Study of vibration suppresion of mechanical systems with elastic elements

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Abstract: - The presents the study of the dampings transmitted in a mechanical system, from a motion way to others, due to the system general motions. In this way, a damper which acts of a degree of freedom, can dissipate the energy that derive from other system degrees of freedom, apparently weak coupled. The relative motion allows even this coupling that makes possible the dampings' remittance on significant degrees of freedom. The study is accomplished on a simple example for a qualitative explanation of the phenomena.

Key-Words: - Multibody systems, Liaison forces, Dampings, Coriolis terms, Mathematical model, Degrees of freedom, Elastic elements, Motion equations.

1 Motion equations of the multibody systems with elastic elements

In the following will be established the motion equations for an elastic finite element with a general motion together with an element of the system. The type of the shape function is determined by the type of the finite element. We will consider that the small deformations will not affect the general, rigid motion of the system. The continuous displacement field $\{f(x,y,z,t)\}$ is approximated, in FEM, by:

$$\{f\} = [N(x,y,z)]\{\delta_e(t)\}, \tag{1}$$

where the elements of matrix [N] (the shape functions), are determined by the type of the finite element used. We consider that, for the all elements of the system, the field of the velocities and of the accelerations are known. We refer the finite element to the local coordinate system Oxyz, mobile, and having a general motion with the part of system considered. We note with $\vec{v}_o(\dot{X}_o, \dot{Y}_o, \dot{Z}_o)$ the velocity and with $\vec{a}_o(\ddot{X}_o, \ddot{Y}_o, \ddot{Z}_o)$ the acceleration of the origin of the local coordinate system. The motion of the whole system is refer to the general coordinate system O'XYZ. By [R] is denoted the rotation matrix. If we apply the Lagrange's equations after some algebraic operations we obtain the motion equations for a single finite element under the compact form[1],[4]:

$$\begin{aligned}&[m_e]\{\ddot{\delta}_e\} + 2[c_e]\{\dot{\delta}_e\} + \left([k_e] + [k_e(E)] + [k_e(\Omega^2)]\right)\{\delta_e\} = \\ &= \{q_e\} + \{q_e^*\} - \{q_e^i(E)\} - \{q_e^i(\Omega^2)\} - [m_{oe}^i][R]^T\{\ddot{r}_o\},\end{aligned} \tag{2}$$

where $\vec{\Omega}$ represent the angular velocity and $\vec{E}$ the angular acceleration with the components in the local coordinate system. These motion equations are referred to the local coordinate system and the nodal displacement vector $\{\delta_e\}$ and the nodal force vector $\{q_e\} + \{q_e^*\}$ are express in the same coordinate system.

2 Assembling procedures and Liaison forces eliminating

The unknowns in the elasto-dynamic analysis of a mechanical system with liaisons are the nodal displacements and the liaison forces. Generally, the relations between the first order derivatives of the nodal displacements can be expressed by the linear formula:

$$\{\dot{\Delta}\} = [A]\{\dot{q}\}. \tag{3}$$

By assembling the motion equations written for each finite element we try to eliminate the liaisons forces and the motion equations will contain only nodal displacements as unknowns [2],[3]. The liaison between finite elements is realized by the nodes where the displacements can be equal or can be other type of functional relations between these. When two finite elements belong to two different elements (bodies) the liaison realized by node can imply relations more complicated between nodal displacement and their derivatives. The system of differential equations obtained after the assembling procedures is nonlinear, the matrix of the left term depending on the configuration of the multi-body system. These equations can be writing under the form:

$$[m]\{\ddot{q}\}+[c]\{\dot{q}\}+([k]+[k_{\varepsilon}]+\lfloor k_{\omega^2} \rfloor)\{q\}=\{f\}, \qquad (4)$$

where $[m]$, $[k]$ and $\lfloor k_{\omega^2} \rfloor$ are symmetric and $[c]$, $[k_{\varepsilon}]$ are skew-symmetric.

2.1 The influence of the Coriolis terms

The matrix $[c]$ is skew-symmetric. If we want to obtain the energy balance by integration, we obtain that the variation of energy due to the term skew-symmetric is null. Consequently, the Coriolis term only transfer the energy between the independent coordinates of the system and had no role in the dissipation of the energy. If we consider now a motion mode on the form:

$$\{q\}=\{A\}\sin(\omega t+\varphi) \qquad (5)$$

and we introduce in the motion equations, where the forces are considered null, we obtain:

$$-\omega^2[m]\{A\}sin(\omega t+\varphi)+\omega[c]\{A\}cos(\omega t+\varphi)+ \\ +([k]+[k_{\varepsilon}]+\lfloor k_{\omega^2} \rfloor)\{A\}sin(\omega t+\varphi)=\{0\}. \qquad (6)$$

If we pre-multiply with $\{A\}^T$ we obtain:

$$-\omega^2\{A\}^T[m]\{A\}sin(\omega t+\varphi)+ \\ +\left(\{A\}^T[k]\{A\}+\{A\}^T[k_{\omega^2}]\{A\}\right)sin(\omega t+\varphi)=\{0\}. \qquad (7)$$

We have considered that $\omega\{A\}^T[c]\{A\}=0$ and $\{A\}^T[k_{\varepsilon}]\{A\}=0$ because [c] and $[k_{\varepsilon}]$ are skew-symmetric. It results:

$$\omega^2=\frac{\{A\}^T[k]\{A\}+\{A\}^T[k_{\omega^2}]\{A\}}{\{A\}^T[m]\{A\}}. \qquad (8)$$

This relation can not express, in a direct way, the influence of the matrix [c] in the eigen-values calculus, but this influence is present by the eigenvectors $\{A\}$. The term $[c]$ has an influence on the values of the eigen-values. Some of the eigen-values increase and the other decrease. These Coriolis effects make possible the transfer of the damping between different degree of freedom.

3 Mathematical model

The model of fig.1 express the situation presented previously. We consider an elastic beam with a planar motion. To study the transmission of the damping we consider a controller with the stiffness k_r and damping is c_r. The system presents a number of degree of freedom determined by the type of finite element description of the system. The study of a system with a large number of degree of freedom is a difficult problem. In the following we make a simplification of the model in order to determine the qualitative effects of the controller. We consider the system presented in fig.2.

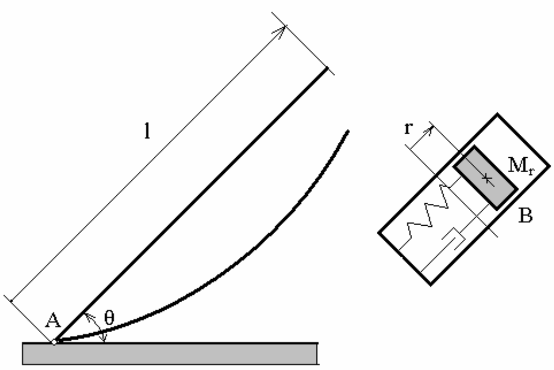

Fig. 1

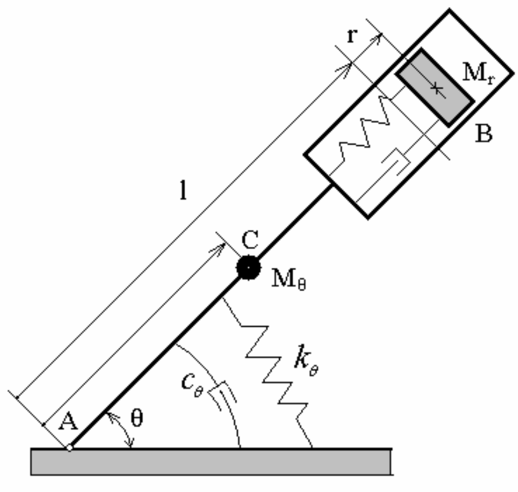

Fig. 2

With the initial conditions and, if we consider k_θ and c_θ equal to zero:

$$\theta_0 = 0; \quad \dot{\theta}_0 = \omega_0; \quad r_0 = 0; \quad \dot{r}_0 = 0, \tag{9}$$

the initial constant momentum of momentum is:

$$K_0 = \left(\frac{M_\theta}{4} + M_r \right) l^2 \omega_0 = J_0 \omega_0 ; \tag{10}$$

$$J_0 = \left(\frac{M_\theta}{4} + M_r \right) l^2 . \tag{11}$$

The kinetic energy is:

$$E_0 = \frac{1}{2} \left(\frac{M_\theta}{4} + M_r \right) l^2 \omega_0^2 = \frac{1}{2} J_0 \omega_0^2 \tag{12}$$

$$\boxed{K_0 = J_0 \omega_0 ; \quad E_0 = \frac{1}{2} J_0 \omega_0^2} \tag{13}$$

The value of the absolute velocity is:

$$v_{abs}^2 = v_{rel}^2 + \left(l\dot{\theta} \right)^2 = \dot{r}^2 + l^2 \dot{\theta}^2 \tag{14}$$

The expression of the moment is:

$$K = K_0 = \left[M_\theta \frac{l^2}{4} + M_r (l+r)^2 \right] \dot{\theta} = J_{red} \dot{\theta} \tag{15}$$

and the kinetic energy:

$$\begin{aligned} E = E_0 &= \frac{1}{2} M_r v_{abs}^2 + \frac{1}{2} M_\theta \frac{l^2 \dot{\theta}^2}{4} + \frac{kr^2}{2} = \\ &= \frac{1}{2} M_r \left[\dot{r}^2 + (l+r)^2 \dot{\theta}^2 \right] + \frac{1}{2} M_\theta \frac{l^2 \dot{\theta}^2}{4} + \frac{kr^2}{2} \end{aligned} \tag{16}$$

$$K = K_0 = J_{red} \dot{\theta} ;$$

$$E = E_0 = \frac{1}{2} M_r \dot{r}^2 + \frac{1}{2} J_{red} \dot{\theta}^2 + \frac{kr^2}{2} \tag{17}$$

From:

$$\dot{\theta} = \frac{K_0}{J_{red}} \tag{18}$$

it result:

$$E_0 = \frac{M_r \dot{r}^2}{2} + \frac{K_0^2}{2J_{red}} + \frac{k_r r^2}{2}, \tag{19}$$

and:

$$\dot{r} = \pm \sqrt{\frac{2}{M_r} \left(E_0 - \frac{K_0^2}{2J_{red}} - \frac{k_r r^2}{2} \right)}, \tag{20}$$

where:

$$E_0 - \frac{K_0^2}{2J_0} = 0 . \tag{21}$$

It result the first order degree differential system:

$$\begin{cases} \dot{r} = \pm \sqrt{\dfrac{2}{M_r} \left(E_0 - \dfrac{K_0^2}{2J_{red}} - \dfrac{k_r r^2}{2} \right)} \\ \dot{\theta} = \dfrac{K_0}{J_{red}} \end{cases} \tag{22}$$

The integration must take into account the change of sign in the expression of velocity of the controller. This representation offers the possibility to study the influence of the damping on the general behavior of the system.

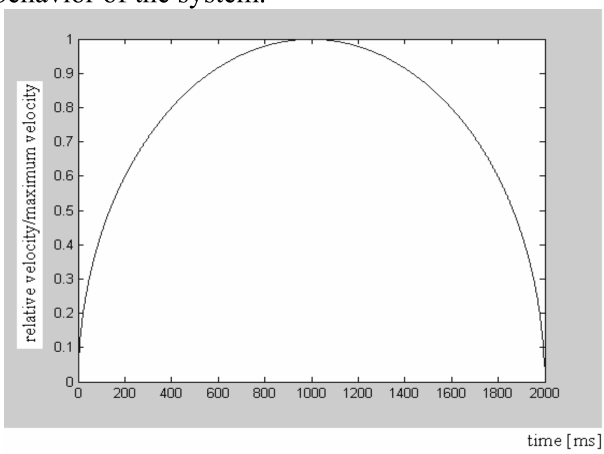

Fig.3. The representation of $\dot{r}$

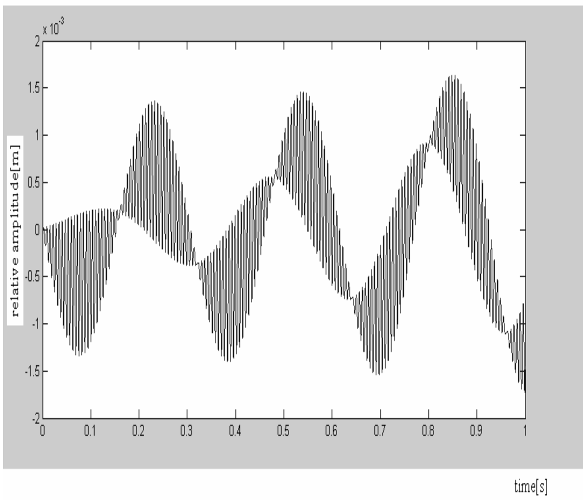

Fig. 4

Fig.4. represent the relative amplitude, transmitted from the controller to the bar, due to the elasticity of the controller.

4 Conclusions

The presented method offers a way to study the influence of a controller in a mechanical system with elastic elements. The difficulty consists in the great number of the degree of freedom of the system, but a numerical study can offer solutions to the problem.

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