Simulation of Kink and Antikink Solitons and Their Behaviors

M. MEHDI KARKHANEHCHI and MOHSEN OLIAEE FAZABAD
Department of Electronics, Faculty of Engineering
Razi University
Taghbostan, Kermanshah
IRAN

Abstract:-The dynamic behavior of the Kink and Antikink solitons has been investigated by Sine- Gorden equation in physical systems. This equation has significant role in various physical branches. In optical fiber, Kink and Antikink solitons keep their forms when they propagate in fiber length. Because of indirect loss effect on nonlinear higher order, numerical simulation of those solitons is necessary. Therefore, Sine- Gorden equation has to be solved analytically.

Keywords: solitons, optical fiber, nonlinear effect, Sine-Gorden equation

1 Introduction

Sine–Gorden equation took from Clyn-Gorden equation. This equation is a relativity nonlinear equation in space-time without dimension 1+1. Before this equation, there was only Korteweg de vries (KDV) in nineteen century [1, 4]. Not only one can use Sine-Gorden equation in relativity field phenomena, but also it can be used in solid state physics, nonlinear optics, and so on. Solitonic solutions of Sine-Gorden are better than KDV and modified KDV solutions [1, 5].
Single solitonic solution is two various cases of Kink and Antikink solutions [8]. But a Kink solution is a solution with boundary conditions in left infinity is zero and in right infinity is 2π. Boundary conditions for an Antikink solution in left infinity are zero and in right infinity is -2π. The Sine-Gorden equation is as follow [2, 3]:

$$\phi_{xx} - \phi_{tt} = \sin\phi \tag{1}$$

To solve this equation, first of all it is assumed that:

$$\phi(x,t) = u(x - vt) = u(\xi)$$

Where v is velocity of soliton. But *u*, *v* are two functions which can be determined. We have:

$$\frac{d^2u}{d\xi^2} - v^2\frac{d^2u}{d\xi^2} = (1 - v^2)\frac{d^2u}{d\xi^2} = \sin u \tag{2}$$

With dividing both sides of the equation (2) by $\mathbf{1 - v^2}$ and multiplying both sides by $\frac{\boldsymbol{du}}{\boldsymbol{d\xi}}$, therefore we have:

$$\frac{d^2u}{d\xi^2}\frac{du}{d\xi} = \frac{\sin u}{(1 - v^2)}\frac{du}{d\xi} \tag{3}$$

With multiplying both sides of equation (3) by $\frac{\boldsymbol{du}}{\boldsymbol{d\xi}}$, one can obtain:

$$\frac{du}{d\xi}\left[\frac{1}{2}\left(\frac{du}{d\xi}\right)^2 + \frac{\cos u}{(1 - v^2)}\right] = 0 \tag{4}$$

One can define A as follow:

$$\frac{1}{2}\left(\frac{du}{d\xi}\right)^2 + \frac{\cos u}{(1 - v^2)} = A \tag{5}$$

First of all, the ordinary differential equation for u is obtained as follow:

$$\frac{du}{d\xi} = \left(2A - \frac{2\cos u}{(1-v^2)}\right)^{1/2} \qquad (6)$$

With assumption $B = \dfrac{A}{(1-v^2)}$ one can write the variable in (6) as follow:

$$\int_{u_0}^{u} \frac{du'}{\sqrt{B-\cos u'}} = \left(\frac{2}{(1-v^2)}\right)^{1/2} \int_{\xi_0}^{\xi} d\xi' \qquad (7)$$

The integral in above equation is a variable function of v and B, where v is soliton velocity, and B is integral constant. With selection of B=1, a single wave with velocity $\mathbf{0} < |\mathbf{v}| < \mathbf{1}$ is obtained. In this case, the left side of equation (7) with assumption $1-\cos u = 2\sin^2\left(\frac{1}{2}u\right)$ may be suggested as follow:

$$\frac{d}{du}\ln\tan\left(\frac{1}{4}u\right) = \frac{1}{2\sin\left(\frac{1}{2}u\right)} \qquad (8)$$

The both sides of equation (7) can be written as follow:

$$u(\xi) = 4\tan^{-1}\left\{\tan^{-1}\left(\frac{1}{4}u\right)\exp\left[\frac{\xi-\xi}{\sqrt{1-v^2}}\right]\right\} \qquad (9)$$

With solving equation for u:

$$\phi(x,t) = 4\tan^{-1}\left\{\pm\exp\left[\frac{x-vt}{\sqrt{1-v^2}}\right]\right\} \qquad (10)$$

The kink and antikink solitons can be determined by this solution [2].

In equation (10) plus sign is for kink soliton and minus sign is for antikink soliton.

The other solution which can be obtained from equation (7) as follow:

$$\phi(x,t) = 4\cot^{-1}\left\{\mp\exp\left[\frac{x-vt}{\sqrt{1-v^2}}\right]\right\} \qquad (11)$$

Again the plus sign is for antikink soliton and the minus sign is for kink soliton.

Here, we investigate the solution of single soliton.

2 Simulation of solitons

a: Kink soliton:

In computer program, the kink velocity v=0.1 is considered and transmission parameter a is defined as follow: [5, 6]

$$a = \sqrt{\frac{1-v_k}{1+v_k}} \qquad (12)$$

The simulation result for a single kink soliton is shown in Fig. (1):

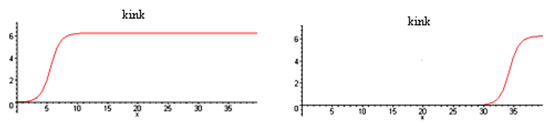

Fig. 1: Two dimensional plot of Sine-Gorden equation for Kink single soliton with v=0.1

b: Antikink soliton

If the transmission parameter in kink soliton is negative, that is:

$$a = -\sqrt{\frac{1-v_k}{1+v_k}} \qquad (13)$$

We have the antikink soliton with the same velocity. The simulation result is shown in Fig. (2). For antikink soliton.

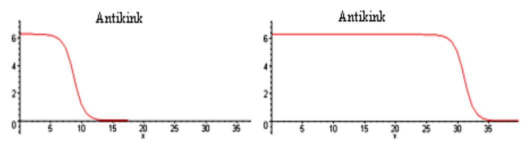

Fig. 2: Two dimensional plot of Sine-Gorden equation for Antikink single soliton with v=0.1

The two solitonic solution is divided in some cases [7]. The interaction of kink with kink soliton, the interaction of kink with antikink solitons, the interaction of kink with antikink soliton and so on which can be shown here.

(a) Interaction of kink with kink soliton.

Here the interaction of two kink soliton with v=0.2 is simulated. The interaction of two single kink-kink soliton which have the same velocity and amplitude, is shown in Fig. 3. It is observed that each soliton keep its shape after the collision.

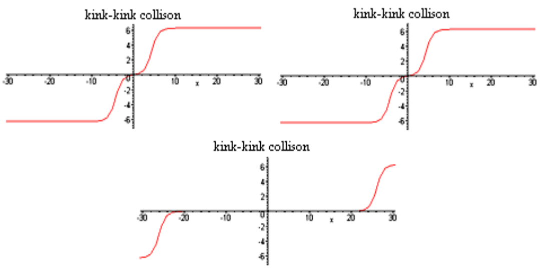

Fig. 3: Interaction of two Kink-Kink solitons with v=0.2

(b) Interaction of kink–antikink soliton:

Here the interaction of kink and antikink soliton with v=0.2 is simulated. Which is shown in Fig.(4) in two dimensions and in Fig.(5) in three dimensions, the solution for Sine-Golden equation for the kink and antikink solitons and the interaction behavior are observed. In Fig. 4, one can see that after the interaction, each of them are continued to move without changing in their shapes. Also, for better understand the solitonic interaction, the three dimensional plot is shown in Fig. 5.

3 Conclusion

In this paper, the propagation of kink and antikink solitons and the interaction between them are investigated.
During the interaction of solitons, the solution can not consider as the linear combination but after the solitons interaction, the solitons again obtain the shapes, and only, the interaction is a phase transmission. The animation and three dimensional plots can help to better understanding of solitonic interactions.

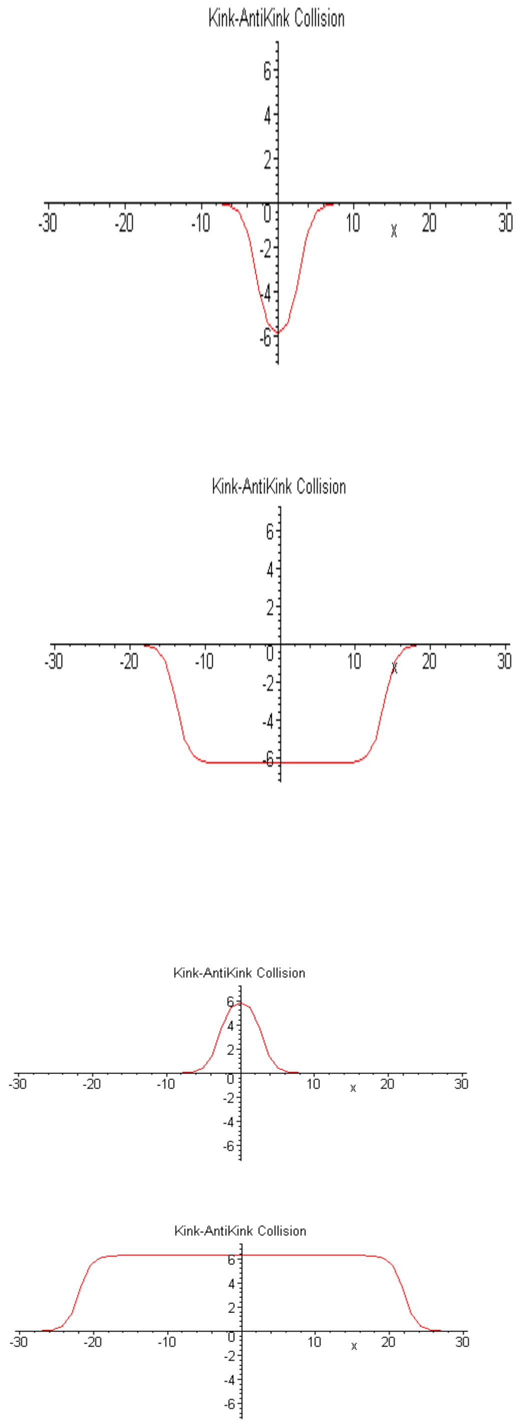

Fig. 4: Collision of two solitons Kink-Antikink with together with velocity v=0.2.

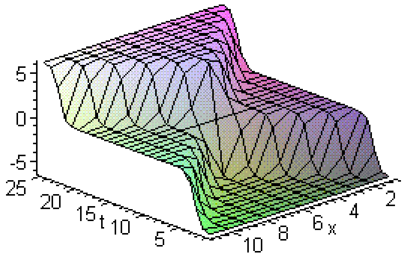

Fig. 5: Three dimensional plot of collision of Kink-Antikink solitons with velocity v=0.2.

References:

1. G. P. Agrawal, "Nonlinear Optical Fiber optics", Second Ed. Academic Press, San Diago,1995.
2. M. Toda, "Nonlinear waves and solitons, 1989.
3. S. R. Chen& N. N. Huang, J. Math. Phys, 2001
4. N. N. Huang, S. Chi, and X. J. Chen, J. Phys. 1999.
5. R. K. Dodd, J. C. Eilbeck, J. D. Gibbon, H. C. Morries, "Solitons and nonlinear wave equations", Academic press, London, 1982.
6. I. Morita, K. Tanaka, N. edagawa, and m. Suzuki, :40 Gb/s single channel soliton transmission over transoceanic distances by reducing Gordon-Haus timing jitter and soliton-soliton interaction", J. Lightwave Techno., 17, 1999.
7. A. N. Pinto, G. P. Agrawal, and J. F. Rocha, "Effect of soliton interaction on timing jitter in communication systems", J. of Lightwave Technol., Vol. 16, No. 4, 1998.
8. D. Marcuse, "An alternative derivation of the Gorden-Haus effect", J. Lightwave, Technol. Vol. 10, No. 2, 1992.