Advanced algorithms for inverse problems in electrical engineering

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Abstract: – This paper presents some theoretical and numerical problems that arise in inverse problems in electromagnetic devices.

The principal objective of the paper is to describe some computational aspects for inverse problems in coupled electromagnetic and thermal fields. We develop the necessary conditions for optimality using Lagrange multipliers. The state equations are mathematical models for magnetic field in case of time-harmonic Maxwell equations in vector magnetic potential formulation for axisymmetric fields. The model for the heat transfer is the heat conduction equation.

The state and co-state equations are discretised by the finite element method using domain decomposition techniques for the analysis domain. The analysis domain is divided into two overlapping subdomains for the two coupled-fields considering physical significance of the pseudo-boundary of the two subdomains.

Key-Words: - Coupled fields; Inverse problems; Finite element method.

1 Introduction

It is well known that the nature is complex in its behaviour and the abstract models do not capture accurately the laws of the nature. We work with abstract models that try to describe the phenomena from nature and the technical devices. But it is a great mistake to think that we have perfect models of the natural phenomena. More, many numerical algorithms are not discovered so that, although we limit our discussion to our actual achievements in this area, we must dream and to seek permanently new and modern approaches for the actual problems in science, technics and life.

Analytical solutions for the electrical engineering problems are limited to some simple applications and ignore some physical phenomena. For complex problems the accurate models are necessary and the numerical solutions are efficient approaches for an optimal design and operation.

With the advent of modern digital computers, many numerical models were developed and they become widely used in the scientific computing. We use the old algorithms and transform them for the new architectures but we must invent new algorithms having in our mind the computational power of the new computers.

The efficient design of the electromagnetic devices has resulted in more stringent specifications and a demand for optimal operation, which is very important in high-performance electrical power systems. More exacting specifications have demanded during the design stage the development of accurate methods of predicting the performance characteristics of these devices. Some of the performance indicators of concern in the design of the power devices are the electromagnetic forces, iron losses, the eddy-currents effects and the heat transfer between the component parts. Prediction of the flux densities and current densities can be used to compute forces and local heating, both of which are of a serious concern to the designer of the devices of high performance.

The equations of the electromagnetic fields and heat dissipation in electrical engineering are coupled because the most of the material properties are temperature dependent and the heat sources represent the effects of the electromagnetic field [5].

1.1. Motivations for optimisation of coupled problems

In practical engineering synthesising the best engineering solution to a given design problem is of great interest. This requirement in electrical engineering is called inverse problem or optimisation problem, and several methods have been developed for this purpose. Among them the deterministic method using design sensitivity analysis has proved to give a proper design in terms of computational efficiency.

Optimisation methods has been efficiently developed and applied to electromagnetic devices and mechanics. Unfortunately, the methods developed always deal with single systems. The reality is the coupled problems are complex because the critical design parameters are in both systems.

In the area in discussion, one of the principal criteria of performance is to control the distribution of the temperature in a device. In inverse problems the heat sources play the role of the control variable of the heat dissipation in an electromagnetic device [2].

With the terminology of the system theory, we identify two kinds of the heat sources (and commands in an inverse problem):

- **Distributed commands** (electrical currents)
- **Boundary commands** (Dirichlet condition, Neumann condition, convection and radiation)

A control of the electromagnetic devices can be done by internal (distributed) commands or/and boundary commands. For the first case the commands are the heat sources (position, amplitude).

In the heating of the electromagnetic devices, the **distributed commands** are the **internal heat sources** (position, amplitude) that are represented by:

- **Ohmic losses** from **driving (source) currents**
- **Ohmic losses** from **eddy currents** induced in conducting materials of the time variable magnetic field
- **Dielectric losses** due to friction in the molecular polarisation process in solid dielectrics that form the insulation of the high-voltage apparatus
- **Hysteresis loss** in magnetic problems. It is due to magnetic domain friction in ferromagnetic materials.

The **boundary commands** can be [3]:

- **Dirichlet command**, that is, an imposed temperature on the boundary of the spatial domain
- **Neumann command** that involves an imposed flux temperature on the boundary of the spatial domain
- **Convective command** (the temperature of the ambient medium or a cooling fluid, a parameter of the cooling fluid as the speed etc)
- **Radiation commands** (the temperature of the ambient medium or other parameters that are outside the spatial domain of the field problem and influences the temperature of a device by radiation phenomenon).

2 The state equations

A complete physical description of electromagnetic field is given by Maxwell's equations in terms of five field vectors: the magnetic field **H**, the magnetic flux density **B**, the electric field **E**, the electric field density **D**, and the current density **J**. In low-frequency formulations, the quantities satisfy Maxwell's equations [5]:

$$\nabla \times \boldsymbol{H} = \boldsymbol{J} \tag{1}$$

$$\nabla \times E = -\frac{\partial B}{\partial t} \tag{2}$$

$$div\ B = 0 \tag{3}$$

$$div\ D = \rho_C \tag{4}$$

with ρ_c the charge density.

For simplicity we gave up to the bold notations for vectors.

The second set of relationships, called the constitutive relations, is for linear materials:

$$B = \mu H;\ D = \varepsilon E;\ J = \sigma E$$

with σ – the electric conductivity and μ the magnetic permeability.

The B-H relationship is often required to represent non-linear materials. The current density **J** in Eq. (1) must represent both currents impressed from external sources and the internally-generated eddy currents.

The formulation with vector and scalar potentials has the mathematical advantage that boundary conditions are more often easily formed in potentials than in the fields themselves. The magnetic vector potential is a vector **A** such that the flux density **B** is derivable from it by the operation ***curl*** or the operator ($\nabla\times$).

3 Inverse problems

In this section we present some computational aspects for optimal control of the heat transfer in solids, both for single system and coupled systems. For the single system we consider the case of the conduction heat transfer using as mathematical model the heat equation in space 2D. The functional cost (objective function) is a quadratic form [4].

3.1. Optimal control by distributed-commands

The general class of the problems dealt with this paper is governed by the following differential equation [5]:

$$\frac{\partial}{\partial x}(k_x \frac{\partial u}{\partial x}) + \frac{\partial}{\partial y}(k_y \frac{\partial u}{\partial y}) + h + f = 0 \tag{6}$$

with boundary conditions:

$$u(x,y)\Big|_{C1} = u_0(x,y) \tag{7}$$

$$\left(\frac{\partial u}{\partial n} + q\right)\Big|_{C2} = 0 \tag{8}$$

$$\left[\frac{\partial u}{\partial n}+\alpha(u-v)\right]\Big|_{C3}=0 \tag{9}$$

$$\left[\frac{\partial u}{\partial n}+g(u,v)\right]\Big|_{C4}=0 \tag{10}$$

where u(x,y) is the temperature in the analysis domain Ω, and $C = C_1 \cup C_2 \cup C_3 \cup C_4$ is the boundary of the domain; h is a known function representing internal heat generation and f is the command (an unknown function).

In (7) u_0 is a known function (Dirichlet condition) and in (8) we have a Neumann condition with q -the flux on the boundary. On the boundary C_3 we have a convective condition (9) with α the convection coefficient and v the ambient temperature. On the boundary C_4 we have a mixed-condition (as for example convection and radiation condition), with g a known function. In (6) k_x , k_y, are the thermal conductivities in the directions of the axes of the co-ordinates system Oxy. In the relationships (7)-(9), ∂/∂n is the directional derivative normal to the boundary C.

We consider a functional cost by the form [3]

$$J(w)=c_0\int_\Omega (u-u_D)^2\,dx\,dy \tag{11}$$

with c_0 - a given positive coefficient and u_D - an imposed internal temperature distribution. This functional penalises the deviation of the temperature from an imposed (desired) distribution.

The problem of the optimal control consists in the minimisation of the functional (11). In other words we seek a command f^* in F (an admissible set) such that [7]:

$$J(f^*)\le J(f);\quad \forall f\in F$$

in the conditions (13)-(17). Frequently, the set of admissible commands is by the form:

$$F=\{f\in L_2(\Omega): f_{\min}\le f\le f_{\max}\}$$

Two practical cases appear:

- the positions of the distributed sources (x_i, y_i) are known and the intensities f_i of these sources are required, that is the command function has the form:

  $$f_i=\sum_{i=1}^{n} f_i\delta(x-x_i)\delta(y-y_i)$$

  with δ the Dirac's function.
- the intensities f_i are known and the positions are required.

The first case is simpler than the second case because it doesn't involve geometrical parameters in the design of the device.

In our examples we considered a fixed position of the heat sources and we tried to seek the amplitudes of the heat sources so that the functional cost has a minimum.

3.1.1. Necessary conditions for optimality

We transform the constrained optimal control problem into an unconstrained problem through the introduction of adjoint function Φ. We define the augmented cost-functional by [3]:

$$L=J(f)+\int_\Omega \Phi(x,y)\left[\frac{\partial}{\partial x}\left(k_x\frac{\partial u}{\partial x}\right)+\frac{\partial}{\partial y}\left(k_y\frac{\partial u}{\partial y}\right) + h + f\right]dx\,dy \tag{12}$$

Necessary conditions for optimality are derived by a variational approach. It is considered a variation δf in the command f that introduces a variation δL. From the first variation of L, results the adjoint equation [2]:

$$\frac{\partial}{\partial x}\left(k_x\frac{\partial \Phi}{\partial x}\right)+\frac{\partial}{\partial y}\left(k_y\frac{\partial \Phi}{\partial y}\right)+2c_0(u-u_D)=0 \tag{13}$$

$$J'(f)=\phi \tag{14}$$

with boundary conditions:

$$\left[\frac{\partial \Phi}{\partial n}+\Phi\frac{\partial g}{\partial u}\right]\Big|_{C4}=0 \tag{15}$$

$$\left[\frac{\partial \Phi}{\partial n}\right]\Big|_{C2}=0 \tag{16}$$

$$\left[\frac{\partial \Phi}{\partial n}+\alpha\Phi\right]\Big|_{C3}=0 \tag{17}$$

To obtain the optimal command f^* (practically, the method of gradient projection), the algorithm proceeds as follows:

1. make an initial guess of the command f_0 and set the iterations counter n to zero;
2. solve the state equation (6) with conditions (7)-(10);
3. solve the adjoint equation (13) with conditions (14)-(17);
4. compute the new command:
   - $$f_{n+1}=f_n - s.J'(f_n) \tag{18}$$

   with s the length of the step in the antigradient direction.
5. repeat the steps 2^0-4^0 until subsequent changes in J are less than a pre-set criterion.

The length of the step s is determined by a one-dimensional search technique [7].

3.2. Optimal control by boundary commands

Another practical case is the boundary control. For example the speed of the cooling medium in an electromagnetic device is the principal command for a desired temperature in device. The general class of the problems dealt with this paper is governed by the following differential equation:

$$\frac{\partial}{\partial x}\left(k_x \frac{\partial u}{\partial x}\right)+\frac{\partial}{\partial y}\left(k_y \frac{\partial u}{\partial y}\right)+f=0 \qquad (19)$$

with specified boundary conditions. In (19) f is a known function that represents internal heat sources-the Joule-Lenz's effect and eddy-current losses.

The boundary conditions are:

$$u(x,y)\,|_{C1}=u_0(x,y) \qquad (20)$$

$$\left(\frac{\partial u}{\partial n}+q\right)|_{C2}=0 \qquad (21)$$

$$\left[\frac{\partial u}{\partial n}+\alpha(u-w)\right]|_{C3}=0 \qquad (22)$$

$$\left[\frac{\partial u}{\partial n}+g(u,w)\right]|_{C4}=0 \qquad (23)$$

where: u (x, y) is the temperature in the analysis domain Ω from the bi-dimensional space R^2 and C $=C_1 \cup C_2 \cup C_3 \cup C_4$ is the boundary of the domain. In (20) u_0 is a known function (Dirichlet condition) and in (21) we have a Neumann condition with q -the flux on the boundary. On the boundary C_3 we have a convective condition (22) with α the convection coefficient and w the ambient temperature. On the boundary C_4 we have a mixed-condition (as for example convection and radiation condition), with g a known function. In (19) k_x, k_y are the thermal conductivities in the directions of the axes of the co-ordinates system Oxy. In conditions (21)-(23), $\partial/\partial n$ is the directional derivative normal to the boundary C.

The mathematical model of the heat equation in space 2D, also is met in axisymmetric field, where the equation (6) becomes:

$$\frac{\partial}{\partial r}\left(k_r r \frac{\partial u}{\partial r}\right)+\frac{\partial}{\partial z}\left(k_z r \frac{\partial u}{\partial z}\right)+rf=0 \qquad (24)$$

In a convective control, w can be chosen as a command variable. We consider a functional cost by the form:

$$J(w)=c_0\int_\Omega (u-u_D)\,dxdy \qquad (25)$$

with: c_0 - a given positive coefficient; u_D-an imposed internal temperature distribution.

The functional cost has a practical significance: it penalises the deviations of the temperature in the domain from the imposed standard (u_D). On the boundary $C_3 \cup C_4$ we apply a command w ε L^2 (C) - the space of the integrable-squared functions, with g a known function. The boundary command w can be the temperature of the cooling medium, which is we have a convective control like in (22) where the coefficient α is supposed constant or depends by the boundary temperature. In another practical case, the command w is the speed of the cooling medium (like in the oil-immersed transformer), and g has the form g (u, w) =α (w) ($u-u_\infty$), where u_∞ is the temperature of the cooling medium (supposed a constant). The dependence of α by w must be known but unfortunately this is a difficult task. It is determined from experimental data and is expressed using nondimensional parameters as Nusselt and Reynolds numbers.

The problem of the optimal control consists in the minimisation of the functional (25). Practically we seek a command w^* ε W (an admissible set) such that:

$$J(w^*)\le J(w) \quad \forall w\in W \qquad (26)$$

in the condition (26), with specified boundary conditions (20)-(23).

Frequently, the set of admissible commands is by the form [7]:

$$W=\{w\in L_2(\Omega): w_{\min}\le w\le w_{\max}\} \qquad (27)$$

3.2.1. Necessary conditions for optimality

We transform the constrained optimal control problem into an unconstrained problem through the introduction of adjoint function Φ. We define the augmented cost-functional by [3]:

$$L=J(w)+\int_\Omega \Phi(x,y)\left[\frac{\partial}{\partial x}\left(k_x\frac{\partial u}{\partial x}\right)+\frac{\partial}{\partial y}\left(k_y\frac{\partial u}{\partial y}\right)+f\right]dxdy \qquad (28)$$

Necessary conditions for optimality are derived by a variational approach. It is considered a variation δw in the command w that introduces a variation δL. From the first variation of L, results the adjoint equation [5]:

$$\frac{\partial}{\partial x}\left(k_x\frac{\partial \Phi}{\partial x}\right)+\frac{\partial}{\partial y}\left(k_y\frac{\partial \Phi}{\partial y}\right)+2c_0(u-u_D)=0 \qquad (29)$$

with boundary conditions:

$$\frac{\partial \Phi}{\partial n}|_{C2}=0$$
$$\left[\frac{\partial \Phi}{\partial n}+\alpha\Phi\right]|_{C3}=0 \qquad (30)$$
$$\left[\frac{\partial \Phi}{\partial n}+\Phi\frac{\partial g}{\partial u}\right]|_{C4}=0$$

The gradient of the cost-functional is:

$$J'(w)=-\Phi\frac{\partial g}{\partial w}|_{C4} \qquad (31)$$

The gradient method [4] can be employed to obtain the optimal command w^* (or the method of gradient projection for the constrained problem).

3.2.2. A numerical model
For obtaining the optimal command w^*, the gradient method can be used with good results, especially for the unconstrained commands. For this case the gradient method proceeds as follows [4]:

- *make an initial guess of the command w_0, and set the iterations counter to zero;*
- *solve the state equation (19) with conditions (20)-(23);*
- *solve the adjoint equation (29) with the boundary conditions (30);*
- *compute the new command:* $w_{n+1} = w_n - s.J'(w_n)$
- *repeat the steps 2^0-4^0 until subsequent changes in J are less than a preset criterion.*

The length of the step s is determined by a one-dimensional search technique. Recent developments allow replacing the step length rule by a trust region method. In the application program developed by the authors, it was used the following rule: an initial value for s is chosen and the functional-cost is calculated and if its value isn't less than the old value, the length of the step is divided to two. This procedure continues until the monotony of the functional is satisfied. The disadvantage of this rule is that requires an iterative method to determine s. The steps 2^0 and 3^0 of the algorithm imply the solution of the state and adjoint equations. The finite element method was used to obtain approximate solutions in finite dimensional subspace.

Finally, by assembling the element equations, results an algebraic equations system. The adjoint equation (27) and cost-functional are discretized in the same manner.

3.3. Optimal control of the heat in electrical cables by boundary commands
As target examples we consider an infinitely long coaxial cable with a stranded inner conductor carrying the direct current. This problem can be treated as a two-dimensional problem. The current density is a constant and this assumption is valid in the analysis and synthesis of electrical devices where the current density J is a specified constant in conductors and zero elsewhere. This inherent approximation becomes more and more valid as we use smaller and smaller triangles. In the alternating current, the skin effect appears but in the most practical systems the conductor is stranded (that is made up several tightly wound strands of conductor insulated from each other). This solution forces the currents to flow through the entire cross section of the conductor and thereby utilise the material better. Hence the validity of assuming uniform density as in direct current systems can simplify the computation. This assumption can lead at some practical applications. For such a system it has seen that the governing equation is (19). With the origin of the co-ordinates system in the centre of the cable, only a part of the entire domain is used. The convective command w is applied on the shield of the cable. The functional cost is by the form (25). We considered an averaged value of the gradient so that we can obtain a sub-optimal command.

In this target example we consider a coaxial cable with a non-uniform current density and two insulation layers. In the figure 1 the analysis domain is presented. The geometrical dimensions are: conductor radius is 15 mm, the outer radius of the first layer is 30 mm and the outer radius of the second layer is 50 mm. The resistivity of the copper was considered at the temperature 75 ^{0}C and equal to 1.78 .10^{-8} Ω/m. The physical properties are: thermal conductivities k_x=k_y=385 W/m.^{0}C in the copper and equal to k1=0.14 W/m.^{0}C and k2=0.175 W/m.^{0}C in the insulation layers; α=12 W/m^2.^{0}C .The current density is 5.0.10^{-5} A/mm^2. The minimum value of J (w) was found to be equal to 5.604 for c_0=0.0001. The number of iterations is 181 with the initial value of the command equal to 40^{0}C. The optimal command is 63.95 ^{0}C for u_D=75 ^{0}C.

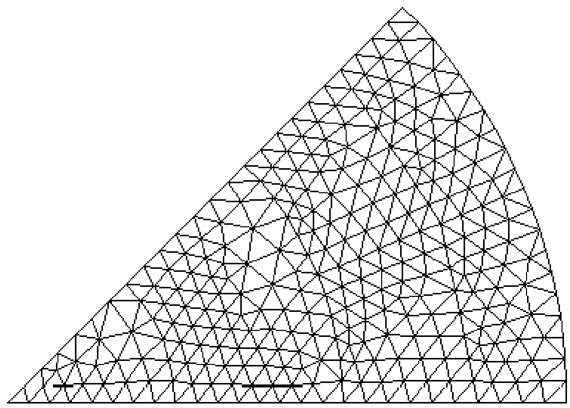

Fig. 1 – *Analysis domain and mesh*

In numerical simulation it is considered a medium value of the gradient on the boundary, that is in the formula (29) the command w, at each iteration step, is a constant (a frequent case in industry where we consider an average value of the command variable). Any case may be treated in the same manner (for example, a piecewise command or a local command).

7 Conclusions
The problem of coupled fields and inverse problems in electrical engineering is a complex problem in terms of computing resources. It is obviously that a command variable can be a function in any system of

the coupled system. For example, the heat source (as command variable) is an electrical parameter determined in the electrical system as Joule-Lenz's effect of the eddy-currents. As output of the coupled system can be either the temperature distribution in the electromagnetic device or the distribution of the flux density B in different parts of the electromagnetic device.

The state and co-state equations are discretised by the finite element method. Domain decomposition techniques can be used at the level of the problem and/or of any physical system. Domain decomposition offers an efficient approach for large-scale problems or complex geometrical configurations ([1]-[10]). This method in the context of the finite element programs leads to a substantial reduction of the computing resources as the time of the processor.

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