A numerical simulation of distributed-parameter systems

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Abstract: - The scope of this paper is to present, both from a theoretical and a practical viewpoint, the problem of the free convection flow in a porous medium bounded by a vertical flat plate (wall).

The objective of the present paper is to compare the numerical results with the analytical solutions presented in the professional literature of the area in the case of the large Rayleigh number.

The numerical results are reported for the problem of steady free convection about a vertical impermeable flat plate in a saturated porous medium. We consider an unbounded porous medium in a gravitational field, saturated with a fluid at at temperature T_∞ containing a longitudinal line heat source. We limit our study to a heated flat plate. The mathematical models was integrated numerically using the Runge-Kutta method.

Keywords:- Fluid mechanics; Similarity solutions; Numerical simulation.

1 Introduction

In transport of the heat transfer through a porous medium, the fluid motion is due to the convection phenomenon. The mathematical model for the fluid flow is described by Navier-Stokes equations that are partial differential equations of hyperbolic type. The solution of this model can be obtained by numerical methods. In some assumptions the mathematical model is reduced to a lumped-parameter non linear equation so that a practical solution can be obtained by numerical procedures. But in the last case there is a large literature so that we have the advantage to use it.

In this paper we present some computational aspects for the classical two-dimensional laminar incompressible boundary layer flow past a flat plate. This case is of great practical importance in many engineering applications (geothermal energy, building thermal insulation, enhanced oil recovery, solid-matrix heat exchangers etc.)

2 Mathematical modelling

Our target example is a heated semi-infinite vertical flat plate embedded in an unbounded porous medium in a gravitational field, saturated with a fluid at temperature T_∞.at rest

We considered a rectangular Cartesian coordinate system with the origin fixed at the leading edge of the veretical surface with the x-axis directed upwards along the wall, and y-axis directed to normal to the surface.

Mathematical model is defined by a system of partial derivative equations [1]:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u=-\frac{k}{\mu}\left(\frac{\partial p}{\partial x}+\rho g\right) \tag{2}$$

$$v=-\frac{k}{\mu}\frac{\partial p}{\partial y} \tag{3}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\lambda\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right) \tag{4}$$

$$\rho = \rho_\infty\left[1 - \beta\left(T - T_\infty\right)\right] \qquad (5)$$

The significances of the variables from Eq. 1 to 5 are:

- u, v – Darcy speed components in the directions Ox and Oy
- ρ, μ, β – density, viscosity and the coefficient of the thermal expansion of the fluid
- k – permeability of th6 porous medium
- λ – coefficient of thermal difusivitty
- p - pressure
- T- temperature
- g – acceleration due to gravity

Indicele ∞ are semnificația valorii mărimii respective la infinit.

Is is proved that at Rayleigh numbers, the most important part of convection takes place in a thin layer around the heated plate. In this case the mathematical model is a third order non linear ordinary differential equation depending on a parameter related to the temperature on the wall.

We distinguish some practical cases:

- The wall temperature is uniform
- The wall temperature is non uniform with a prescribed law
- The heat flux is uniform
- The heat flux is non uniform

The mathematical models have different forms for each case.

Case a. The uniform temperature of the wall

As first example we consider the case of the uniform temperature of the wall. The Blasius equation for steady-state regime with dimensionless variables is:

$$f''' + \frac{1}{2} ff'' = 0 \quad w \in [0,\infty) \quad (1)$$

with the boundary conditions:

$$\begin{aligned} &f(0)=0 \\ &f'(0)=1 \qquad (2) \\ &f'(\infty)=1 \end{aligned}$$

This equation gives a solution to the Prandtl boundary layer equations with f' the temperature of the fluid; practically it is a particular case of Falkner-Skan model [2]. It is obtained in some assumptions:

- Vertical component of the velocity and temperature distribution are of the same shape
- The vanishing of the vertical velocity and the fluid temperature at infinity

The Balsius equation with the boundary conditions (2)-(3) is a bi-local problem that can be solved by iterative procedures. Practically, we transform the bi-local problem in a Caughy problem with initial values:

$$f''' + \frac{1}{2} ff'' = 0 \quad w \in [0,T) \qquad (3)$$

$$\begin{aligned} &f(0)=a \\ &f'(0)=b \qquad (4) \\ &f''(0)=c \end{aligned}$$

where [0,T] is the maximum interval of existence of the solution, a and b belong to real numbers and c is real number that must be determined. The selection problem of the initial condition f''(0) influences the convergence speed of the algorithm.

In this formulation the problem depends by a single parameter c for a and b fixed.

Case b. The non-uniform temperature of the wall

Another interesting case corresponds to a non-uniform wall temperature by the form:

$$T(x) = T_\infty + A.x^\alpha \qquad (5)$$

In this case the mathematical model is the following equation:

$$f''' + \frac{\alpha+1}{2} ff'' - \alpha(f')^2 = 0 \qquad (6)$$

with the boundary conditions:

$$\begin{aligned} &f(0) = 0 \\ &f'(0) = 1 \qquad (7) \\ &f'(\infty) = 0 \end{aligned}$$

We introduce the initial values Caughy problem for (6) by conditions:

$$\begin{aligned} &f(0) = 0 \\ &f'(0) = 1 \qquad (8) \\ &f''(0) = \mu \end{aligned}$$

The value of the parameter μ must be selected so that the final value of f' must be zero.

3 Numerical results

As target example we considered the heat transfer from a surface embedded in a porous medium through which a liquid is flowing. The surface temperature is uniform or non-uniform and is defined by the law from the relationship (5).

Case a. The uniform temperature of the wall

The Eq. (1) can be solved iteratively using a single-step method as Runge-Kutta of the fourth order. For this we transform the Eq. (1) in a system of the first order. With the new variables {z1,z2,z3}={f,f',f''}, we must solve the system:

$$\begin{aligned} &z1' = z2 \\ &z2' = z3 \qquad (9) \end{aligned}$$

$$z3' = -\frac{1}{2} z1 \cdot z3$$

with the initial conditions:

$$z1(0)=a$$
$$z2(0)=b \qquad (10)$$
$$z3(0)=c$$

In the case in discussion, the parameters a and b are fixed and the parameter c is choosen so that the final value for f' is reached.

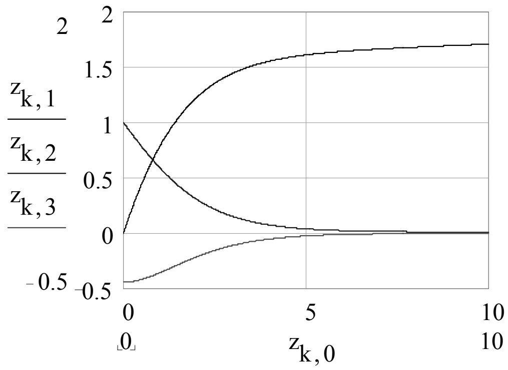


Fig. 1 – The variation of f, f', f'' in Blasius model

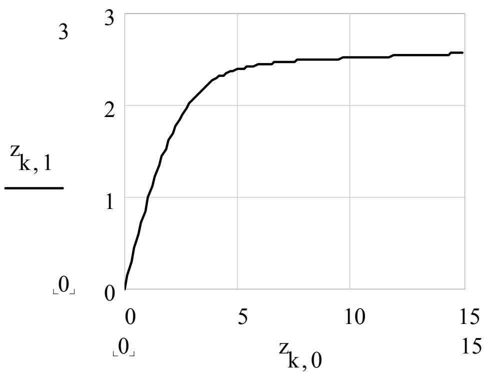
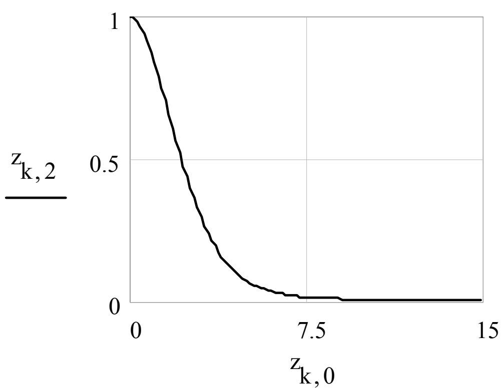


Fig. 2.- The variation of f' in Blasius model

In our target example the initial value of f''(0) is free and we tried to find this value so that the problem defined by Eq. 1 to have a solution both physical and mathmatical. The value c was found as –0.44. The value was obtained by an iterative



procedure by an approximation row that converges to the solution.

In figures 1 to 3 the curves for the variables f, f' and f'' are plotted. It is obviously that the mathematical solution is a physical solution.

Fig. 3- The curve f' in Blasius model

Case b. The non-uniform temperature of the wall

The differential equation (6) can be rewritten as a system of differential equations of the first order. For this we define the set {z1,z2,z3}={f,f',f''}. The equations system is the following:

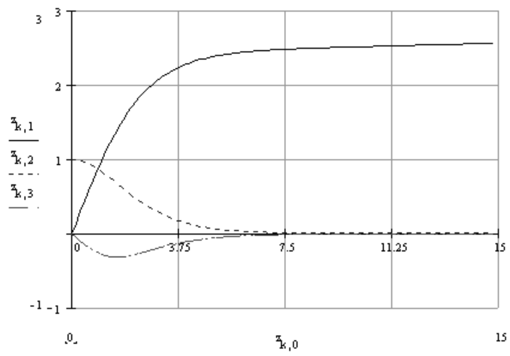
$$z1' = z2$$
$$z2' = z3$$
$$(11)$$
$$z3' = -\frac{1+\alpha}{2} z1 \cdot z3 + \alpha \cdot (z2)^2$$

with the initial conditions:

$$z1(0)=0$$
$$z2(0)=1$$
$$z3(0)=\mu$$

where μ is a real parameter that is selected so that the condtions (7) are fullfilled.

With the notations $z1(t_k)=z_{k,1}$, the curves for the unknowns are plotted in the figure 4. In Fig. 5 the curve for f' is plotted. It can ve seen that the



conditions defined by relationships (7) are fullfilled.

Fig.4. The variation of f,f',f''

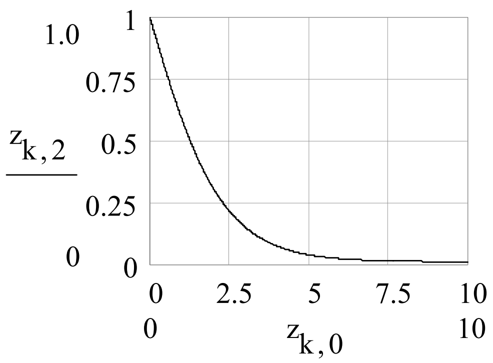


Fig.5- Variation of f'

4 Conclusion

In this work we presented some aspects in numerical simulation of the fluid flow. Mathematical model is a differential equation with partial derivatives. In some assumptions this model was approximated by a lumped-parameter model with imposed boundary conditions. Our approximations were solved by uni-step method as Runge-Kutta of the first order. The bilocal problem was transformed in a Caughy problem. The numerical model was solved by an iterative procedure. The convergence of the method depends on the first initial approximation of Caughy problem. We limited discussion at some practical examples from the open literature.

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