

Optimal Design of Multi-loop PI Controllers for Enhanced Disturbance Rejection in Multivariable Processes

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Abstract: - The proposed method presents a new design for multi-loop PI controllers in MIMO systems in two cases: set-point tracking and disturbance rejection. It can identify the tuning parameters of multi-loop PI controllers by starting from the generalized IMC-PID approach [1], which is extended from single input, single output (SISO) systems to multiple input, multiple output (MIMO) systems. Control parameters obtained from the IMC-PID approach have shown good performance for several control systems. However, there is not enough robustness in systems which contain a lot of noise and disturbance. A new design can solve this problem by applying the magnitude of sensitivity (*Ms*) theory. A simulation study is performed for the well-known process model and the response performances compared favorably with some famous tuning methods. The results show that the proposed method is superior to existing techniques for multi-loop processes.

Key-Words: - Multi-loop PI controller, MIMO system, IMC-PID approach, *Ms* Criterion.

1 Introduction

The multi-loop PID/PI controller has been studied for many decades. In the 1980s, the famous tuning method for calculating multi-loop PID controller parameters was the Internal Model Control (IMC) [2], it was published by C.G. Economou and M. Morari. A typical method which related to the multi-loop IMC design method was proposed by M.S. Basualdo and J. L. Marchetti [3], which considers to the interactions between the control loops. The biggest log modulus (BLT) tuning design method [4] was published by W. L. Luyben and it is still popular in process control today. In the 1990s, Loh *et. al* [5] studied the auto-tuning procedure for improving the closed-loop frequency responses in MIMO systems, and Jung *et. al* [6] presented the decentralized lambda tuning (DLT) design method with the same goal of improving stability and robustness. Recently, the generalized IMC-PID approach is designed for the multi-loop PID control systems by Lee *et. al* [1]. This approach is a variation of Lee *et. al* [7] which admitted to SISO systems. Many multi-loop tuning design methods exist for set-point tracking problems today. However, there are few methods available for disturbance rejection despite the fact that disturbance rejection is a more serious problem in industry.

Therefore, we proposed a new design method which proceeds from the generalized IMC-PID approach and *Ms* criterion. The aim of this method is to design a multi-loop PI controller that enhances disturbance rejection as well as set-point tracking.

In the multi-loop IMC control systems, the performance and robustness of the closed-loop system largely depends on the closed-loop time constant (λ). The optimal value for the closed-loop time constant can be obtained by using *Ms* criteria. The proposed method can be compensated the influence of disturbance effectively by compensating the dominant poles in the diagonal element of the process transfer function.

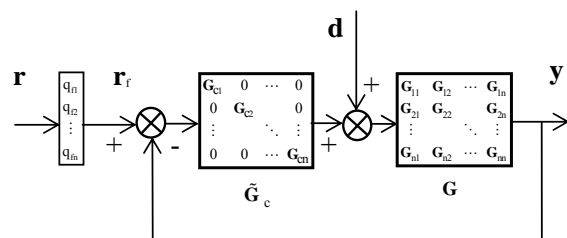


Figure 1. Block diagram for the multi-loop control system

2 The multi-loop PI controller design

In the $n \times n$ multi-loop feedback control system shown in Fig. 1, the closed-loop response to the set-point change is

$$\mathbf{y}(s) = \mathbf{H}(s)\mathbf{r}(s) = (\mathbf{I} + \mathbf{G}(s)\tilde{\mathbf{G}}_c(s))^{-1} \mathbf{G}(s)\tilde{\mathbf{G}}_c(s)\mathbf{r}(s) \quad (1)$$

where $\mathbf{H}(s)$ is the closed-loop transfer function; $\mathbf{G}(s)$ is the process transfer function which is open-loop stable; $\tilde{\mathbf{G}}_c(s)$ is the multi-loop controller with diagonal elements only; $\mathbf{y}(s)$ and $\mathbf{r}(s)$ are the controlled variable and the set-point, respectively.

Suppose that the desired closed-loop response of the diagonal elements in the multi-loop system is given by

$$\tilde{\mathbf{R}}(s) = \text{diag} [R_1, R_2, \dots, R_n] \quad (2)$$

According to the design strategy of the IMC controller [1], the desired closed-loop response R_i of the i th loop is presented by

$$\frac{y_i(s)}{r_i(s)} = R_i(s) = \frac{G_{ii+}(s)(\beta_i s + 1)}{(\lambda_i s + 1)^{n_i}} \quad (3)$$

where G_{ii+} is the non-minimum part of G_{ii} and chosen to be the all pass form; λ_i is an adjustable constant for system performance and robustness; n_i is chosen for the IMC controller to be realizable.

β_i is designed to cancel the dominant poles in the diagonal process element.

$$\left| 1 - \frac{G_{ii+}(s)(\beta_i s + 1)}{(\lambda_i s + 1)^{n_i}} \right|_{s=p_i} = 0 \quad (4)$$

Note that λ_i is analogous to the closed-loop time constant and thus determines the speed of the closed-loop response. The multi-loop controller $\tilde{\mathbf{G}}_c(s)$ with integral term can be expressed in a Maclaurin series as

$$\tilde{\mathbf{G}}_c(s) = \frac{1}{s} [\tilde{\mathbf{G}}_{c0} + \tilde{\mathbf{G}}_{c1}s + \tilde{\mathbf{G}}_{c2}s^2 + \mathcal{O}(s^3)] \quad (5)$$

where $\tilde{\mathbf{G}}_{c0}$, $\tilde{\mathbf{G}}_{c1}$, $\tilde{\mathbf{G}}_{c2}$ can be considered as the integral, proportional, and derivative terms of the multi-loop PID controller, respectively.

As indicated from (5), the impact of proportional and derivative terms (*i.e.*, $\tilde{\mathbf{G}}_{c1}$, $\tilde{\mathbf{G}}_{c2}$) dominates at high frequencies and thus they should be designed based on the process characteristics at high frequencies. On the other hand, the integral term $\tilde{\mathbf{G}}_{c0}$ is dominating at low frequencies and thus needs to be designed based on the characteristics at low frequencies.

In the multi-loop system, the characteristic of the closed-loop interaction is changed according to frequency range. Using this frequency-dependent properties of the closed-loop interactions, analytical design of the multi-loop PID controller can be largely simplified while it still takes the interaction effect fully into account as follows [1]:

At high frequencies, the magnitude of open loop gain becomes $|\mathbf{G}(j\omega)\tilde{\mathbf{G}}_c(j\omega)| \ll 1$ and thus $\mathbf{H}(s)$ can be approximated to

$$\mathbf{H}(s) = (\mathbf{I} + \mathbf{G}(s)\tilde{\mathbf{G}}_c(s))^{-1} \mathbf{G}(s)\tilde{\mathbf{G}}_c(s) \approx \mathbf{G}(s)\tilde{\mathbf{G}}_c(s) \quad (6)$$

It indicates that $\tilde{\mathbf{G}}_{c0}$ and $\tilde{\mathbf{G}}_{c1}$ can be designed by considering only the diagonal elements in $\mathbf{G}(s)$, which means the generalized IMC-PID method for the SISO system [7] can be applied to the design of the proportional and derivative terms in the multi-loop PID controller. Therefore, at high frequencies, the ideal multi-loop feedback controller to give the desired closed-loop response $\tilde{\mathbf{R}}(s)$ is given by

$$\tilde{\mathbf{G}}_c(s) = \tilde{\mathbf{G}}^{-1}(s)\tilde{\mathbf{R}}(s)(\mathbf{I} - \tilde{\mathbf{R}}(s))^{-1} \quad (7)$$

where $\tilde{\mathbf{G}}(s) = \text{diag} [G_{11}, G_{22}, \dots, G_{nn}]$

Accordingly, the ideal multi-loop controller of the i th loop can be designed by

$$G_{ci}(s) = \frac{(G_{ii-}(s))^{-1}(\beta_i s + 1)}{(\lambda_i s + 1)^{n_i} - G_{ii-}(s)(\beta_i s + 1)} \quad (8)$$

where G_{ii-} is the minimum part of G_{ii} .

Since $G_{ii+}(0)=1$, (8) can be rewritten in a Maclaurin series with an integral term as

$$G_{ci}(s) = \frac{1}{s} (f_i(0) + f_i'(0)s + \frac{f_i''(0)}{2}s^2 + \mathcal{O}(s^3)) \quad (9)$$

where $f_i(s) = G_{ci}(s)s$

The standard PID control algorithm is given by

$$G_{ci}(s) = K_{ci} \left(1 + \frac{1}{\tau_{Di}s} + \tau_{Di}s \right) \quad (10)$$

Comparing (9) with (10) gives the analytical tuning rules for the proportional gain of the multi-loop PI controller as follows:

$$K_{ci} = f_i'(0) \quad (11)$$

At low frequencies, according to the design of the integral term $\tilde{\mathbf{G}}_{c0}$, the interaction effect between the control loops can not be neglected. Expansion of $\mathbf{G}(s)$ in a Maclaurin series gives

$$\mathbf{G}(s) = \mathbf{G}_0 + \mathbf{G}_1 s + \mathbf{G}_2 s^2 + \mathcal{O}(s^3) \quad (12)$$

where $\mathbf{G}_0 = \mathbf{G}(0)$; $\mathbf{G}_1 = \mathbf{G}'(0)$; $\mathbf{G}_2 = \mathbf{G}''(0)/2$

By substituting (5) and (12) into (1), one can obtain $\mathbf{H}(s)$ as

$$\mathbf{H}(s) = \mathbf{I} - (\mathbf{G}_0 \tilde{\mathbf{G}}_{c0})^{-1} s + \mathcal{O}(s^2) \quad (13)$$

Furthermore, the desired closed-loop response $\tilde{\mathbf{R}}$ can also be written in Maclaurin series as

$$\tilde{\mathbf{R}}(s) = \tilde{\mathbf{R}}(0) + \tilde{\mathbf{R}}'(0)s + \mathcal{O}(s^2) \quad (14)$$

where $\tilde{\mathbf{R}}(0) = \mathbf{I}$ because $G_{ii+}(0) = 1$

By comparing the diagonal element of $\mathbf{H}(s)$ in (13) and $\tilde{\mathbf{R}}(s)$ in (14) for the first-order s term, one can get the analytical tuning rule for the integral time constant of the multi-loop PID controller as follows

$$\tau_{ii} = - \frac{(G'_{ii+}(0) - n_i \lambda_i + \beta_i) K_{ci}}{(\mathbf{G}^{-1}(0))_{ii}} \quad (15)$$

Tuning formulae by (11) and (15) provide an important advantage to solve the optimization problem for finding the PID parameter values: for a given process, all the PID parameters can be expressed by a single design parameter λ_i and thus the dimension of the search space for optimization is greatly reduced.

The lead term by $(\beta_i s + 1)$ in (3) can cause an excessive overshoot in the set-point response. The two degree of freedom structure can overcome this problem by designing a set-point filter q_i as

$$q_i(s) = \frac{1}{(\beta_i s + 1)} \quad (16)$$

3 M_s criterion for the MIMO systems

M_s tuning is the frequency-domain method which relates to the resonant peak M_s . The M_s values are related to the resonant peak of the sensitivity function. The relative stability and robustness of a stable closed-loop system can be suggested by the magnitude of M_s . In 1996, Skogestad and Postlethwaite [8] employed M_s as a tool for measuring system robustness.

In 1998, Astrom *et. al* [9] proposed that the desirable values of M_s for SISO systems are in the range of 1.2 to 2. M_s tuning provides a limit for the closed-loop time constant for a model, and it allows the optimal controller parameters to be found.

The sensitivity function in the multi-loop control system can be represented by

$$\mathbf{S}(s) = (\mathbf{I} + \mathbf{G}(s)\mathbf{G}_c(s))^{-1} \quad (17)$$

The sensitivity frequency response can be found by setting $s = j\omega$ in term of ω and λ as follows

$$\mathbf{S}(j\omega, \lambda) = [\mathbf{I} + \mathbf{G}(j\omega, \lambda)\mathbf{G}_c(j\omega, \lambda)]^{-1} \quad (18)$$

The sensitivity function can be expressed by the matrix form as

$$\mathbf{S}(j\omega, \lambda) = \{S_{ij}\} = \begin{bmatrix} S_{11} & S_{12} & \cdots & S_{1n} \\ S_{21} & S_{22} & \cdots & S_{2n} \\ \vdots & \vdots & & \vdots \\ S_{n1} & S_{n2} & \cdots & S_{nn} \end{bmatrix} \quad (19)$$

The maximum sensitivity M_s is obtained as the maximum value of the sensitivity function over frequencies

$$M_s = \{M_{S_{ij}}\} = \max_{\lambda, \omega \geq 0} |S_{ij}(j\omega, \lambda)| \quad (20)$$

The peak magnitude of the sensitivity function can be expressed by the matrix form as

$$\mathbf{M}_s = \{M_{S_{ij}}\} = \begin{bmatrix} M_{S_{11}} & M_{S_{12}} & \cdots & M_{S_{1n}} \\ M_{S_{21}} & M_{S_{22}} & \cdots & M_{S_{2n}} \\ \vdots & \vdots & & \vdots \\ M_{S_{n1}} & M_{S_{n2}} & \cdots & M_{S_{nn}} \end{bmatrix} \quad (21)$$

The proposed M_s tuning method is aimed to improve the performance and robustness of closed-loop frequency responses in the multi-loop control system by finding an optimal λ . The multi-loop control system can also be made to meet the stability bounds and all the multi-loop PID parameters can be expressed by a single design parameter λ_i . This optimization problem in the frequency domain is

$$\begin{aligned} & \min_{\lambda, \omega \geq 0} \left(\sum_i M_{S_{ij}} \right) \\ & \text{s.t. } M_{S_{ii}} \geq M_{S_{low}} \end{aligned} \quad (22)$$

where $M_{S_{low}}$ is the lower bound of the diagonal \mathbf{M}_s and it also can be considered as optimizing value to minimize the integral absolute error (IAE). Fig. 2 shows the effects of $M_{S_{low}}$ on the overall performance in the OR column [11]. It implies that at small values of $M_{S_{low}}$, the IAE values are large. However, when $M_{S_{low}}$ increases to high values, the IAE values also increase. Our extensive simulation study shows that the desirable value of $M_{S_{low}}$ lies between 1.8 and 2. This range of $M_{S_{low}}$ can be used for the trade-off

between sluggish, overshoot, oscillation, and minimizing IAE.

According to (22) it is easy to find the optimal value of λ which makes multi-loop control systems stable and robust not only for set-point tracking but also for disturbance rejection.

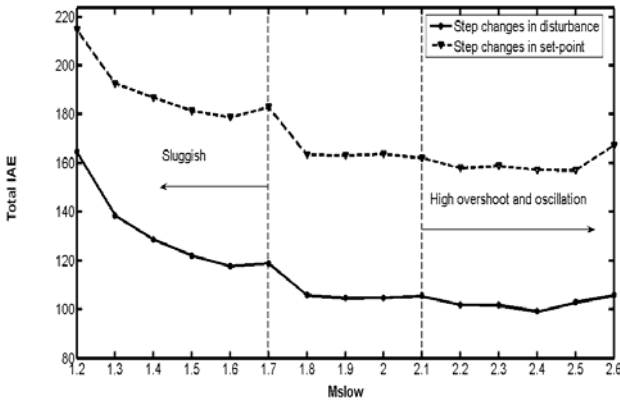


Figure 2. Effects of M_{slow} on the IAE: OR column.

4 Simulation study

In the following case studies, we demonstrate our tuning rules with 3x3 system from the open literature, Ogunnaike and Ray (OR) column, a multi-product plant distillation column for separation of a binary ethanol-water mixture, was modeled experimentally in Ogunnaike *et al.* [11]. The proposed method is also compared with several well-known tuning methods such as BLT and DLT tuning methods.

The transfer function matrix of the OR column is given by

$$G(s) = \begin{bmatrix} \frac{0.66e^{-2.6s}}{6.7s+1} & \frac{-0.6e^{-3.5s}}{8.64s+1} & \frac{-0.0049e^{-s}}{9.06s+1} \\ \frac{1.11e^{-6.5s}}{3.25s+1} & \frac{-2.36e^{-3s}}{5s+1} & \frac{-0.01e^{-12s}}{7.09s+1} \\ \frac{-34.68e^{-9.2s}}{8.15s+1} & \frac{46.2e^{-9.4s}}{10.9s+1} & \frac{0.89(11.61s+1)e^{-s}}{(3.89s+1)(18.8s+1)} \end{bmatrix} \quad (23)$$

By using (22), the optimum λ_i values were found as 10.07, 8.78, and 2.3 for each loop, respectively. Step changes in set-point and disturbance were sequentially made in the individual loops. n_i in (3) was chosen as 2 for all loops according to the process model order. The value of 1.9 was chosen for M_{slow} . All tuning parameters are listed in Table 1. The set-point filters can be found as

$$q_{f1,2,3}(s) = \left\{ \frac{1}{(5.48s+1)}, \frac{1}{(3.43s+1)}, \frac{1}{(3.39s+1)} \right\}$$

TABLE 1
Tuning results by the proposed PI method and various methods: OR column

	Proposed	BLT	DLT
K_c	1.62,	1.51,	0.61,
	-0.32,	-0.29,	-0.14,
	9.43	2.63	0.39
τ_I	9.32,	16.4,	8.00,
	7.27,	4.18,	6.50,
	12.40	6.61	6.85
λ	10.07,		
	8.78,	-	-
	2.3		
Step changes in set-point			
IAE_1	18.15,	32.06,	35.86,
	24.2,	70.22,	55.49,
	81.48	149.85	975.54
IAE_2	3.51,	6.19,	6.99,
	15.76,	45.71,	35.74,
	24.13	43.07	297.52
IAE_3	0.07,	0.12,	0.12,
	0.06,	0.15,	0.10,
	2.32	3.59	27.58
IAE_t	169.68	350.96	1434.96
Step changes in disturbance			
IAE_1	6.06,	10.86,	13.12,
	8.22,	11.9,	17.46,
	34.37	90.96	463.50
IAE_2	2.95,	5.28,	8.12,
	22.22,	60.93,	46.51,
	34.39	94.68	505.40
IAE_3	0.05,	0.01,	0.08,
	0.07,	0.02,	0.08,
	1.32	2.51	22.83
IAE_t	109.54	277.15	1077.10

IAE_i : IAE for the step change in loop i . IAE_t : sum of each IAE_i .

Figures 3 and 4 show the closed-loop responses for sequential step changes in set-point and disturbance, respectively. Sequential step changes of magnitude 1, 1, and 10 were made to each loop.

The BLT method shows high overshoot and oscillation in the closed-loop responses while the DLT method leads to very sluggish and unbalanced ones with large IAE values.

The proposed method provides fast and balanced responses through the illustrated example. The superiority of the proposed method is also demonstrated by comparison of the IAE values.

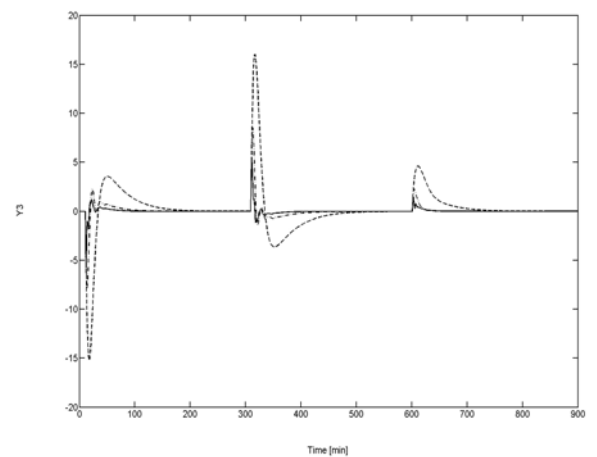
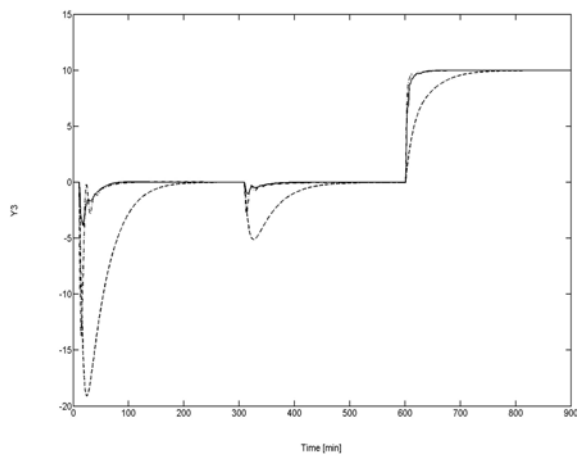
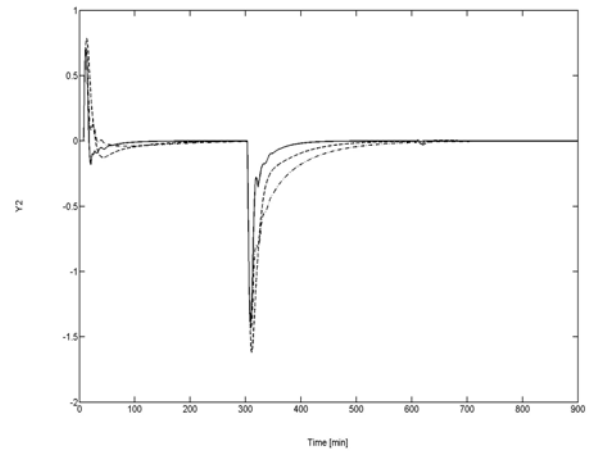
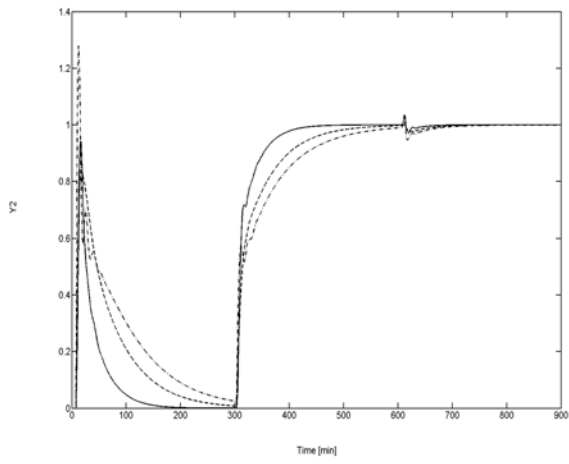
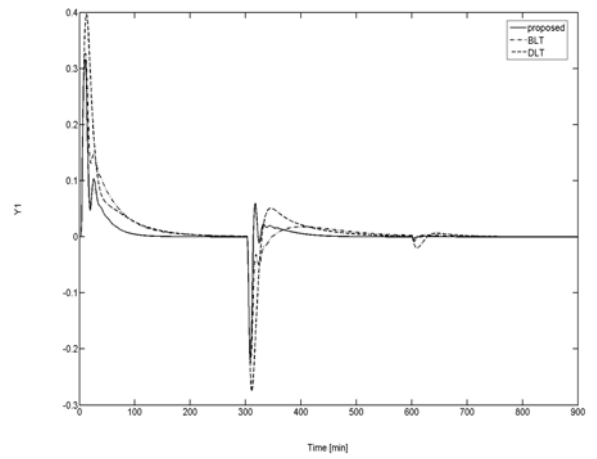
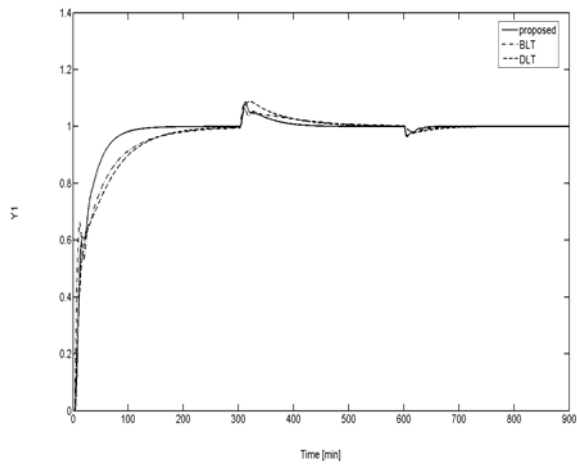


Figure 3. Closed-loop response to sequential step changes in set-point for OR column.

Figure 4. Closed-loop response to sequential step changes in disturbance for OR column.

5 Conclusion

The tuning of multi-loop PI controller for MIMO process is usually a complex problem. In this paper, the generalized-IMC approach is used to develop a simple but efficient design method for the multi-loop PI controller. The proposed method has several clear advantages.

Firstly, the method is straightforward and it can be easily implemented in multivariable control systems. Secondly, M_s criterion is very suitable for achieving good stability and robustness in multi-loop PI control systems. Furthermore, it is provided the minimizing of IAE values, while other control parameters are well-balanced. The simulation results show that the proposed method is very effective both set-point tracking and disturbance rejection in the multi-loop control system.

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