Position predictive control for an induction motor

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Abstract: -This paper presents a position predictive control scheme for an induction motor. The non-linear differential equations, which describe the dynamics of the motor, are represented by a d-q model. The design of a Generalised Predictive Control is obtained as a simplified model. On the other hand, an observer is used in open loop in order to obtain state measurements. The main advantages offered by the proposed scheme are: the position is the only measurement required and the simplicity of the control law allows a simple and straightforward implementation. The efficiency of the controller is demonstrated through simulations.

Key-Words: - Nonlinear System, Control Theory, Predictive Control, Induction Motor,

1 Introduction

Induction motors are robust, not expensive and require low maintenance when compared with direct current motors. In this work the position control of an induction motor based on predictive control is presented. The type of motor considered is the one with squirrel-cage motor, which dynamics are described by non-linear differential equations [1] [2]. Some of the difficulties faced are due to uncertainties in the parameters measurements in addition to the inherently non-linear behaviour. Various applications of no-linear control of induction motors based on parametric methods have been published. De Luca [3] proposed a controller with position feedback and Marino [4] considered an adaptive no linear control, while Kim Donf-II [5] adopted a linearization and output feedback.

The relevance of our proposed scheme lies on the simplicity of the controller when comparing it to previous designs, with the fact that the only measurement used is the position.

2 Dynamics of Induction Motors

It is necessary to determine the main characteristics of the induction motor, which can be represented by the "d-q" model in [1] and [2].

$$\begin{bmatrix} v_d^s(t) \\ v_q^s(t) \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} (R_s + L_s p) & 0 & Mp & 0 \\ 0 & (R_s + L_s p) & 0 & Mp \\ Mp & nMw_r & (R_r + L_r p) & nL_r w_r \\ -nMw_r & Mp & -nL_r w_r & (R_r + L_r p) \end{bmatrix} \begin{bmatrix} i_d^s(t) \\ i_q^s(t) \\ i_d^r(t) \\ i_q^r(t) \end{bmatrix} \quad (1)$$

$$\dot{w}_r = (-Dw_r + T_e)/J$$

$$T_e = nM[i_d^s(t) i_q^r(t) - i_q^s(t) i_d^r(t)]$$

$$\dot{q} = w_r$$

with

$$v_d^s = \sqrt{2} V_s \cos(wt)$$

where

$$v_q^s = \sqrt{2} V_s \sin(wt)$$

v_d^s, i_d^s Instantaneous stator direct axis voltage and current

v_q^s, i_q^s Instantaneous stator quadrature axis voltage and current

i_d^r, i_q^r Instantaneus rotor direct and quadrature-axis currents.

V_s Supply voltage amplitude

p Operator d/dt

w_r Rotor angular velocity

T_e Instantaneous electromagnetic torque

R_s, R_r Stator and rotor resistences. $R_s = 60 ohms$, $R_r = 37.36 ohms$

M Peak stator-rotor mutual inductance. M =1.6h

J, D Equivalent Inertia and viscous friction $J = .0186 kg - m^2$, $D = .0261$ newton-m-sec/rad

L_s, L_r Stator and rotor self-inductance $L_s = 1.699h$ $L_r = 1.68h$

w Excitation frequency w=377rad/sec

n Number of pole-pairs. n=2

l_s, l_r Stator and Rotor Leakage Inductance $l_s = .0991h$, $l_r = .0804h$

A simple form can be derived if the average torque is considered [1] and [2]. In such a case the dynamics of the motor are reduced to the following form:

$$T_{em} = \frac{2nR_r V_s^2 /(w\phi)}{(R_s + R_r/\phi)^2 + (w(l_s + l_r))^2} \quad (2)$$

Replacing $u = V_s^2$ we obtain

$$T_{em} = f(w_{r_m})u$$

where

$$f(w_{r_m}) = \frac{2nR_r /(w\phi)}{(R_s + R_r/\phi)^2 + (w(l_s + l_r))^2} \quad (3)$$

The control input is voltage amplitude, $V_s = \sqrt{u}$. And therefore the mechanical part of the motor is reduced to:

$$\dot{w}_{r_m} = (-Dw_{r_m} + f(w_{r_m})u)/J \quad (4)$$

$$\dot{q}_m = w_{r_m}$$

$$\phi = 1 - \|s - 1\|$$

Where ϕ represents a normalisation of the slip s, which can be written as

$$s = \frac{w_s - w_{r_m}}{w_s} \quad (5)$$

with $w_s = w/n$

Where w_s is defined as the synchronous speed of the motor..

3 Development of the NCGPC

The development of the Nonlinear Continuous Time Generalized Predictive Control (NCGPC) [7, 8]was carried out following the receding horizon strategy of its linear counterpart [6], which principles can be summarised as follows:

1. Predict the output over a range of future times.
2. Assuming that the future setpoint is known, choose a set of future controls which minimize the future errors between the predicted future output and the future setpoint.
3. Use the first element $u(t)$ as a current input and repeat the whole procedure at the next time instant; that is, use a receding horizon strategy.

3.1 System Description

The Nonlinear Continuous Time Generalized Predictive Control (NCGPC) considers nonlinear dynamics systems with the state-space representation:

$$\dot{x}(t) = f(x) + g(x)u$$

$$y(t) = h(x) \quad (6)$$

where f, g y h are differentiable N_y times with respect to each argument $x \in R^n$, is the vector of the state variables $u \in R$ is the manipulated input and $y \in R$ is the output to be controlled.

3.2. Prediction of the output

In this section the output prediction is obtained following the idea of CGPC [6]. The output prediction is approximated for a Maclaurin series expansion of the system output as follows.

$$y^*(t,T) = y(t) + \dot{y}(t)T + y^{(2)}(t)\frac{T^2}{2!} + \cdots + y^{(N_y)}(t)\frac{T^{N_y}}{N_y!} \quad (7)$$

or

$$y^*(t,T) = T_{N_y} Y_{N_y} \quad (8)$$

where

$$Y_{N_y} = [y \quad \dot{y} \quad y^{(2)} \quad \cdots \quad y^{(N_y)}]^T \quad (9)$$

and

$$T_{N_y} = [1 \quad T \quad \frac{T^2}{2!} \quad \cdots \quad \frac{T^{N_y}}{N_y!}] \quad (10)$$

The predictor order N_y is chosen less than the number of the times that the output has to be differentiated in order to obtain terms not linear in u. But in this paper the output will be differentiated until obtain u^2.

3.3 Prediction of the reference trajectory

The objective of the control is to drive the predicted output along a desired smooth path to a set point. Such a path is called a reference trajectory. The reference trajectory following [6] is given by

$$w_r^*(t,T) = [p_0 + p_1 T + p_2 \frac{T^2}{2!} + \cdots + p_r \frac{T^{N_y}}{N_y!}][w - y(t)] + y(t) \quad (11)$$

where w is the set point, or rewriting this equation

$$w_r^*(t,T) = T_{N_y} w_r + y(t) \quad (12)$$

where

$$w_r = [p_0 \quad p_1 \quad \cdots \quad p_r]^T (w - y(t)) \quad (13)$$

and T_{N_y} is given by (5)

3.4 Derivative emulation

The NCGPC is based in taking the derivatives of the output, which are obtained as follows

$$\dot{y}(t) = L_f h(x)$$
$$y^{(2)}(t) = L_f^2 h(x)$$
$$\vdots$$
$$y^{(r)}(t) = L_f^r h(x) + L_g L_f^{r-1} h(x) u(t)$$
$$y^{(r+1)}(t) = S_1(x) + J_1(x)u(t) + L_g L_f^{r-1} h(x)\dot{u}(t)$$
$$y^{(r+2)}(t) = S_2(x) + J_2(x)u(t) + I_1(x)\dot{u}(t) + L_g L_f^{r-1} h(x) u^{(2)}(t)$$
$$\vdots$$
$$y^{(N_y)}(t) = S_{(N_y-r)}(x) + J_{(N_y-r)}(x)u(t) + I_{(N_y-r)}(x)\dot{u}(t) + I_{(N_y-r+1)}(x)u^{(2)}(t) + I_{(N_y-r-1)}(x)u^{(N_y-r-1)}(t) + L_g L_f^{r-1} h(x) u^{(N_y-r)}(t) \quad (14)$$

Where $L_f h(x)$ represents the Lie derivative S_i, J_i and I_i, are some functions of x (and not u). These output derivatives are obtains from the system of equation (1) and N_y is chosen less than the number of the times that the output has to be differentiated in order to obtain terms not linear in u, r is the relative degree. Output and its derivatives can be rewritten by

$$Y_{N_y}(t) = O(x(t)) + H(x(t))u_{N_y} \quad (15)$$

where

$Y_{N_y} = [y \quad \dot{y} \quad y^{(2)} \quad \cdots \quad y^{(N_y)}]^T$ and

$u_{N_y} = [u \quad \dot{u} \quad u^{(2)} \quad \cdots \quad u^{(N_y-r)}]^T$

$$O = \begin{bmatrix} y \\ L_f h(x) \\ L_f^2 h(x) \\ \vdots \\ L_f^r h(x) \\ S_1(x) \\ S_2(x) \\ \vdots \\ S_{(N_y-r)}(x) \end{bmatrix} \quad H = \begin{bmatrix} 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ L_g L_f^{r-1} h(x) & 0 & \cdots & 0 \\ J_1(x) & L_g L_f^{r-1} h(x) & \cdots & 0 \\ J_2(x) & I_1(x) & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ J_{N_y-r}(x) & I_{N_y-r}(x) & \cdots & L_g L_f^{r-1} h(x) \end{bmatrix} \quad (16)$$

3.5 Cost function minimization

The function is not defined with respect current time, but respect a moving frame, which origin is in time t. Where T is the future variable. Given a predicted output over a time frame the CGPC calculates the future controls. The first element $u(t)$ of the predicted controls is then applied to the system and the same procedure is repeated at the next time instant. This makes the predicted output depend on the input $u(t)$ and its derivatives, and the future controls being function of $u(t)$ and its N_u-derivatives. The cost function is:

$$J(u_{N_y}) = \int_{T_1}^{T_2} [y^*(t,T) - w_r^*(T,t)]^2 \, dT \quad (17)$$

With the substitution of Eqs. 8 and 12 the cost function becomes

$$J(u_{N_y}) = \int_{T_1}^{T_2} [T_{N_y} O + T_{N_y} H u_{N_y} - T_{N_y} w_r)]^2 \, dT \quad (18)$$

and the minimization results in

$$u_{N_y} = K(w_r - O) \quad (19)$$

where

$$T_y = \int_{T_1}^{T_2} T_{N_y}^T T_{N_y} \, dT \quad \text{and} \quad K = [H^T T_y H]^{-1}[H^T T_y] \quad (20)$$

As explained above, just the first element of u_{N_y} is applied. Then, the first row of, which will be called, the control law is given by

$$u(t) = k[w_r - O] \quad (21)$$

4 Predictive Control for an Induction Motor

In this section, a predictive control for the position of an induction motor, described by Eq. 1 is presented. The design of the controller was based on the use of the simplified model described in Eqs. 2 to 5. The output variable is the angular position q, and the control variable is V_s. The dynamics of the motor were neglected, while the average torque is considered. We can observe in the Figs. 1 and 2 the instantaneous torque T_e using the model "d-q" given in Eq. 1 and the average torque T_{em} using the simplified model given in Eq. 2.

Figure 1 shows the simulation of the motor rotating in negative direction, while Fig. 2 illustrates the rotation in the positive direction.

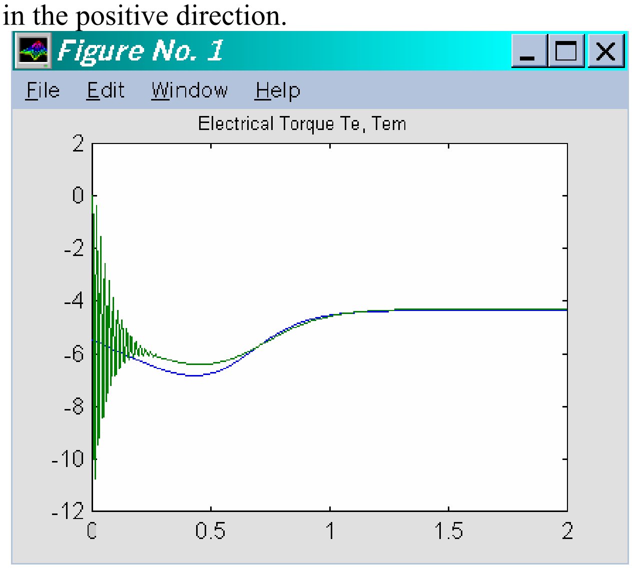

Figure 1 Electrical Torque Te, Tem, negative direction

To obtain the predictive controller it was necessary to get the derivatives of output of the simplified model. In this case until the second derivative was gotten, this is the relative degree of the simplified model.

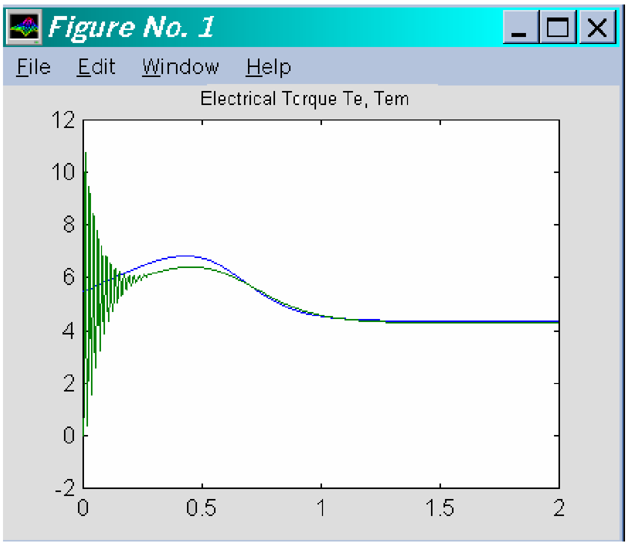

Figure 2 Electrical Torque Te, Tem, positive direction

Obtaining

$$y_m = q_m$$
$$\dot{y}_m = w_{rm} \quad (22)$$
$$\ddot{y}_m = -Dw_{rm} / J + T_{em} / J$$

When the predictor is equal to the relative degree, el NCGPC is converted in a state feedback linearization, and the control law is obtained as follow:

$$u = \frac{-L_f^2 h(x_m) + \{r_0(y_{ref} - y)\frac{10}{3T^2} + [r_1(y_{ref} - y) - w_{rm}]\frac{5}{2T} + r_2(y_{ref} - y)\}}{L_g L_f h(x_m)} \quad (23)$$

where

$$L^2{}_f h(x_m) = -Dw_{rm} / J \quad (24)$$
$$L_g L_f h(x_{xm}) = T_{em} / J \quad (25)$$

and T_{em} is given by Eq. 2, y_{ref} is reference trajectory.

Note that the simplified model has to be simulated in parallel (open loop observer) Fig 3, in order to obtain the rotor velocity w_{rm}.

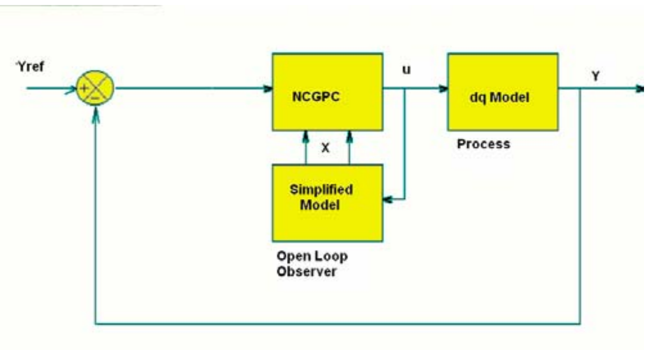

Figure 3 NCGPC Control Squeme

Figure 4 show that the position of the motor described by Eq. 1 reached 10 rad achieving the objective, while the position q_m of the simplified model has not reached the reference. Figures 5, 6 and 7 show the velocities of motor rotor represented by the model "d-q" and the simplified model w_r and w_{rm}, the torques T_{em} and T_e , and finally the amplitude of the applied voltage V_s, which is used to control both models

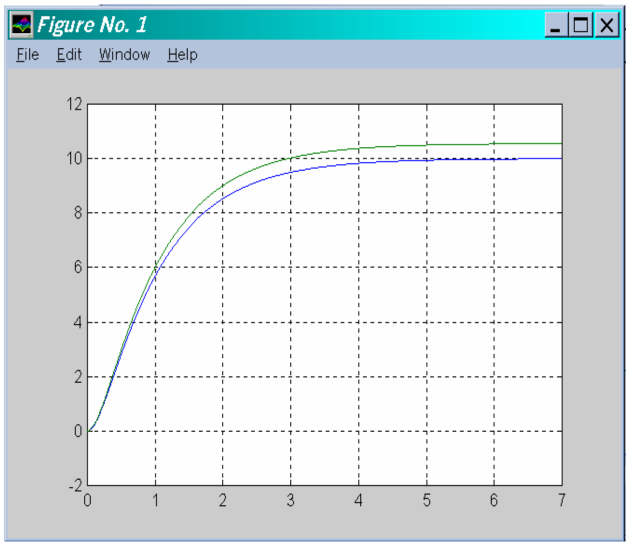

Figure 4 Angular positions q and q_m

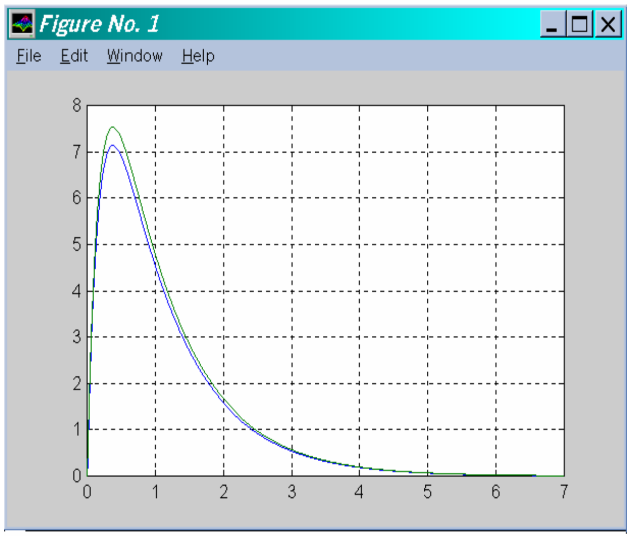

Figure 5 Rotor angular velocities w_r and w_{rm}.

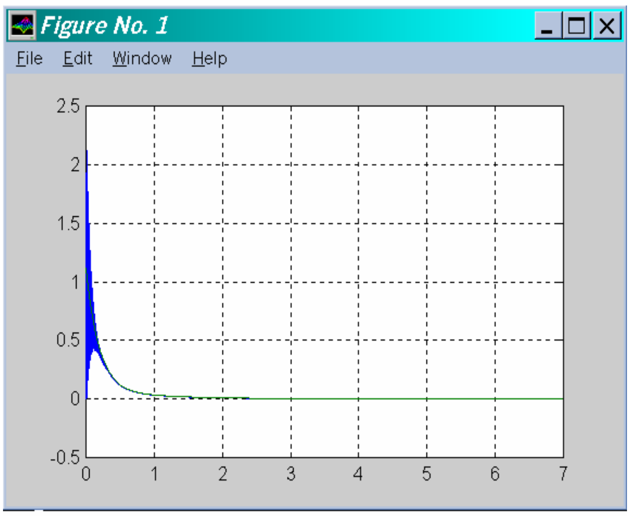

Figure 6 Electrical Torque T_{em} and T_e.

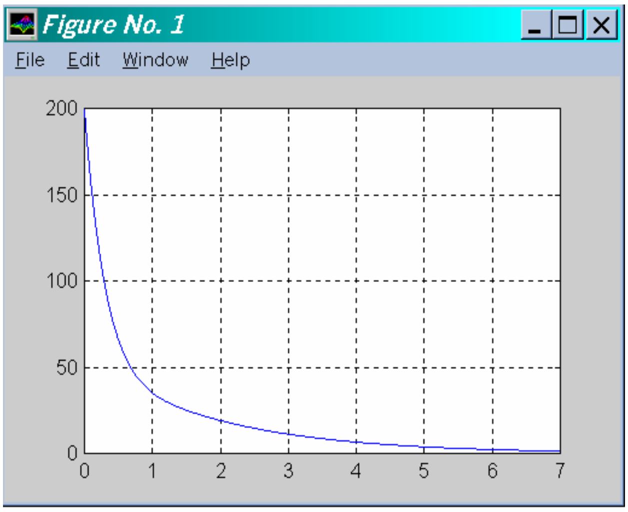

Figure 7 Supply voltage amplitude V_s .

4 Conclusions

The paper has proposed and a position predictive control scheme for the induction motors, described in the model Eq. 1. The design of the Nonlinear Continuous Time Generalized Predictive Control (NCGPC) was obtained using the simplified model described by Eqs. 2 to 5 and it is used as well as an observer in open loop in order to obtain state measurements. The amplitude of the supply voltage is used as control input. The main advantages offered by the proposed scheme are: the position is the only measurement required and the simplicity of the control law allows a simple and straightforward implementation. The efficiency of the controller is demonstrated through simulations, wich show that the objectives of the controller are achieved and the immediate work is dedicated to the implementation and the adaptation of the proposed scheme to the real process.

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