Resonance at Descent in the Mars's Atmosphere of Analogue of the Beagle 2 Lander

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Abstract: - The uncontrolled motion of analogue the Beagle 2 Lander is considered in the rarefied Mars's atmosphere. Such reentry vehicle has small lengthening and the blunted form for effective braking. Such Lander can have three balancing positions of a spatial angle of attack: $\alpha^*=0, \alpha^*\neq 0, \alpha^*=\pi$ depending on position of the center mass. It can result in a resonance at change of a dynamic pressure at descent in an atmosphere. It is shown, that numerical integration of motion equations do not allow receiving authentic results because of probabilistic character of transients connected to a resonance. Conditions of stability of movement are received for various areas of movement at a resonance. Calculation of the top and bottom estimations of parameters of movement is offered to spend with use of the averaged equations. Researches are shown, that the resonance could be cause of accident the Beagle 2 Lander.

Key-Words: - Uncontrolled Reentry Vehicle, Resonance, Stability, Separatrice, Atmosphere, Mars

1 Introduction

One of the principal causes, resulting in abnormal behavior Reentry vehicle in an atmosphere, it is considered a parametrical resonance [1, 2] which arises at presence of small asymmetry when movement concerning the center of mass depends on two angular variables: a spatial angle of attack and a angle of own rotation. If frequency of fluctuation of the angle of attack and average angular speed of own rotation under action of indignations become multiple to the relation of simple integers then the resonance arises. The resonance as the phenomenon of big change of amplitude fluctuations can to arise when small asymmetry does not have also movement depends on one angular variable: a spatial angle of attack, if coefficient of the aerodynamic static moment $m_\alpha(\alpha)$ addresses in a zero in three points on a interval $[0,\pi]$. In this case on a phase portrait $\dot{\alpha}=\dot{\alpha}(\alpha)$ three areas divided separatrices [3] can to take place. The dynamic pressure changes with height of flight and the phase portrait any more does not answer conservative system. In connection by it the evolution of phase trajectories takes place. As a result, these trajectories can to cross separatrices and fall into various phase portrait areas, which is followed by qualitative changes in the motion character. This is a resonance.

2 Problem Formulation

Uncontrolled reentry vehicle has small lengthening of the blunted form, which provides effective braking for descent in a rarefied atmosphere of Mars. This paper considers spatial motion around a re-entry's center of mass with the angle of attack dependence of the coefficient static moment having form of a biharmonical series

$$m_\alpha(\alpha)=a\sin\alpha+b\sin 2\alpha .$$

Such the angle of attack dependence of the coefficient static moment is typical for uncontrolled reentry vehicles of segmentally-conical, blunted conical, and other shapers (Soyuz, Mars, Apollo, Viking, Beagle 2 Lander). The presence of second harmonic in the moment characteristics causes the possibility of appearance of an additional equilibrium position of a reentry vehicle in the angle of attack, i.e., an additional singular point on a phase portrait $\alpha^*\in(0,\pi)$ of the system, which causes the transient mode – resonance. Fig. 1 shows a segmentally - conic body (analogue of the Beagle 2 Lander) and dependences of the coefficient static moment on the spatial angle of attack $m_\alpha(\alpha)$ at various positions of the center of the mass $\overline{x}_T=x_T/L$, counted from nose of a body (L - reference length), received on the shock theory of Newton.

For considered reentry vehicles position $\alpha=0$ is stability. If the condition

$$|2b|>|a| \tag{1}$$

take place, then there is an intermediate position of balance $\alpha^*\in(0,\pi)$.

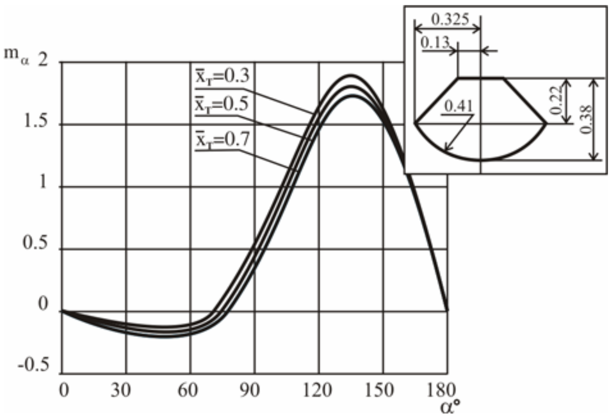

Fig. 1

The purpose of the given report - to show an opportunity of occurrence of resonance, to find conditions of stability of the perturbed motion, to construct procedure of calculation of the top and bottom estimations of parameters of movement with use of the average equations [3].

3 The equations of motion and the phase portrait

The motion of an axial-symmetric body around the center of mass at descent in an atmosphere is described by the system with slowly varying parameters of type [2]

$$\ddot{\alpha} + F(\alpha, z) = -\varepsilon m_z(z)\dot{\alpha}, \qquad (2)$$
$$\dot{z} = \varepsilon \Phi_z(\alpha, z),$$

$$F(\alpha) = \frac{(G - R\cos\alpha)(R - G\cos\alpha)}{\sin^3\alpha} -$$
$$- A\sin\alpha - B\sin 2\alpha,$$

$$A = \frac{aSL}{I}q, \qquad B = \frac{bSL}{I}q,$$

$$z = (R, G, V, \theta, H), \quad M_\alpha = m_\alpha qSL / I$$

where z are the vector of slowly varying parameters; α is the angle of attack, ε is the small parameter; *R, G* are projections of a kinetic moment vector on the longitudinal axis body, on of the velocity vector ; *V* is absolute value of the velocity vector, θ is the angle of the trajectory deviation, *H* is height of flight, q is the dynamic pressure, *S* is the vehicle reference area, M_α is the static moment, *I* is the transverse moment of inertia.

Evolution of motion occurs under action of disturbance at $\varepsilon \neq 0$. The disturbance system (2) is reduced to non-perturbed system with one degree of freedom at $\varepsilon = 0$

$$\ddot{\alpha} + F(\alpha) = 0. \qquad (3)$$

It is possible to find connection between three balancing positions $\alpha^* = 0, \alpha^* \neq 0, \alpha^* = \pi$ and positions of balance of the system (3) on a phase portrait at performance of conditions (1). The energy integral of system (3) has the form of

$$\dot{\alpha}^2 / 2 + W(\alpha) = E, \qquad (4)$$

where

$$W(\alpha) = \int F(\alpha)\, d\alpha = \frac{G^2 + R^2 - 2GR\cos\alpha}{2\sin^2\alpha} +$$
$$+ A\cos\alpha + B\cos^2\alpha$$

is potential energy.

We introduce a new variable $u = \cos\alpha$, and then the energy integral (4) takes the form

$$\frac{\dot{u}^2}{2(1-u^2)} + \frac{G^2 + R^2 - 2GRu}{2(1-u^2)} + Au + Bu^2 = E$$

or

$$\dot{u}^2 - f(u) = 0,$$
$$f(u) = 2(1-u^2)(E - Au - Bu^2) + \qquad (5)$$
$$+ 2GRu - G^2 - R^2.$$

Research of potential energy $W(u)$ is executed in [2]. Function $W(u)$ has no points of excess on an interval $(-1, 1)$ at performance of the condition

$$B \geq -\left[\min_{-1 \leq u \leq 1}\left(0.5 W_g''(u)\right)\right] \equiv B^*,$$

where $W_g''(u) = \dfrac{d^2 W_g(u)}{du^2} = \dfrac{d}{du}\left(\dfrac{G^2 + R^2 - 2GRu}{2(1-u^2)}\right)$.

The phase plane divided the separatrice into three areas: external A_0 and two internal A_1 and A_2, if the condition satisfies

$$W'(u_{*1}) \cdot W'(u_{*2}) < 0,$$

where u_{*1}, u_{*2} are roots of the equation:

$$W''(u) = \frac{d^2 W(u)}{du^2} = 0.$$

If $E > W_*$, where W_* is value $W(u)$ in saddle point $u = u_*$, then motion occurs in external area A_0, as can been from fig. 2. Otherwise ($E < W_*$) motion can to take place in any of internal areas A_1 or A_2 depending on entry conditions. Equality $E = W_*$ satisfies to motion on the separatrice.

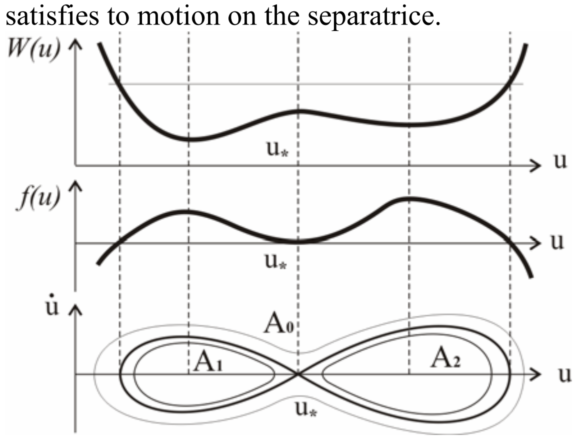

Fig. 2

4 Disturbed motion stability

We investigate the disturbed motion research in areas A_0, A_1, A_2. Movement can begin both in external area A_0, and in any of internal areas A_1 and A_2. If the area in which began movement, is unstable, the phase trajectory will cross through separatrice some limited time by virtue action of disturbances. It is obvious, that at the moment of crossing separatrice two situations take place: two areas are unstable, one is stable and, on the contrary, one is unstable, and two are stable. At action of small disturbances the average of full energy $\bar{E}$ and of potential energy W_* in saddle point $u = u_*$ slowly changes. For definition of stability it is enough to calculate derivatives on time from these functions [2]. The internal area (A_1 or A_2) will be stable, if in neighborhood separatrice the following condition satisfies

$$\dot{\bar{E}}(z) < \dot{W}(u_*, z). \qquad (6)$$

For external area A_0 the condition of stability looks like:

$$\dot{\bar{E}}(z) > \dot{W}(u_*, z). \qquad (7)$$

Function (5) in saddle point $u = u_*$ is equal

$$f_* \equiv f(u_*, z) = 2(1-u_*^2)[\bar{E}(z) - W(u_*, z)]. \qquad (8)$$

In neighborhood separatrice take place

$$\bar{E}(z) - W(u_*, z) = O(\varepsilon), \quad \dot{u}_*(z) = O(\varepsilon)$$

Differentiation of function (8) on time gives the following result to within of the order infinitesimal ε^2

$$\dot{f}_* = 2(1-u_*^2)[\dot{\bar{E}}(z) - \dot{W}(u_*, z)]. \qquad (9)$$

From (9) follows, that conditions (6) and (7) are answered with the following conditions, accordingly (fig. 3, 4)

$$\dot{f}_* < 0, \qquad \dot{f}_* > 0.$$

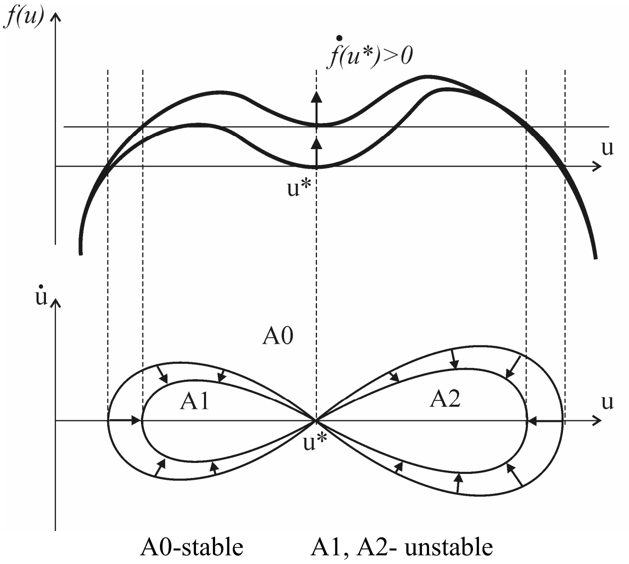

Fig. 3

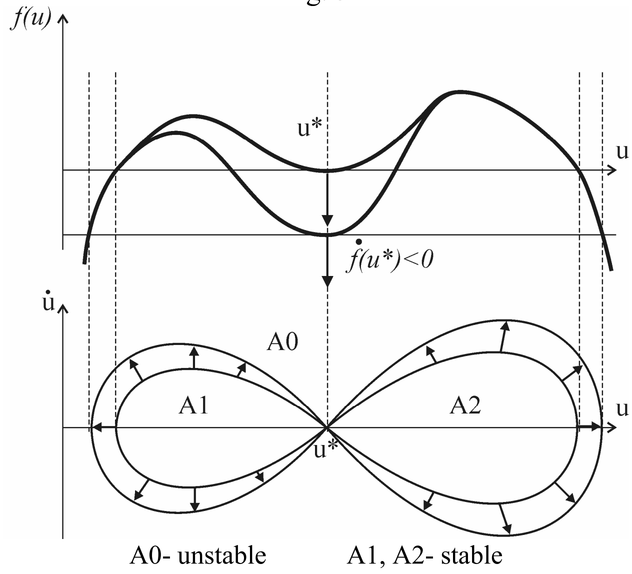

Fig. 4

From integral of energy (4) follows, that at $\dot{\alpha} = 0$

$$\bar{E}(z) = W(\alpha_m, z),$$

where $\alpha = \alpha_m$ is amplitude of attack angle.

For system (2) average equations of motion, are received in [3]. We calculate derivatives $\dot{\bar{E}}(z)$ and $\dot{W}(\alpha_*, z)$ by virtue of the average equations:

$$\dot{\bar{E}}(z) = \left.\frac{\partial W}{\partial \alpha}\right|_{\alpha=\alpha_m} \cdot \dot{\alpha}_m + \left.\frac{\partial W}{\partial z}\right|_{\alpha=\alpha_m} \cdot \dot{z}$$

$$= F(\alpha_m, z) \cdot \dot{\alpha}_m + \left.\frac{\partial W}{\partial z}\right|_{\alpha=\alpha_m} \cdot \dot{z},$$

$$\dot{W}(\alpha_*, z) = \left.\frac{\partial W}{\partial z}\right|_{\alpha=\alpha_*} \cdot \dot{z}.$$

For definition of stability of the disturbed motion in neighborhood separatrice we introduce new criterion

$$\Lambda \equiv F(\alpha_m, z) \cdot \dot{\alpha}_m + \left.\frac{\partial W}{\partial z}\right|_{\alpha_*}^{\alpha_m} \cdot \dot{z}, \qquad (10)$$

Then conditions of stability (6) for internal area (A_1 or A_2) and (7) for external area A_0 will become, accordingly

$$\Lambda < 0, \qquad \Lambda > 0. \qquad (11)$$

On the basis of the carried out analysis it is possible to offer the following procedure of calculation of the top and bottom estimations of motion parameters with use of the average equations [3]. Numerical integration of the average equations is carried out from an initial point belonging to one of areas till the moment of crossing separatrice. Then it is calculated criterion (10) for each of areas A_0, A_1, A_2, and with the help of conditions (11) stability of disturbed motion is defined. The area from which there is an exit on separatrice, always is unstable, therefore there can be or one, or two stable areas. In the first case, numerical integration proceeds in stable area. In the second case, numerical integration for each stable area is carried out, in result is received the top and bottom estimations of the decision.

As an example the uncontrolled motion of analogue the Beagle 2 Lander is considered in the rarefied Mars's atmosphere. On fig. 5 two branches of decisions for angle attack are shown: $A_0 \rightarrow A_1$ and $A_0 \rightarrow A_2$.

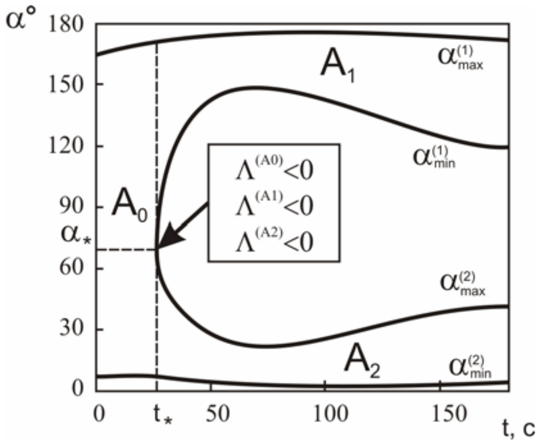

Fig. 5

5 Conclusion

Thus, we have shown, that exist transitive modes (resonance) at which parameters of motion considerably change at descent in the atmosphere of Mars for an axial-symmetric bodies having the biharmonic static moment. Criteria of stability of transitive modes are found and procedure is offered for the analysis of motion uncontrolled reentry vehicles of blunted conical shaper. It is shown, that if not to carry out the similar analysis of stability it is possible to overlook one of branches possible decisions, hence, to receive not genuine result.

Work is executed at support of the Russian Foundation for Basic Research (Project № 06-01-00355-a).

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