Numerical simulation of coupled fields in high-voltage insulation

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Abstract: – Dielectric heating is caused by losses due to friction of the molecular polarisation process in dielectric materials. A polluted dielectric has a finite resistance so that the leakage current in the dielectric heats the dielectric. The problem of heating is a coupled thermal-electric problem. The paper presents an algorithm based on a 2D model for coupled fields in the insulation of a large power cable. The heat transfer in insulation is described by the heat conduction equation where the heat sources are both internal sources generated by the leakage current in a resistive dielectric, and the boundary heat sources of the convective and Dirichlet/Neumann type.

Key-Words: - Coupled fields; Finite element method.

1 Introduction

This work deals with the heat generated by ohmic losses generated by the electric field in high-voltage cables. The problem is described by a coupled thermal-electric set of equations. The coupling between the two fields is the thermal effect of the electrical current or a material property as the electrical conductivity. A computation algorithm is presented for coupled problems in two dimensions.

A numerical algorithm based on the finite element method (FEM) is presented describing the solution of two-dimensional systems. In our example we consider only steady-state regime for the electric field although many transient regimes appear in the behaviour of the electromagnetic devices. The assumption is acceptable because the time constants for the electric phenomenon are less than the time constants for the thermal field.

The problem of dielectric heating involves two approaches [2]:

1.The capacitive case. In this case, all the involved insulation media can be assumed to be perfect dielectric without free charges. The mathematical model is given by Laplace's equation, written with the potential V.

2.The resistive case. In this case the resistive contribution is not negligible.

We consider a coaxial cable with two insulation layers that can be imperfect dielectrics. At the application of a voltage U, the field changes from a purely capacitive distribution to a purely resistive field. The field between the initial and final has a time variation. Generally speaking there is no perfect dielectric insulation so that a leakage current exists. Ohmic losses cause the dielectric heating. A parallel-plane model can be used to compute the electric and thermal fields.

2 Mathematical model

The electric field distribution can be obtained by approximation of the Maxwell equations. These approximations take different forms in accordance with material properties of the equipment. In modelling of these physical systems we must consider both perfect dielectrics and imperfect (or polluted) dielectrics.

In our target example the analysis domain is plotted in the Fig. 1. The mesh with triangular elements is presented.

The static field distribution can be modelled by the following equations [1]:

$$\nabla \times \overline{E} = 0; \quad \overline{E} = \rho \overline{J}$$

with: ρ - the material resistivity, E - the electric strength and J – the current density.

A 2D-field model was developed for a resistive distribution of the electric field. An electric vector potential P is introduced by the relation:

$$\overline{J} = \nabla \times \overline{P}$$

Laplace's equation describes the field distribution (for anisotropic materials):

$$\frac{\partial}{\partial x}(\rho_x \frac{\partial P}{\partial x}) + \frac{\partial}{\partial y}(\rho_y \frac{\partial P}{\partial y}) = 0 \qquad (1)$$

Mathematical model for the thermal field is the conduction equation [2]:

$$\frac{\partial}{\partial x}(kx \frac{\partial T}{\partial x}) + \frac{\partial}{\partial y}(ky \frac{\partial T}{\partial y}) + q = \gamma c \frac{\partial T}{\partial t} \qquad (2)$$

with: T (x, y, t) - temperature in the point with co-ordinates (x, y) at the time t; kx , ky – thermal conductivities; γ-specific mass; c – specific heating; q – heating source.

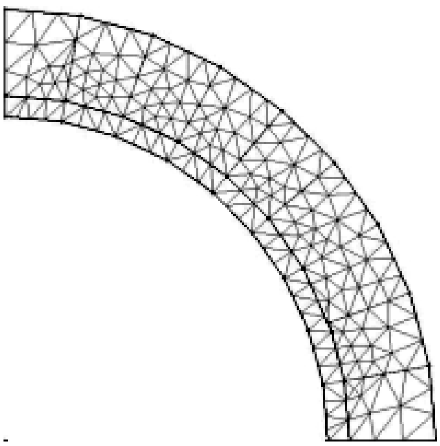

Fig. 1. The analysis domain

It is obviously that there is a natural coupling between electrical and thermal fields. Thus, the resistivity in equation (1) is a function of T, and the heating source q in (2) depend on J. Numerical models for the two field problems can be obtained by the finite element method. In dielectric applications we consider the dependency of the temperature by the form [2]:

$$\sigma = \sigma_0 \exp(\alpha T) \exp(\gamma E)$$

where: σ_0 stands for the conductivity at a temperature of 0^0 C and field strength of 0 kV/mm; α denotes the temperature dependency coefficient and γ denotes the field dependency coefficient.

3 Numerical modelling

The differential model can not be solved analytically. A numerical model can be obtained by Galerkin's procedure.

In general the time dependent problems after a spatial discretization can lead to a lumped-parameter model. For example, the heat equation, after spatial discretization, lead to a system of ordinary differential equations by the form [3]:

$$[S]\left\{\frac{\partial T}{\partial t}\right\} + [R]\{T\} + \{b\} = 0 \qquad (3)$$

where [R] and [S] are matrices and b is the vector of the free terms.

The algorithm in pseudo-code has the following structure [3]:

Choose the initial value of the temperature
Repeat
{Computations for electrical field}

- *Compute the resistivity ρ*
- *Solve the numerical model for electric potential P*

{Computations for thermal field}

- *Compute the heating source q by (2)*
- *Solve the numerical model for the temperature*

Until the convergence_test is TRUE

The convergence test is the final time of the physical process.

We present the numerical model for the heat equation. A spatial discretization leads to the ordinary differential equation (3). The time discretization of the temperature can be obtained by a formula of finite difference:

$$\frac{\partial T}{\partial t} = \frac{T^{(k)} - T^{(k-1)}}{\delta t}$$

With this approximation, the heat equation (2) becomes:

$$(\gamma c)T^{(k)} = \nabla(k \nabla T^{(n-1)}) + \frac{(c\gamma)T^{(k-1)}}{\delta t} + q^{(k-1)}$$

A refinement of the numerical algorithm in pseudo-code can have the following form:

- *Put the iteration counter k on 0 and the initial time t_0.*
- *k=k+1*
- *Compute the resistivity value.*
- *Solve the numerical model for the electric potential P*
- *Compute the heat source q in Eqn. 2.*
- *Update the numerical model for the temperature*
- *Solve the numerical model for the thermal field. The result is the temperature at the moment t_k.*
- *Increase the time with the step δt in order to obtain the following step t_k.*

- *If the time t_k is less than the imposed limit of the time, then jump to the step 2, else stop.*

2 Numerical results

Our example is a high-voltage direct-current (HVDC) cable with two insulation layers. The leakage current in dielectric is caused by the finite resistivity of the dielectric insulation.

The geometrical properties of the cable are: internal radius of the first layer is 15 [mm]; internal radius of the second layer is 16 [mm]; external radius of the second layer is 20 [mm].

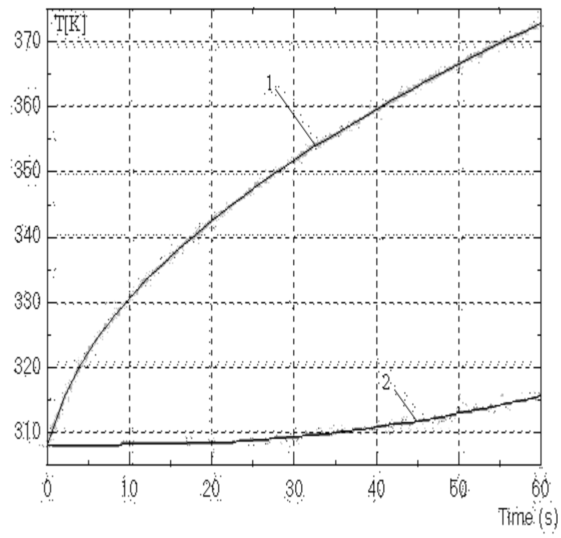

Fig. 2. Temperature versus time

The physical electrical properties are: voltage of the cable is U=150 [kV]; resistivity of the first layer is $1.\ 10^{9}$ [Ω. m]; resistivity of the second layer is 2.10^{10} [Ω. m]

The thermal properties are: thermal conductivity of the first layer is 0.271[W/K.m]; specific heat c=1800 [J/Kg.K]; specific mass γ=1300 [Kg/m^3]; thermal conductivity of the second layer is 0.17[W/K.m]; specific heat c=1600 [J/Kg.K]; specific mass γ=1200 [Kg/m^3].

At the application of a high voltage the field has a capacitive distribution initially [4]. This distribution is for a short time so that it is not interest for the temperature distribution. Finally the field has a resistive distribution. Between these limits there is an intermediate field that can be computed by an iterative procedure.

The analysis domain is the insulation space. The symmetry of the problem can reduce the analysis domain to a quarter (Fig.1).

The heat source is the thermal effect of the current in the dielectric insulation and the load current of the cable. It is obviously that the ohmic losses in the cable conductor are the most important heat source.

A. Constant heat flux

In our first case we consider that there is a constant heat flux on the interface conductor-insulation. The source of this flux is the Joule-Lenz's effect of the load in the cable. The mathematical model for the heat transfer is the conduction equation (2). The boundary conditions are Neumann's condition at the interface conductor-insulation, and convective condition at the boundary insulation-environment.

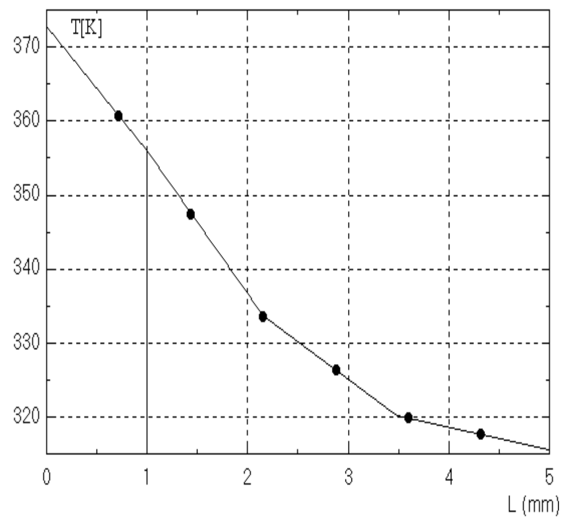

Fig. 3. Final temperature in radial direction

The Neumann's condition can be computed by the conductor losses in the case the cable was loaded before switching of the step voltage, that is the current in the cable has been raised long before and the temperature distribution in the cable is stable. In this case the value of the heat flux is computed with the relation [3]:

$$p = \frac{P_{cond}}{2\pi r_0}$$

with P_{cond} - the ohmic losses per cable meter in the inner conductor as Joule-Lenz's effect.

Thus, Neumann's condition is:

$$\left.\frac{\partial T}{\partial n}\right|_{C_1} = -p$$

with C_1 – the interface of the cable conductor and insulation.

At the interface insulation-environment we consider a convective condition by the form:

$$\left.\frac{\partial T}{\partial n}\right|_{C_2} = h(T - T_\infty)$$

with h – the convective coefficient, T_∞ - the ambient temperature and C_2 – the boundary of the cable and the external medium.

In Fig.2 the temperatures vs. time at external surface of the conductor the curve 1 (green), and environment surface – the curve 2 (red) are plotted [4]. The time interval was 60 [s]. The convection coefficient h was 12 for an environment temperature 308 [K] (35 ^{0}C).

In the Fig. 3 the final distribution of the temperature in the radial direction is plotted [4]. The width of the insulation is 5 [mm].

B. Constant temperature

Another practical assumption in electrical engineering is a Dirichlet boundary condition at the interface conductor- insulation. For our target example we considered a constant temperature of the conductor surface and a convective condition at the boundary insulation - environment. In numerical simulation the conductor temperature was considered as 100 ^{0}C (373.16 K).

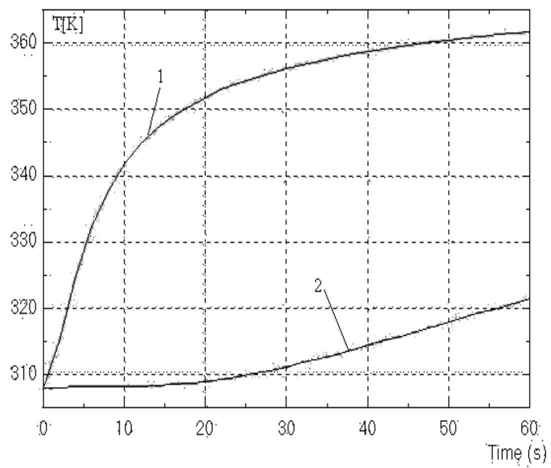

Fig. 4. Temperature versus time for Dirichlet condition

In Fig. 4 the temperature versus time is plotted in two interest points. The first curve denoted 1 (green) is the temperature at the external surface of the first layer. The curve 2 (red) represents the temperature at the external surface of the cable.

From the engineering viewpoint the assumption of Neumann's condition seems more realistic. The heat flux at the conductor surface can be estimated more accurate than the conductor temperature.

An accurate model can be obtained by including the conductor in the analysis domain [2]. This approach increases the computational effort.

In these two examples we considered a steady-state regime of the electric field. This is a practical situation but there are cases where the voltage has step variations so that a transient regime appears as a natural situation.

3 Conclusion

In this paper we presented an algorithm for coupled electric and thermal fields in the insulation of the large power cables. A parallel-plane model was considered both for electrical field and thermal field. The numerical models were obtained by the finite element method in a 2D space.

As target example we considered a cable with two-insulation layer. The resistivity of the insulation was considered as finite value. In this case the ohmic losses of the leakage current in insulation generate supplementary losses. The principal heat source remains the losses in the cable conductor.

As first example we considered a constant heat flux on the interface conductor-insulation. In practice, the heat flux is dependent on the temperature of the cable conductor. By the numerical simulation we can consider all practical cases in the operating regimes of the cable. In the second example we considered a constant temperature of the conductor.

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