

# Time-Frequency Analysis in Structures Monitoring Subject to Extreme Dynamic Loads

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*Abstract:* The structures monitoring subject to actions produced by extreme dynamic loads (earthquakes, strong wind, tornados, sea waves etc.) need to account for temporal evolution of their frequency content. Separate time analysis and frequency analysis by themselves do not fully describe the nature of these dynamic loads. As consequence, it is required to develop and use some non-stationary spectral analysis techniques. Significant efforts in order to permit the temporal evolution of non-stationary spectral characteristics representation have been done in the last years. The objective of this paper is to present some techniques currently available with possible application in earthquake engineering. Finally, some experimental results obtained in the analysis of the earthquake records during the Vrancea seism, August 1986, Romania, are presented.

*Key-Words:* Time-frequency analysis, short-time Fourier transform, spectrogram, Wigner-Ville distribution, Choi-Williams distribution, reduced interference distribution, earthquake signals.

## 1 Introduction

The spectral analysis using the Fourier's integral transform represented one of the most important and used tool in the study of vibration phenomena in the dynamics of the structures and in seismic engineering. The results have underlined the limits of this technique, especially in the case of non-stationary signals analysis and in the study of the nonlinear systems behavior. Moreover, the techniques used to generate artificial excitation signals for Monte-Carlo simulations, have to take into account the non-stationary frequency characteristics of these signals. Also, the capture of the evolutive characteristics of the response for linear and nonlinear systems, for some non-stationary actions, is not possible in the case of classical stationary spectral analysis. As consequence, it is required to develop and use some non-stationary spectral analysis techniques; in this sense have been done significant efforts in order to permit the temporal evolution of non-stationary spectral characteristics representation.

The object of this paper consists in vibration time-frequency analysis with application to monitoring of the structures subject to some extreme dynamic loads using dynamic measurements. This approach will create the necessary ambient for the analysis of the structures subject to extreme dynamic loads in order to raise security ensuring in their life time.

## 2 Time-Frequency Analysis

The basic objective of Time-Frequency Analysis (TFA) is to develop a function able to describe how the energy density of a signal is distributed simultaneously at time,  $t$ , and frequency,  $\omega$ . It consists in a set of transforms that maps a one-dimensional time domain signal into a two-dimensional representation of energy versus time and frequency. There are a number of different transforms available for TFA, to compute Time-Frequency Distributions (TFD), [1]. Each transform type shows a different time-frequency representation. The Short-Time Fourier Transform (STFT) is the simplest TFA transform and the easiest to compute. This suffers from an inherent coupling between time resolution and frequency resolution (increasing the first decreases the second, and vice versa). This coupling can skew the measurements that can be derived from the transform, such as average instantaneous frequency. Other TFA methods and transforms can yield a more precise estimate of the energy in a given frequency-time domain. The Short-Time Fourier Transform (STFT) was used to generate the Spectrogram (SP) and the Wigner-Ville distribution (WVD).

In order to achieve fine simultaneous time-frequency resolution in a non-stationary signal, the uncertainty principle has been introduced, [2]. It imposes restricts to obtain an arbitrarily fine resolution

simultaneously in both time and frequency domain:  $\Delta t \Delta \omega \geq \frac{1}{4\pi}$ , in which the selection of time resolution,  $\Delta t$ , and frequency,  $\Delta \omega$ , are not arbitrary parameters. A trade-off between them should be considered in order to reach a "good" resolution. An overview and a brief description of the advantages/limitations of the time-frequency distributions introduced above are presented in [3].

## 2.1 General Concepts

Linearity is a desirable property of the TFD; however, quadratic TFD's have been proposed and interpreted as time-frequency energy distributions, or instantaneous "power" spectra. The TFD, known as Cohen's shift invariant class distributions, combines the concepts of the instantaneous power and the spectral energy density. Special cases of this general class are the SP, the WVD, the Choi-Williams distribution (CWD) and the binomial distribution (BD). The last two distributions belong to the so-called reduced interference distribution (RID) and they also belong to the Cohen's class, which is by itself an extension of the WVD, [4]. The WVD has been of special interest since it satisfies a large number of important properties. Every member of Cohen's general class may be interpreted as two-dimensional filtered WVD. The SP lacks the time resolution, even though the frequencies are fairly well defined. The WVD offers a significant improvement in time-frequency resolution, but it suffers from cross-terms interference when applied to multi-component signals such as earthquake data. The CWD overcomes the WVD limitation suppressing in great measure the cross-term interference, but some time-frequency resolution is lost. Also, when different time and/or frequency components are present at the same frequency and/or time synchronisation effects occur producing singularities in the time-frequency distribution. The RID overcomes these problems to a significant extent, even though some small synchronisation and cross-terms may be present. For these reasons and the enhancement in the time-frequency resolution, the RID appears to preferred to be analyze earthquake waves.

## 2.2 General Approach and the Kernel Method

Cohen, [1], generalized what is known as Cohen's class of TFD's. This can be expressed in terms of a product of a kernel and a time "autocorrelation" function. The general requirements of an appropriate time-frequency distribution are non-negativity, realness, time and frequency marginal, instantaneous frequency group delay, time and frequency support, and

time and frequency shift properties. From the Cohen's class TFD definition stated in equations (1) and (2), it should be noted that Cohen's class is a bilinear transformation of the signal, and can be interpreted as the 2D Fourier transform of a weighted version of the ambiguity function. Specifically the TDF  $P(t, \omega)$  of the signal  $x(t)$  is given by

$$P(t, \omega) = \frac{1}{4\pi^2} \int \int A(\theta, \tau) \Phi(\theta, \tau) \exp^{-j\theta t - j\tau \omega} d\theta d\tau \quad (1)$$

where  $A(\theta, \tau)$  is the symmetrical ambiguity function of the signal  $x(t)$  and is given by

$$A(\theta, \tau) = \int x\left(t + \frac{\tau}{2}\right) x^*\left(t - \frac{\tau}{2}\right) \exp^{j\theta t} dt \quad (2)$$

$\Phi(\tau, \theta)$  is the kernel of the time frequency distribution, and  $\theta$  and  $\tau$  are frequency and time dummy variables of integration. The kernel of the SP corresponds to the ambiguity function of the localization window  $h(t) : \int h^*(u - \frac{1}{2}\tau) \exp^{-j\theta u} h(u + \frac{1}{2}\tau) du$ , the WVD has a kernel value of 1, the kernel for the CWD is a Gaussian function with the degree of the cross-terms suppression depending on the value of  $\tau$ . Finally, the kernel for the RID is equivalent to a two-dimensional low pass filter.

## 3 Experimental Results

The earthquake accelerations used in this case study were recorded for three components (horizontal North-South, East-West and vertical) of the ground motion during the Vrancea seism, Romania, August 30, 1986. The data were sampled with a sampling frequency of 0.02 seconds for 40 seconds and were previously corrected for instruments effects to provide absolute values of acceleration for ground motions.

As stated in the above section the time-frequency analysis was performed with four well known and generally used TFD's: the spectrogram (SP), the Wigner-ville distribution (WVD), the Choi-Williams distribution (CWD) and the reduced interference distribution (RID), implemented in [5]. We present and discuss the TDF analysis only for horizontal E-W and vertical components of the seismic motion. In Fig. 1 and in Fig. 2 these components and its frequency domain representation (| FFT |) are shown.

In the frequency domain representation of the horizontal E-W component (Fig. 1) at least three frequency components located at the following values of the normalized frequency: 0.02, 0.03 and 0.05 or 0.5, 0.75 and 1.25 Hz are clearly seen. Of these frequency components, the dominant is located at 0.5

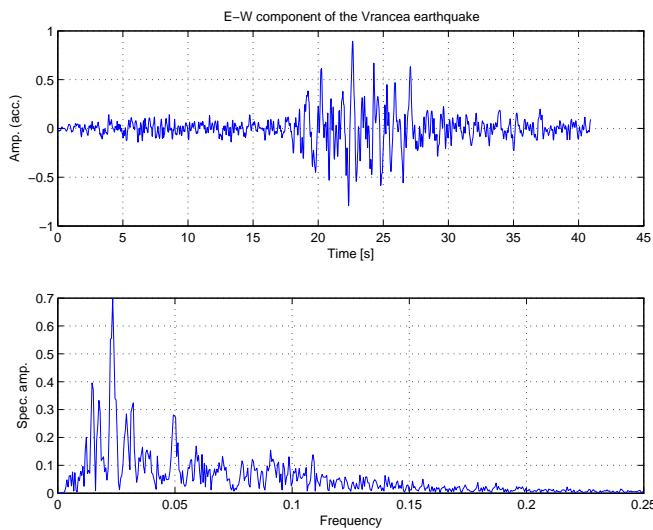


Figure 1: E-W component of Vrancea earthquake and Fourier spectrum

Hz. Beyond 2.5 Hz the spectral amplitudes are small. For the same component, the time domain representation indicates that the maximum amplitudes is located around 22 s. The frequency appears to change from high to low. A low frequency component may be observed almost all along the record. However, in certain small time window intervals a high frequency components is also evident.

For the vertical component (Fig. 2), four frequency components, located at values of normalized frequency: 0.02, 0.05, 0.07 and 0.085, or 0.5, 1.25, 1.75 and 2.2 Hz, are clearly seen. Of these frequency components, the dominant is located at 1.25 Hz. Beyond 3 Hz the spectral amplitudes are small. The maximum amplitude of the same component is located at around 22 s. A high frequency dominates the whole record.

The relevant frequency components to be analyzed in time-frequency domain for E-W horizontal component, as mentioned above, are the normalized frequency values 0.02, 0.03 and 0.05, or 0.5, 0.75 and 1.25 Hz. The Fig. 3 corresponds to the TFD of the spectrogram. In this figure at least three smooth well-defined frequency components can be observed. The strongest amplitude and large duration is located at the lowest frequency 0.5 Hz. In this case we must be careful because of the well-known limitations of the spectrogram: lacking in time resolution, even for a satisfactory frequency resolution.

A significant increase in time and frequency resolution is obtained when WVD is applied (Fig. 4), but this increase in resolution is blurred by cross-term interference, which will put problems in a realistic interpretation of the results.

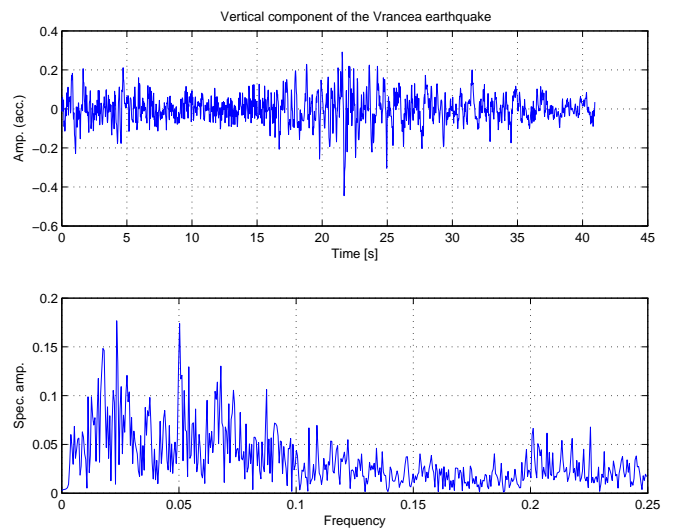


Figure 2: Vertical component of Vrancea earthquake and Fourier spectrum

In the case of CWD application (Fig. 5) it can be noted that many of the cross-term have been removed, but some of the details gained in WVD are lost due to the trade-off between suppression of the cross-terms and the auto-components terms.

Fig. 6 shows the TFD obtained with the RID. In this case the cross-terms are smaller than observed in WVD and CWD, respectively. This improvement facilitates the identification/interpretation of the frequency components of the seismic motion. In this case is possible to interpret some seismic signals in terms of frequency dispersive characteristics or to observe that the times when the maximum amplitudes occur in time domain of the signal correspond to the times where several time frequency characteristics of the signal "converge".

The results for the TFD analysis of the vertical component of the same seismic motion are presented in Fig. 7, Fig. 8, Fig. 9 and Fig. 10, for the SP, WVD, CWD and RID, respectively. The normalized frequencies of interest were 0.02, 0.05, 0.07 and 0.085, or 0.5, 1.25, 1.75 and 2.2 Hz.

It can be noted that the four frequency components are of larger time duration, compared with the ones observed in horizontal E-W component and are located between 0.5 and 2.2 Hz. As in the case of horizontal E-W component a similar pattern is observed regarding the increase/reduction of the time frequency resolution, and the suppression of cross-term interference.

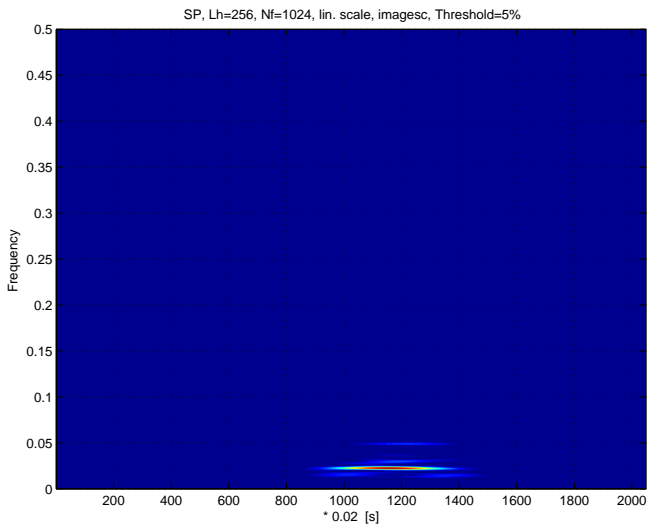


Figure 3: Spectrogram for E-W component of Vrancea earthquake

## 4 Conclusions

Advanced time-frequency analysis techniques are used to observe the shifting of the natural frequencies of nonlinear structures and the changes on modal damping. It is clear that that these techniques can be useful in health monitoring and structural control of structures subject to non-stationary signals, such as earthquake, and other extreme loads (blast, wind, ocean waves, etc.) based on these evolutionary power spectra.

The experimental results in seismic waves analysis based on TFD analysis, presented here, could provide interesting features from the seismologist interpretation point of view. In the case of SP time-frequency distribution, the global characteristics about the frequency distribution are quite clearly evident, but details about time duration are limited, as well as any interference related with energy concentration. In the case of WVD application, energy concentrations are possible to be observed, due to the significant increase of the time-frequency resolution, but in the same time a large amount of cross-term interference masks the true signal of the seismic motion. While using the CWD, the cross-term interference is reduced, making possible to identify more clearly the spots where the concentration of energy occurs. The RID time-frequency distribution is the one that appears to offer the best trade-off between the signal time-frequency distribution and suppression of numerical artifacts (cross-term interference and synchronization effects) of the TFD's algorithms.

The presented results are only some preliminary results. The physical interpretation of TFD's and char-

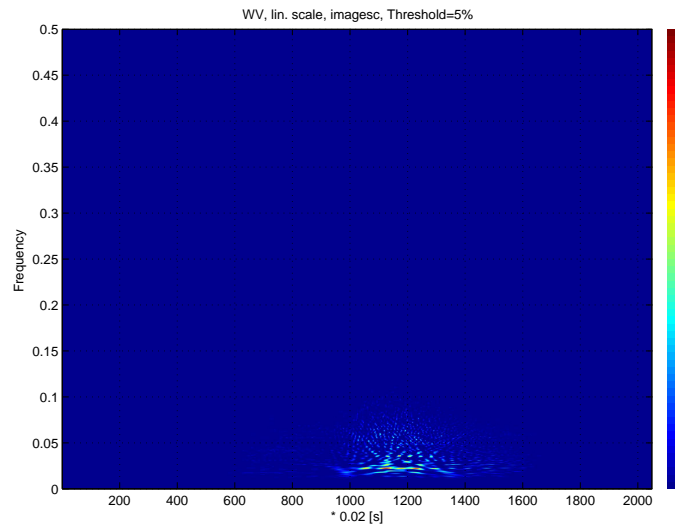


Figure 4: Wigner-Ville distribution for E-W component of Vrancea earthquake

acteristics of seismic waves in time-frequency domain need further studies, to provide new physical insight into propagation phenomena and soil properties for these waves.

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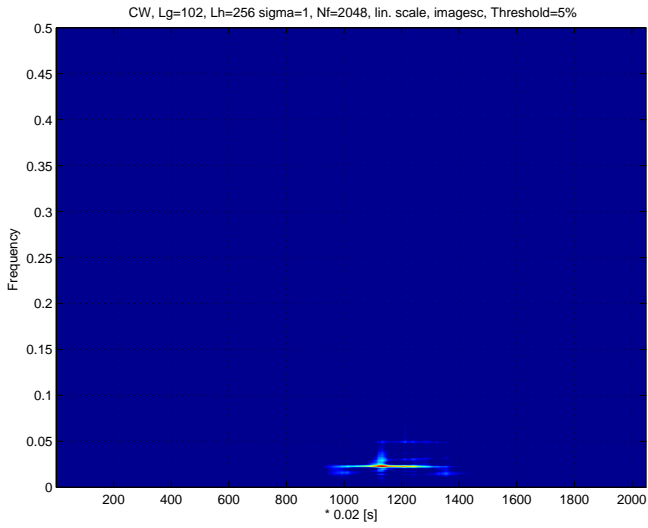


Figure 5: Choi-Williams distribution for E-W component of Vrancea earthquake

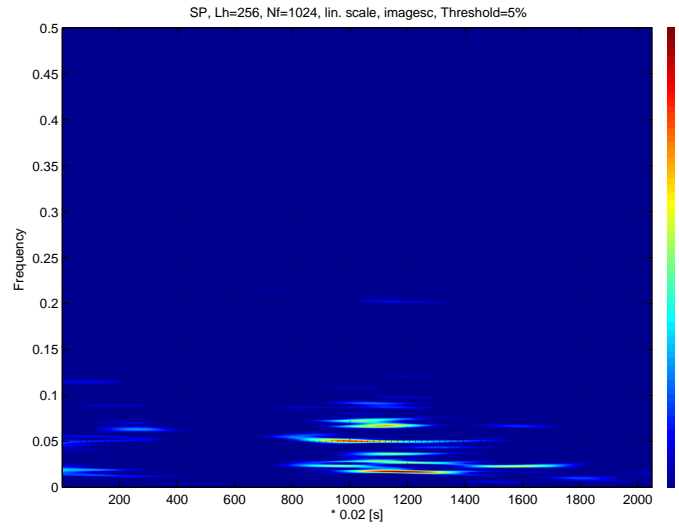


Figure 7: Spectrogram for vertical component of Vrancea earthquake

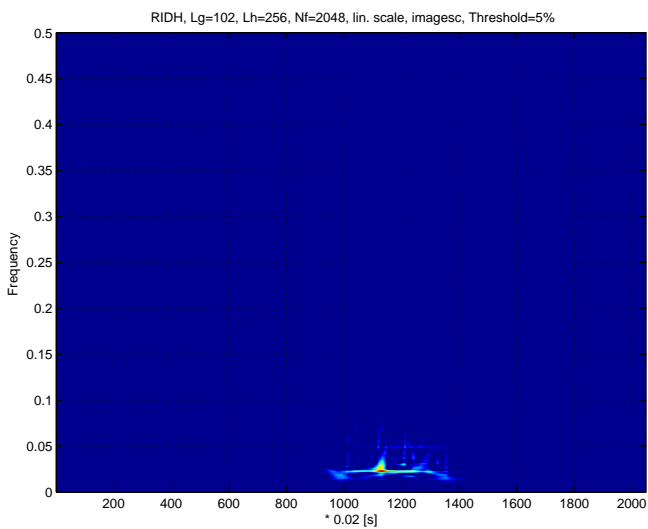


Figure 6: Reduced interference distribution E-W component of Vrancea earthquake

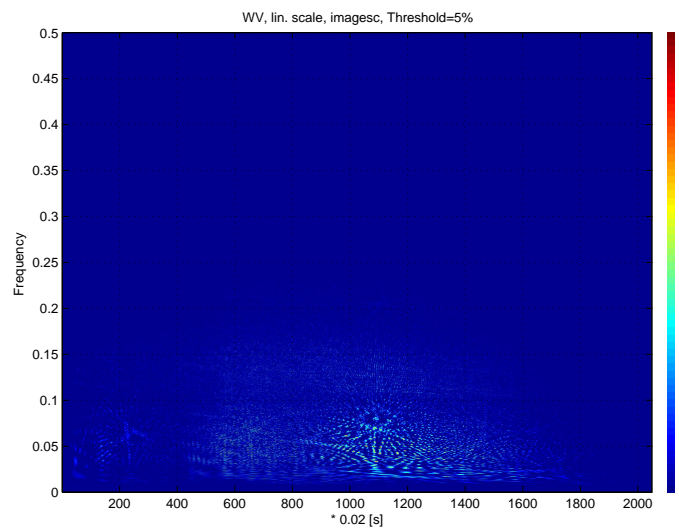


Figure 8: Wigner-Ville distribution for vertical component of Vrancea earthquake

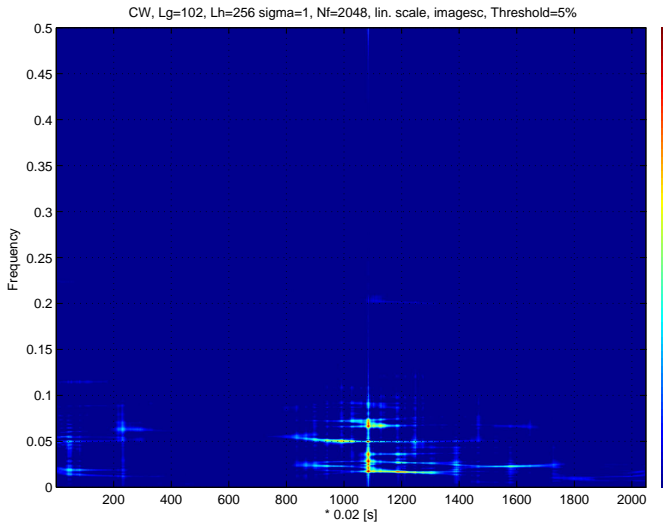


Figure 9: Choi-Williams distribution for vertical component of Vrancea earthquake

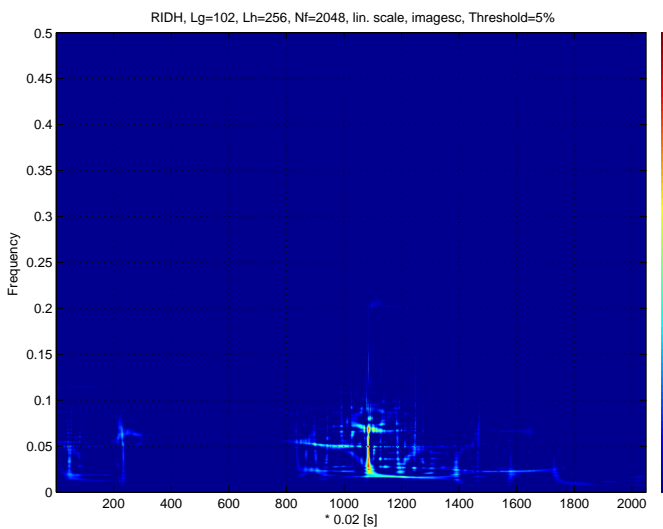


Figure 10: Reduced interference distribution vertical component of Vrancea earthquake