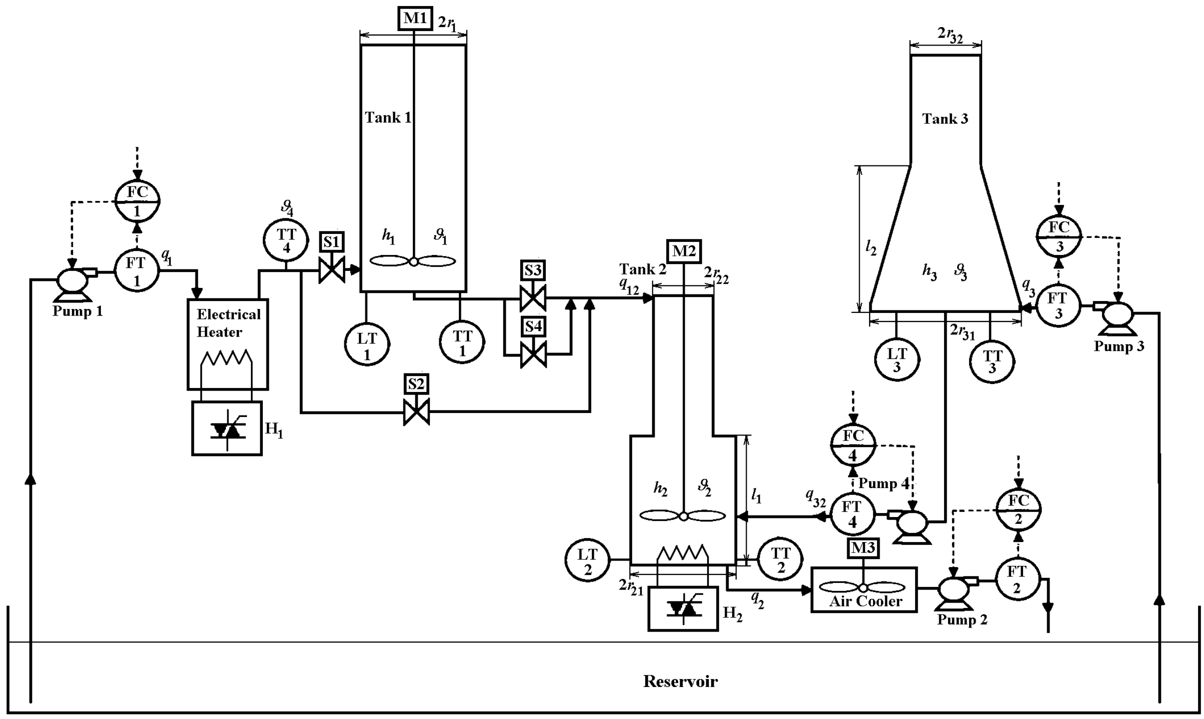

Fig. 2 Structure of the plant (FT, LT, TT stands for flow, level and temperature transmitter respectively, FC – flow controller, S–solenoid valve, M – motor, r_1= 5.6 cm, r_{21}= 4.7 cm, r_{22}= 2.7 cm, r_{31}= 6 cm, r_1= 3.2 cm), tank height 80 cm, l_1=42 cm, l_2=40 cm

3. Laboratory scale plant exhibiting hybrid phenomena

Plant structure is sketched in Fig. 2. Plant photo is shown in Fig. 3. Similarly as in most process control applications, the measured and controlled variables are water levels, temperatures and flows. Basic components of the plant are three water tanks. Tanks 2 and 3 have special shapes that introduce dynamics changes. Their behaviour is described by switched models. The tanks are thermally insulated to make the heat losses negligible (Thermal insulation hides tank shapes and therefore all tanks look the same in the plant photo). Water from the reservoir mounted under the plant is drawn by pumps 1 and 3 to the respective tanks. The delivery rates can be continuously changed and the flow rates are measured with turbine flow-meters. The flow from pump 3 is fed directly to tank 3. The flow from pump 1 goes through a storage water heater and it is further controlled by a solenoid valve S1. The power consumption of the heater is changed continuously. Another continuously controlled heater is mounted at the bottom of tank 2. Besides pumps 1-4 whose delivery rates can be changed continuously, the plant includes discrete valued actuators: solenoid valves. The flow from tank 1 is controlled by valves S3, S4 (k_v=5 l/min each), and it can be changed in three steps: no valve open, one open, both valves open. Closing and opening the valves is instantaneous. Tank 1 can be by-passed by closing S1 and opening S2. The purpose of the air-water heat exchanger with cooling fan at the output from tank 2 is to keep the water temperature in the reservoir roughly constant during the experiments. The plant is controlled from a PC using two data acquisition boards (11 analogue inputs, 6 analogue outputs, 6 digital outputs).

Fig. 3. Photo of the laboratory scale plant

First principles model of the plant is derived using mass and energy balance equations of individual tanks. The liquid (water) can be considered incompressible. Torricelli's law is used to compute the flow from tank 1 to tank 2. Constant liquid heat capacity c, negligible heat losses and well mixed tank are assumed for energy balance equations. The plant model is then given by

$$\dot{h}_1(t) = (1/A_1)\left(q_1(t) - 0.1k_v\sigma_1(t)\sqrt{gh_1(t)}\right) \quad (1)$$

$$\dot{\vartheta}_1(t) = q_1(t)\left(\vartheta_4(t) - \vartheta_1(t)\right)/A_1h_1(t) \quad (2)$$

$$\dot{h}_2(t) = \begin{cases} (1/A_{21})\begin{pmatrix} 0.1k_v\sigma_1(t)\sqrt{gh_1(t)} \\ +q_{32}(t) - q_2(t) \end{pmatrix} & \text{if } m_1 = 0 \\ (1/A_{22})\begin{pmatrix} 0.1k_v\sigma_1(t)\sqrt{gh_1(t)} \\ +q_{32}(t) - q_2(t) \end{pmatrix} & \text{if } m_1 = 1 \end{cases} \quad (3)$$

$$\dot{\vartheta}_2(t) = \begin{cases} \dfrac{\begin{pmatrix} 0.1k_v\sigma_1(t)\sqrt{gh_1(t)}\left(\vartheta_1(t) - \vartheta_2(t)\right) \\ +q_{32}(t)\left(\vartheta_3(t) - \vartheta_2(t)\right) + \dfrac{P(t)}{\rho c} \end{pmatrix}}{A_{21}h_2(t)} & \text{if } m_1 = 0 \\ \dfrac{\begin{pmatrix} 0.1k_v\sigma_1(t)\sqrt{gh_1(t)}\left(\vartheta_1(t) - \vartheta_2(t)\right) \\ +q_{32}(t)\left(\vartheta_3(t) - \vartheta_2(t)\right) + \dfrac{P(t)}{\rho c} \end{pmatrix}}{A_{21}l_1 + A_{22}(h_2(t) - l_1)} & \text{if } m_1 = 1 \end{cases} \quad (4)$$

$$\dot{h}_3(t) = \begin{cases} \dfrac{q_3(t) - q_{32}(t)}{\pi(r_{31} - h_3(t)\Delta r/l_2)^2} & \text{if } m_2 = 0 \\ \dfrac{q_3(t) - q_{32}(t)}{A_{32}} & \text{if } m_2 = 1 \end{cases} \quad (5)$$

$$m_1(t^+) = \begin{cases} 0 & \text{if } h_2(t) \le l_1 \\ 1 & \text{if } h_2(t) > l_1 \end{cases} \quad (6)$$

$$m_2(t^+) = \begin{cases} 0 & \text{if } h_3(t) \le l_2 \\ 1 & \text{if } h_3(t) > l_2 \end{cases} \quad (7)$$

where $A_i = \pi r_i^2$, $\Delta r = r_{31} - r_{32}$, m_1 and m_2 are discrete state variables, discrete valued input σ_1 assumes values 0,1,2 (no valve open, S3 open, S3 and S4 open), P is power output of the heater located at the bottom of tank 2, k_v is the flow coefficient of solenoid valves S3 and S4. Equation for ϑ_3 is not included in the model. As tank 3 is not heated, ϑ_3 is roughly equal to the ambient temperature, and the mixing dynamics of tank 3 are not important.

Vector field defined by (1) to (5) involves both controlled and autonomous switching. Controlled switching due to changes of discrete valued input σ_1 results in vector field discontinuity in (1) and (3). Autonomous switching due to changes of h_2 and h_3 results in vector field discontinuity in (3). The dynamic behaviours of (4) and (5) are also switched, but the vector field remains continuous.

Plant model described by (1) to (7) can be compared with general state equations of hybrid system as given by Branicky in [2]. These general state equations have the form

$$\begin{aligned} \dot{\boldsymbol{x}}(t) &= \boldsymbol{f}\left(\boldsymbol{x}(t), \boldsymbol{m}(t), \boldsymbol{u}(t)\right) \\ \boldsymbol{y}(t) &= \boldsymbol{g}\left(\boldsymbol{x}(t), \boldsymbol{m}(t), \boldsymbol{u}(t)\right) \end{aligned} \quad (8)$$

$$\begin{aligned} \boldsymbol{m}(t^+) &= \boldsymbol{\varphi}\left(\boldsymbol{x}(t), \boldsymbol{m}(t), \boldsymbol{u}(t), \boldsymbol{\sigma}(t)\right) \\ \boldsymbol{o}(t^+) &= \boldsymbol{\varphi}\left(\boldsymbol{x}(t), \boldsymbol{m}(t), \boldsymbol{u}(t), \boldsymbol{\sigma}(t)\right) \end{aligned} \quad (9)$$

$$\boldsymbol{x}(t^+) = \boldsymbol{\psi}\left(\boldsymbol{x}(t), \boldsymbol{m}(t), \boldsymbol{u}(t), \boldsymbol{\sigma}(t)\right) \quad (10)$$

where $\boldsymbol{x}(t)$, $\boldsymbol{u}(t)$ and $\boldsymbol{y}(t)$ are continuous state, input and output respectively. Unlike state equations of a purely continuous system these equations include discrete state $\boldsymbol{m}(t)$. This state indexes the vector fields $\boldsymbol{f}(.,.,.)$. The development of the discrete state is described by (9) where $\boldsymbol{\sigma}(t)$ is discrete input and $\boldsymbol{o}(t)$ is discrete output. Equation (10) models the state jumps (if there are any). It is evident that plant model given by (1) to (7) includes all features of a general hybrid system model except for state jumps. However, state jumps are only present in certain mechanical systems (e.g., systems involving collisions). State variables in process control applications (temperature, liquid level, concentration etc.) cannot be abruptly changed in steps.

4. Practical evaluation of switching control performance

The laboratory scale plant that was described in the previous section is considerably flexible and it allows us to define many control scenarios of varying complexity. For the purpose of this paper a moderately complex control task will be used (for more complex suggestions see e.g. [5]).

We will consider water level control in tanks 2 and 3. The main controlled variable is h_2 and manipulated variable is flow q_3. Standard procedure to avoid tank overflow as described in [3] is applied to the control of flow through pumps 4 and 2. The flow from tank 3 to tank 2 is directly proportional to water level h_3 and the outflow from tank 2 is directly proportional to water level h_2.

$$q_{32}(t) = k_3h_3(t); \quad q_2(t) = k_2h_2(t) \quad (11)$$

The values of coefficients k_2 and k_3 are determined in such a way that outflows from the tanks are equal to maximum flow rate achievable by pump 3 if respective water levels are close to their maximum values. Thus, tank overflow is avoided. Maximum flow rate achievable by the pump is 4.5 l/min, tank height is 0.8 m. If both coefficients are equal to 6 l.m/min, water level will never exceed 0.75 m. Mathematical model of this system is derived by connecting (11) with (3) and (5)

$$\dot{h}_2(t) = \langle \begin{matrix} (1/A_{21})(k_3h_3(t) - k_2h_2(t)) & \text{if } h_2(t) \le l_1 \\ (1/A_{22})(k_3h_3(t) - k_2h_2(t)) & \text{if } h_2(t) > l_1 \end{matrix} \quad (12)$$

$$\dot{h}_3(t) = \langle \begin{matrix} \dfrac{q_3(t) - k_3h_3(t)}{\pi(r_{31} - h_3(t)\Delta r/l_2)^2} & \text{if } h_3(t) \le l_2 \\ \dfrac{q_3(t) - k_3h_3(t)}{A_{32}} & \text{if } h_3(t) > l_2 \end{matrix} \quad (13)$$

Tank 2 is modelled by two switched linear models and it can also be written in the form

$$\begin{aligned} \tau_{21}\dot{h}_2(t) + h_2(t) &= \frac{k_3}{k_2}h_3(t) \text{ if } h_2(t) \le l_1 \\ \tau_{22}\dot{h}_2(t) + h_2(t) &= \frac{k_3}{k_2}h_3(t) \text{ if } h_2(t) > l_1 \end{aligned} \quad (14)$$

where the time constants are $\tau_{21} = A_{21}/k_2 = 69.4\,s$ and $\tau_{22} = A_{22}/k_2 = 22.9\,s$. Model of tank 3 is linear for $h_3 > l_2$. For $h_3 < l_2$ it can be linearized around working points determined by the steady state characteristics that is linear in this case $q_{3S} = k_3h_{3S}$

$$\frac{\pi(r_{31} - h_{3S}\Delta r/l_2)^2}{k_3}\Delta\dot{h}_3(t) + \Delta h_3(t) = \frac{1}{k_3}\Delta q_3(t) \quad (15)$$

with time constant given by

$$\tau = \pi(r_{31} - h_{3S}\Delta r/l_2)^2/k_3 \quad (16)$$

Considering the values of plant parameters that are given in previous pages this time constant changes from 113.1 s at h_3=0 to 32.17 s at h_3=l_2=0.4 m. The relationship between steady state water level and time constant is shown in Fig.4. To replace (15) with a switched model the whole range of h_3 up to l_2 must be divided into several sub-ranges. This division will be done in such a way that the ratio between the maximum and minimum time constant within the sub-ranges would be the same. The nominal values are computed as geometrical mean of the extreme values. If range from 0 to l_2 is divided into two sub-ranges, the following switched models are obtained:

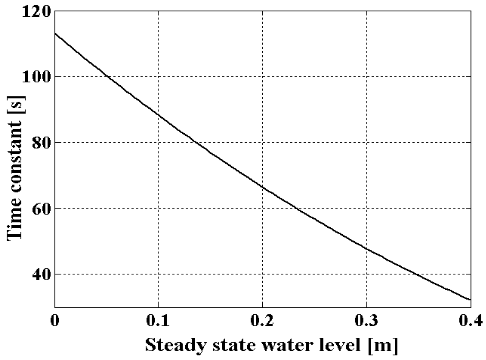

Fig. 4 Time constant as a function of h_{3S}

model 1 valid for $0 \le h_3 \le 0.23$ m, τ_{31}=82.72 s;
model 2 valid for $0.23 < h_3 \le 0.4$ m, τ_{32}=44,1 s;
model 3 valid for $0.4 < h_3 \le 0.8$ m, τ_{33}=32,17 s.

Defining state vector $\boldsymbol{x}(t) = [h_2(t) \quad h_3(t)]$, h_2 as an output y and q_3 as a input u the whole system is described by switched model of the form

$$\begin{aligned} \dot{\boldsymbol{x}}(t) &= \boldsymbol{A}_{ij}\boldsymbol{x}(t) + \boldsymbol{b}_{ij}u(t) \\ y(t) &= \boldsymbol{c}\boldsymbol{x}(t) \end{aligned} \quad (17)$$

$$\boldsymbol{A}_{ij} = \begin{bmatrix} -1/\tau_{2i} & k_3/k_2\tau_{2i} \\ 0 & -1/\tau_{3j} \end{bmatrix} \boldsymbol{b}_{ij} = \begin{bmatrix} 0 \\ 1/k_3\tau_{3j} \end{bmatrix} \quad (18)$$
$$\boldsymbol{c} = [1 \quad 0]; \quad i = 1..3,\ j = 1..2$$

Validity regions of the partial models (altogether six in number) are evident from the previous paragraphs. Model (17), (18) belongs to a special but practically very important class hybrid systems. It is a piece wise affine (PWA) system or in this particular case even more specially piecewise linear system. It can be seen from Fig. 5 that this model represents the original system (12), (13) with a good accuracy.

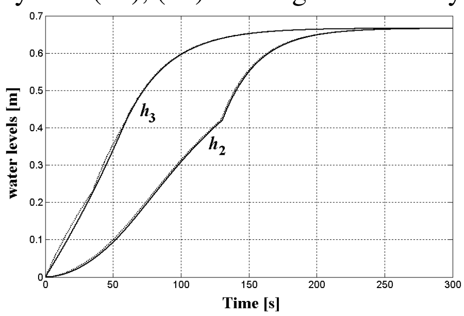

Fig. 5 Comparison of step responses of original model (12), (13) and PWA approximation (17) original solid lines, approximation dotted lines

The design of candidate controllers used within the framework of switching control can be based on very different principles. In our approach we usually use state-feedback control. Model (17) is a second order system. In order to achieve zero steady state error even in the case of system parameter changes basic state feedback structure must be augmented with an integrator. State feedback controller is thus designed for a third order system. As the maximum flow rate achievable by the pumps is limited, some measures to prevent integrator wind-up must be taken. Standard technique of dynamic limitation of manipulated variable is sufficient for this purpose.

The controllers were designed in such a way that the closed loop poles in positions -0.06; -0.07 and -0.1. The evaluation results are in Figs 6, 7 and 8. Fig. 6 shows how a non switched controller that was designed to achieve good control performance at high water levels behaves at low water levels (set point is 0.12 m). Significant control performance deterioration is clearly marked by big overshoots of controlled variable h_2 and very long settling time.

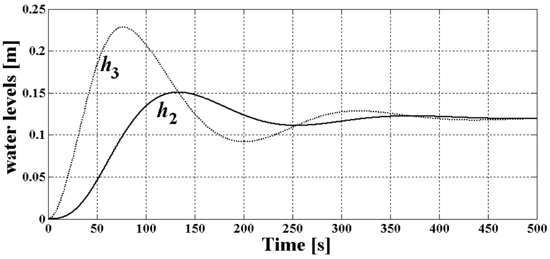

Fig. 6 Step responses with non-switched controller

The same response is shown in Fig. 7 with switched controller. It can be seen that the response of h_2 is closed to ideal. If the set-point is set to higher levels (0.5 m), the switched controller is able to cope with the changed dynamics and the set-point response remains very good. This response is shown in Fig. 8. It remains fast and non-oscillatory. It is somewhat slower than the response in Fig. 7, however this difference is caused by physical constraints. Because of the limited maximum flow rate of the pump the maximum achievable speed of the response is also limited. Still, this physical constraint cannot be removed by any controller.

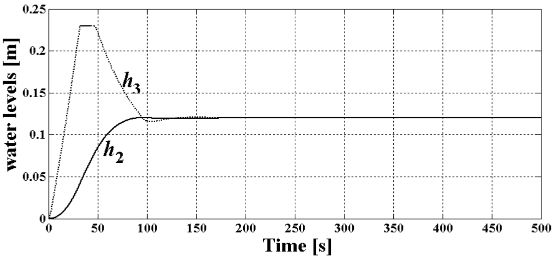

Fig. 7 Switching control, low set-point

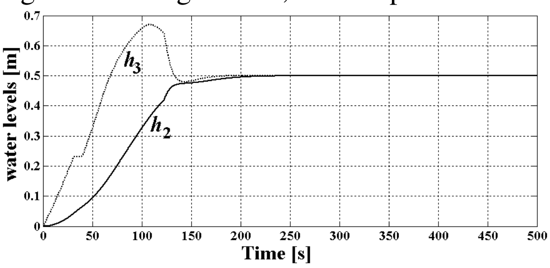

Fig. 8 Switching control, higher set-point

5. Conclusion

This paper was focused on experimental evaluation of performance of switched controllers. It went a different path than most other papers devoted to this subject. Most papers on switching control largely rely on pure theoretical analysis or in a better case they use simulation with simple academic examples of controlled systems. This paper used experimental plant that was designed in such a way that it exhibits most hybrid phenomena that can be encountered in process control applications and as such it represents a real challenge to control theory. In this paper, this plant was used in a moderately complex configuration. The testing results are promising and they show the advantageous properties of switching control. Further testing of this control architecture will be done using more complex configurations of this laboratory scale plant.

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