Digital Sampling Method in the Measurements of Electrical Power and Energy

COSTIN CEPISCA
Electrical Engineering Faculty, University POLITEHNICA of Bucharest
313 Splaiul Independentei, sector 6
ROMANIA
www.electro. pub.ro
GEORGE SERITAN
Electrical Engineering Faculty, University POLITEHNICA of Bucharest
313 Splaiul Independentei, sector 6
ROMANIA

CATALIN IOAN CEPISCA
Electrical Engineering Faculty, University POLITEHNICA of Bucharest
ROMANIA

SORIN DAN GRIGORESCU
Electrical Engineering Faculty, University POLITEHNICA of Bucharest
313 Splaiul Independentei, sector 6
ROMANIA

Abstract: - The paper presents the results of application of digital sampling method and a study of the errors caused in the measurement of active power and energy with digital meters. The presence of non-sinusoidal regimes makes the use of the Shannon theorem more difficult. The models of digital sampling operations and Matlab applications show which the sampling frequency for a minimum of errors is.

Key-Words:- digital sampling, method, meters, energy, errors

1. Introduction

Measurement and calculation of electrical quantities based on digital sampling method of voltage and current have been discussed for some time, e.g. by Clark and Stockton 1982 [1]. The advantage of this system is that it is comparatively easy to calibrate, and that digital operation is precise and does not cause linearity problems.

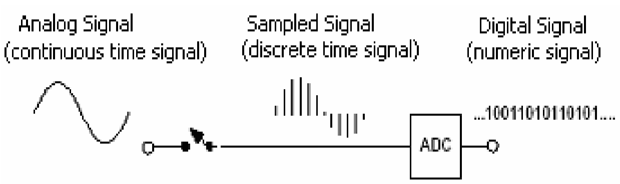


Fig.1. Analog-to-Digital Conversion

The digital meter relies on a process called analog-to-digital conversion [2]. The ADC, analog-to-digital converter, takes samples of the analogue signals at discrete instances of time. These discrete time signals are in turn converted to numeric values by the ADC as shown in figure 1.
Once in this numeric or digital format, digital circuits, e.g., microprocessors, can easily and reliably process these signals. Figure 2 shows a graphical representation of this digital processing.
The amount of sampling rate is very important for the measurement incertitude of electrical power and energy [3]. A basic rule of sampling theory states that the frequency of sampling must be at least twice the highest frequency content of the signal. This is called the Nyquist rate.

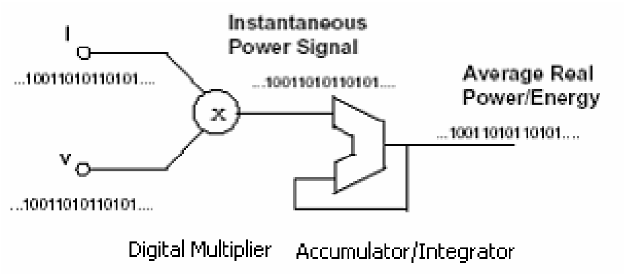
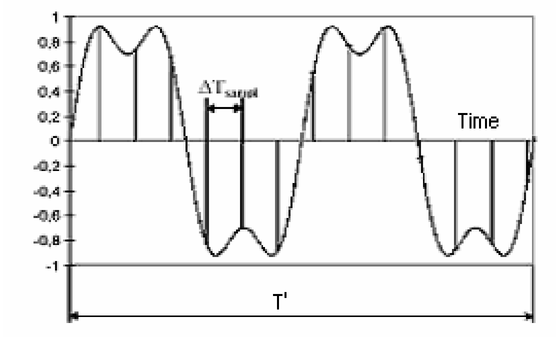


Fig.2. Digital Signal Processing

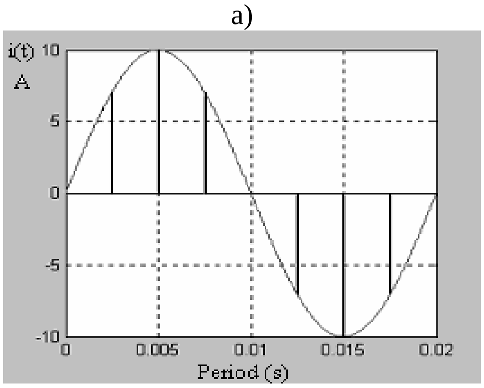
In energy metering current ANSI and IEC specifications [4] call for accurate measurement up to the 20th harmonic (1 kHz), but in the electrical systems the order of harmonics is greater.
This paper presents a study for the accurate choice of sampling frequency to digital meters use in the electrical systems with non-sinusoidal signals.

2. Digital sampling of ac quantities

As early as 1967 Davis et al. [5] showed that for a sinusoidal signal and with phase locked sampling, the RMS value of the signal and the active power of a linear load connected to this signal could be calculated by a numerical integration of as few as three samples taken during precisely one period.



a)



b)

Fig.3. The sampling rate: a) theoretical; b) modeled with MATLAB

It is well known that a Fourier transform can be made on all stationary periodic signals. According to the Nyquist theorem the sampling rate must then be kept higher than twice the highest harmonic contained in the periodic signal, if a correct result from such a frequency analysis should be obtained [6]. The sampling rate *fs* will determine the highest frequency that can be measured, *fs*/2, while the total measurement time *T '* will determine the lowest frequency that can be resolved and the frequency resolution will be 1/*T* – see Figure 3.

3. Accuracy of the digital measurement of electrical energy

The active energy will be:

$$W_a=\int_0^{T'}p(t)dt=T'UI\cos\varphi-\frac{UI}{2\omega}\left[\sin\left(2\omega T'+\alpha+\beta\right)\right]=$$

$$=\frac{2\pi}{k\omega}NUI\cos\varphi-\frac{UI}{2\omega}\left[\sin\left(\frac{2\pi}{k}N+\alpha+\beta\right)-\sin\left(\alpha+\beta\right)\right] \qquad (1)$$

where the measurement length is $T' = NT_e = N\frac{2\pi}{k\omega}$.

In the digital domain, elementary active energy between two sampling moments depends on simultaneous current and voltage sampling $u[n]$ and $i[n]$:

$$\Delta w_{ae} = T_e\cdot u[n]\cdot i[n]=\frac{2\pi}{k\omega}u[n]\cdot i[n]$$

$$\Delta w_{ae} = \frac{4\pi}{k\omega}UI\sin\left(\frac{2\pi i}{k}+\alpha\right)\sin\left(\frac{2\pi i}{k}+\beta\right) \qquad (2)$$

In these conditions, the total energy obtained with help of the digital method is:

$$W_{ae}=\frac{4\pi}{k\omega}UI\sum_{i=0}^{N-1}\sin\left(\frac{2\pi i}{k}+\alpha\right)\sin\left(\frac{2\pi i}{k}+\beta\right)=$$

$$=\frac{2\pi}{k\omega}UI\cos\varphi-\frac{2\pi}{k\omega}UI\sum_{i=0}^{N-1}\cos\left(\frac{4\pi i}{k}+\alpha+\beta\right) \qquad (3)$$

The result of energy measurement error is then:

$$\Delta W_a = W_{ae}-W_a = A\text{-}B$$

where:

$$A=\frac{2\pi}{k\omega}UI\left\{\frac{k}{4\pi}\left[\sin\left(\frac{4\pi}{k}N+\alpha+\beta\right)-\sin\left(\alpha+\beta\right)\right]\right\} \qquad (4)$$

$$B=\frac{2\pi}{k\omega}UI\sum_{i=0}^{N-1}\left(\frac{4\pi i}{k}+\alpha+\beta\right)=\frac{2\pi}{k\omega}UI\frac{\sin\left(\frac{2N}{k}\pi\right)\cos\left(\frac{2(N-1)}{k}\pi\right)}{\sin\left(\frac{2}{k}\pi\right)}$$

The conditions for minimizing the error are:

$$\frac{4\pi}{k}N=2m\pi,\quad m\in Z\quad\Rightarrow\quad 2T^{'}=mT \tag{5}$$

$$\sin\left(\frac{2N}{k}\pi\right)=0\ \Rightarrow\ \frac{2N}{k}\pi=m\pi\qquad 2T^{'}=mT,\quad m\in Z$$

$$\cos\left(\frac{2(N-1)}{k}\pi+\varphi\right)=0\ \Rightarrow\frac{2(N-1)}{k}\pi+\varphi=(2p+1)\frac{\pi}{2},\quad p\in Z$$

$$\frac{4\pi}{k}i\neq 2\mu\pi\quad\Rightarrow\quad\frac{2}{k}i\neq\mu\quad\Rightarrow\quad T_e\neq\frac{\mu}{2i}T,\quad \mu\in Z$$

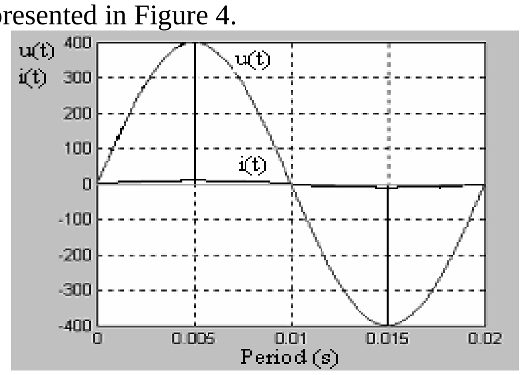
Our study was made taking in consideration the conditions (5).

4. Models and simulations

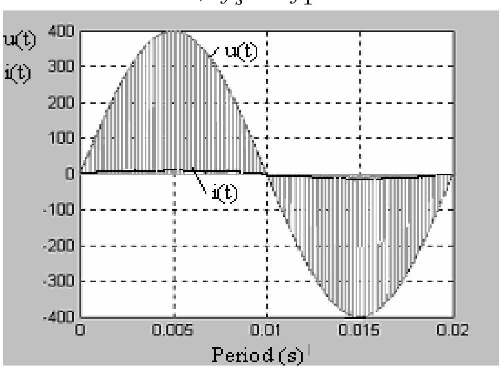
The results based on a Matlab model of digital sampling for sinusoidal signals:

$$u(t)=400\sin 314t\qquad i(t)=10\sin 314t$$

are presented in Figure 4.



a) $f_s=3f_1$



b) $f_s=199f_1$

Fig.4. Digital sampling, $T'=T$

For the fundamental frequency f_1= 50 Hz, the error due to frequency sampling f_s is presented in Figure 5.

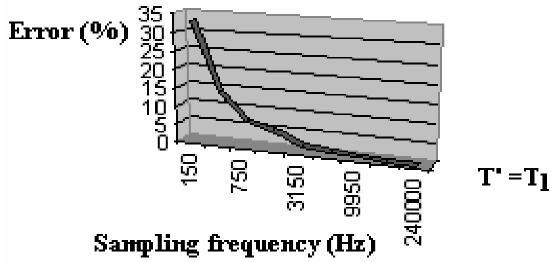


Fig.5. Error due to sampling, $T'=T_1$

In the case of a longer period of measurement ($T'=10T_1$) the error variation curve is presented in Figure 6. The errors are lower in this situation but the values of errors are greater for the correct measurement of electrical energy.

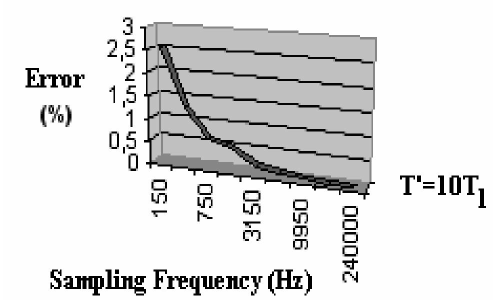


Fig.6. Error due to sampling, $T'=10T_1$

The presence of nonsinusoidal regime in electrical circuits is a reason of additionally errors when using digital meters. The simulation utilize two non-sinusoidal signals, for the current *i(t)* and voltage *u(t)*:

$$u(t)=400\sin 314t+160\sin 3\cdot 314t+80\sin 5\cdot 314t$$
$$i(t)=10\sin 314t+4\sin 3\cdot 314t+2\sin 5\cdot 314t$$

The result of the MATLAB simulation is presented in Figure 9. The additional errors exceed the imposed metrological limits.

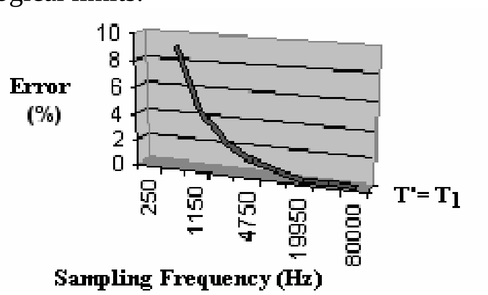


Fig.9. Error due to sampling, $T'=T_1$

4. Conclusion

The error decreases with the number of samples taken, however just by a factor ~ $1/\sqrt{n}$. Therefore with shorter conversion times and higher computer efficiency given by the modern technology it is possible to get a reasonable accuracy with this method, even if it takes more than one million samples to get an uncertainty of less than 0.1%.

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