## Experimental Study Of Distributed Control For Heat Transfer Processes

VINATORU Matei, VINATORU Corneliu, IANCU Eugen Automation and Mechatronics Department University of Craiova 13, A.I. Cuza str., DOLJ , RO 200678 ROMANIA http://www.automation.ucv.ro

*Abstract:* - Using a given mathematical model, a distributed parameters state observer has been implemented experimentally to reconstruct transient temperature profiles of the aluminium slab during heating and cooling. The observer was used to implement a state feedback controller. The studies had in view the time response of the controller, the control error and the form of distributed control. It is also presented an algorithm for model reference adaptive control for distributed parameters system with a command spatially distributed. The algorithm and control structure are verified on an experimental installation that permits the study of the heat transfer processes, realising a certain temperature profile along an aluminium slab. This is a sample of the format of your full paper.

Key-Words: - Distributed parameters control systems, adaptive control, Applications: Process Control

### **1** Introduction

Most of heat transfer processes in industry may be described as distributed parameter systems since the heat transfer varies not only in time but also along a spatial coordinate. Most of control systems for heat transfer processes today are approximating this type of processes as lumped parameter systems, in order to be able to apply well established control laws and algorithms, though a distributed control system may be better suitable for the control.

Some applications of distributed parameters control theory have been given by Ray (1977, 1981), Lausterer and Ray (1979) and after this a lot of reported applications were developed, but most of them were purely theoretical since the computational power involved exceeded the computer power of those times. The evolution of numerical control methods and computer equipment allows today the implementation of distributed control algorithms for distributed parameters systems.

There are a lot of heat transfer applications in industry that may benefit from the use of distributed control, such as rubber curing processes or steel milling processes, where the heat transfer process takes place along a spatial coordinate. In these applications where the command is a spatially distributed function, there are necessary adequate control structures, which can assure a rigorous control of process variables both as time and space variations. In this paper are presented some algorithms for distributed control for a distributed parameter system.

The first algorithm is using a distributed parameter observer, presented by Vinatoru (2001) to implement a feedback controller using estimated state of the process. The second algorithm is using a reference model adaptive control.



Fig. 1. Experimental installation

# 2 Mathematical model of the experimental installation.

In order to verify the proposed algorithms, an experimental installation was built, based on the one used by Mader 1975) (fig. 1). This installation may be used to study the behaviour of distributed, one-dimensional heat transfer processes.

The main process consists of an aluminium slab, heated by a series of lamps placed along the slab. The lamps can be independently controlled (trough continuous voltage or with a pulse series). Temperature sensors are placed in the slab at different points. The mathematical model, given by (1), represents the dynamic equation of the conductive heat transfer:

$$\frac{\partial x(z,t)}{\partial t} = \alpha \frac{\partial^2 x(z,t)}{\partial z^2} - \beta x(z,t) + \gamma u(z',t), t > 0, 0 < z < 1$$
(1)

with boundary conditions:

$$\frac{\partial x}{\partial z} = 0 \quad pt \ z = 0 \quad si \ z = l \tag{2}$$

where u(z,t) is a function of the power input to the heating lamps.

The mathematical model (1) was verified trough simulation and it was compared with the real temperature profile of the experimental installation.

#### **3** Observer based control algorithm.

Although the installation actually has M=11 temperature sensors that can provide an accurate temperature profile x(z,t), since the temperature sensors in industrial process tend to be rather expensive, the goal is to minimize the number of measurement locations. Therefore, state observer was developed by Vinatoru (2001) for the real-time reconstruction of the slab temperature profile using a limited number of temperature measurements, usually three.

An asymptotic state estimator was designed, whose state  $\hat{x}(z,t)$  approaches the real state x(z,t) of the

state x(2, t) approaches the real state x(2, t) of the system (1) with an arbitrary convergence rate. As described in Kohne (1976) the observer equation can be written as:

$$\frac{\hat{\partial x}(z,t)}{\partial t} = \alpha \frac{\partial^2 \hat{x}(z,t)}{\partial z^2} - \beta \hat{x}(z,t) + \gamma u(z,t) + g^T(z)Q(t) \left\{ y(t) - \hat{y}(t) \right\}$$
$$z \in (0,1)$$
(3)

$$\frac{\partial \hat{x}(z,t)}{\partial t} = 0 \text{ for } z=0 \text{ and } =1; \hat{x}(z,0) = \hat{x}_0(z)$$
(4)

The elements  $g_k(z)$  of the vector g(z) are weighting functions. The matrix Q(t) is introduced for a better tuning of the observer parameters.

Six basis functions  $\phi_j(z)$  were considered for experiments.

Initia

$$\hat{x}(z,t) = \sum_{j=1}^{6} \varphi_j(z) \zeta_j(t) = \sum_{j=1}^{6} \cos(j\pi z) \zeta_j(t)$$
(5)

And the command u(t), using a projection of the input vector on the basis functions, can be generated:

$$u_j(t) = g_{1j} = \int_0^1 u(z,t) \cdot \varphi_j(z) \cdot dz \tag{6}$$



Fig. 2 Feedback control using estimated state

In figure 2 is presented the block diagram of the control structure using the distributed state observer. The controller  $G_R(s)$  consists of six digital controllers, which use the estimated temperature in six points to generate six commands.

A state observer using Dirac weighting functions was used, since it proved to be the most accurate in approximating the real temperature profile (Vinatoru, 2001).

The first set of experiments was performed considering the following conditions:

-the initial state of the process  $x_o(z)$  is a lot different than the initial state of the observer (considered zero)  $\hat{x}_o(z) = [0\ 0\ 0\ 0\ 0\ 0];$ 

-the process output in the measurement points  $y_o(z_i) = x_o(z_i)$  does not coincide with the estimated output  $\hat{y}_o(z_i) = \hat{x}_o(z_i)$ , for the measurement points  $z_i$  ( $z_1=0, z_2=0.5, z_3=1$ );

-The controller is tuned such as the response time of the system is minimized for small changes of setpoint w(z).



Fig.3 System response for set-point w=  $0.5 + \sin(6\pi/2)$ 



Fig. 4. System response for set-point w= $0.5+\sin(6*pi/2*z)$  – tuned observer

# 4 Reference model adaptive distributed control system

In order to improve the system response, we experimented an adaptive control law with reference model. The experimental installation model (1) can be rewritten as:

$$\frac{\partial \mathbf{x}(\mathbf{z}, \mathbf{t})}{\partial \mathbf{t}} = \mathbf{q}_I \frac{\partial^2 \mathbf{x}(\mathbf{z}, \mathbf{t})}{\partial z^2} + \mathbf{q}_2 \mathbf{x}(\mathbf{z}, \mathbf{t}) + \mathbf{u}(\mathbf{z}, \mathbf{t}) \tag{7}$$

$$\frac{\partial x}{\partial t} = 0 \quad pt \quad z = 0 \quad si \quad z = l \tag{8}$$

where  $q=(q_1,q_2)$  is the unknown parameters vector.

We'll consider a reference model given by:

$$\frac{\partial \mathbf{v}(\mathbf{z}, \mathbf{t})}{\partial \mathbf{t}} = \mathbf{a}_0 \frac{\partial^2 \mathbf{v}(\mathbf{z}, \mathbf{t})}{\partial z^2} + \mathbf{b}_0 \mathbf{v}(\mathbf{z}, \mathbf{t}) + \mathbf{w}(\mathbf{z}, \mathbf{t}) \tag{9}$$

$$\frac{\partial \mathbf{v}}{\partial z} = 0 \text{ pt } z = 0 \text{ si } z = 1$$
 (10)

where  $a_0$ ,  $b_0$  >0,  $v_0=v(z,0)\in L_2(0,1)$  and  $t\rightarrow w(\cdot,t)\in L_2(0,T;V^*)$  for each T>0.

We'll consider operator  $A_0 \in \mathcal{L}(V, V^*)$  given by  $A_0=A(q^*)$  where  $q^*=(a_0,b_0)$ . In this way we have rewritten the system (7), (8) in form (1) and the reference model (9), (10) in form (3), (4). So we can apply the control law (6) in the form:

$$u(t) = A(q)x(t) - A_0x(t) + w(t)$$
(11)

The adaptation law given by (11) can finally be written as:

$$\frac{\mathrm{dq}_{I}}{\mathrm{dt}} = -\frac{1}{\omega_{I}} \int_{0}^{I} \frac{\partial^{2} \mathrm{e}^{2}}{\partial \mathrm{z}^{2}} \mathrm{dz}$$
(12)

$$\frac{\mathrm{dq}_2}{\mathrm{dt}} = -\frac{1}{\omega_2} \int_0^l \mathrm{e}^2 \mathrm{d}z \tag{13}$$

### **5** Experimental results

We have implemented the control law (11) trough spatial discretization and adaptive laws (12), (13) trough a quadrature method. The reference model (9) was implemented through spatial discretization and the differential equations obtained this way were integrated using a Runge-Kutta algorithm of 4<sup>th</sup> order. The Runge-Kutta method was preferred due to its higher precision and since the process time constants are long, the computing time was not crucial.

The simulation results are presented next: In fig. 5 is presented the time response for some points along the slab (z=0;0.25;0.5;0.75;1), and for comparison, the response of the reference model. For same reasons, in fig. 6 is represented the command evolution for the same spatial points.

As it can be seen, the response time was reduced considerably compared with the classic PID control using state observer, from 500s to 50s. The stationary error is improved as well. Since the reference model response is faster than the real process response allows the real time control of the distributed parameters system. Figures 5 and 6 show the adaptation process, in which the unknown parameters  $a_0$  – conduction heat transfer coefficient and  $b_0$  - convection heat transfer coefficient are determined starting from arbitrary values.

This control structure proves very suitable for processes with known mathematical models but having unknown parameters or parameters that change in time. The best example of such processes are the heat exchangers, for which the heat transfer coefficients depend of the exchanger materials and the fluids used as thermal agents, and which change in time due to the solid deposits on the pipes and exchanger walls.

## **6** Conclusions

This paper presented the possibilities of distributed control of a heat transfer process as distributed parameters system. A linear distributed parameter observer could be easily implemented on a relatively small process computer. The computing time is very small compared with the principal time constant of the system. The implemented observers provide good estimates with measurements taken from a small number of points and can be used to implement a feedback control law using state estimates instead of real process measurements.

An adaptive law, using a reference model was also developed and was verified on an experimental installation.

The experimental results show that the proposed algorithms can be used in practice, with good results, maintaining the computational resources low.

The success of the experimental study should encourage the implementation of distributed state observers in industrial applications for example in heating systems, rubber curing and steel milling processes and in high speed ground transportation systems and the adaptive control of distributed parameter systems. References:

- [1] Lausterer G.K., W.H. Ray, (1979) Distributedparameter state estimation and optimal feedback control – an experimental study in two space dimensions", *IEEE Trans. on Aut. Contr.AC24*
- [2] Kohne M., (1977) "Zustandsbeobachter fur Systeme mit verteilten Parametern – Theorie und Anwendung", *Fortschritt-Berichte der VDI nr.* 26
- [3] Mader H.F., (1975) "Experimenteller Aufbau einer temperatur - Profilregelung", *Regelungstechnische Praxis17*
- [4] Ray W.H., (1977) "Some recent applications of distributed parameter control theory – a survey", Proc. 2<sup>nd</sup> IFAC Symp. Distributed Parameter Control Theory, Coventry, UK
- [5] Ray W.H., (1981) Advanced Process Control, McGraw-Hill, New York
- [6] Vinatoru M, Iancu E, Vinatoru C., (2001) Experimental Study Of Adaptive Control In Conductive Heat Transfer Systems, *IASTED Conf.*. *Innsbruck*
- [7] Skormin, V.A.; Park, K, Distributed control of hot bar thermode in fine pitch surface mountassembly, *Electronics Manufacturing Technology Symposium*, 1995. apos; Manufacturing Technologies - Present and Futureapos;, *Seventeenth IEEE/CPMT International, Volume, Issue*, 2-4 Oct 1995 Page(s):318 - 323
- [8] Faikin, G. M. Approximate model of a heat exchanger with distributed parameters, *Journal* of Engineering Physics and Thermophysics, Springer New York, Volume 12, Number 4, pag. 260-262, SpringerLink Date Wednesday, December 22, 2004
- [9] Zerihun Desta, T, A. Van Brecht, S. Quanten, S. Van Buggenhout, J. Meyers, M. Baelmansand D. Berckmans, Modelling and control of heat transfer phenomena inside a ventilated air space, *Energy and Buildings*, Volume 37, Issue 7, July 2005, Pages 777-786
- [10] Bleris Leonidas G., Kothare Mayuresh V., Reduced order distributed boundary control of thermal transients in Microsystems, IEEE *Transactions on Control Systems Technology* (*IEEE trans. control syst. technol.*) **ISSN** 1063-6536, 2005, vol. 13, n°6, pp. 853-867



Fig. 6. System response for setpoint w=0,5+0.2\*sin(6\*pi/2\*z)