On the Controller Design for the Outpouring Phase of the Pouring Process

M. P. TZAMTZI, F. N. KOUMBOULIS, M. G. SKARPETIS
Department of Automation
Halkis Institute of Technology
34400 Psahna, Evia
GREECE

Abstract: - The performance of a two stage control design scheme on the control of liquid outpouring and the suppression of liquid sloshing during the outpouring phase of the pouring process is studied for the case of liquid containers carried by manipulators. The designed scheme, that does not require measurements of the liquid’s motion within the tank, combines a partial inverse dynamics controller with a PID controller, tuned with the use of a “metaheuristic” search algorithm. Both controllers are designed ignoring the loss of liquid’s mass due to outpouring. The controller performance is studied using simulation results, where the liquid’s motion is modeled using a simplified pendulum-type model whose parameters vary with time accordingly with the loss of liquid’s mass.

Key-Words: - Robotic applications, Robot Control, Nonlinear Control, Pouring Process, Sloshing Control

1 Introduction
Control of liquid pouring is a highly demanding task, that is met in several industrial applications, as for example metal casting and steel industries. The control design goals that appear in such industrial applications concern suppression of liquid sloshing during liquid transfer, as well as control of liquid outpouring with simultaneous sloshing suppression during the pouring process. The pouring process may be divided in three phases: the forward tilting phase, the outpouring phase and the backward tilting phase. During the first phase, the tank tilts until it reaches the maximum critical angle at which no liquid outpouring takes place provided that the liquid’s surface remains horizontal. During the outpouring phase, additional tilting takes place to cause liquid outpouring. When the desired amount of liquid has outpoured the tank, the tank tilts backward to the equilibrium position. This is the third and last phase of the pouring process.

Control of liquid pouring is complicated due to several factors, as the nonlinearity of the pouring process model, the lack of direct actuation of the liquid’s motion and the lack of measurements of the liquid’s displacement. During the outpouring phase, the pouring process model is time-varying, fact that complicates significantly the control design.

Several control design approaches, as for example input shaping control, application of time varying filter gain and hybrid shape approach, have been proposed in order to control liquid vibrations for several cases of liquid transfer and pouring (see for example [1]–[8]).

In [7] and [8] a two stage control scheme has been applied, that combines a partial inverse dynamics controller with a heuristically tuned PID controller for the case of a liquid container carried by a manipulator. The proposed controllers do not require measurements of the liquid’s motion within the tank. In [7] the two stage control scheme have been successfully applied for sloshing suppression during liquid transfer, while in [8] a similar scheme has been appropriately adjusted to prevent undesirable liquid outpouring and suppress sloshing during the forward and backward tilting phases of the pouring process. The liquid’s motion is approximated by a pendulum-type model [1]-[3], [5]-[8]. More specifically, the liquid’s vibrations are approximated by the oscillations of a pendulum, whose mass and length are determined by the mass of the liquid and the natural frequency of liquid’s oscillations, respectively [2], [4]. In the case of liquid transfer [7], the tank’s rotations are used exclusively to suppress sloshing. However, during the pouring process [8], the tank has to rotate appropriately so as to cause or prevent liquid outpouring. Hence, in this case sloshing suppression has to be achieved simultaneously with command
following for the tank’s rotation.

As already mentioned, control of the outpouring phase is more demanding due to the fact that the pouring process model becomes time-varying during this phase due to loss of liquid’s mass. More specifically, time variation of the pendulum’s mass and equivalent length has to be considered in the pendulum–type model that approximates the liquid’s motion within the tank.

In the present work, the performance of the two stage control design scheme proposed in [8] is studied, for the control of liquid outpouring and the suppression of sloshing during the outpouring phase of the pouring process, for the case of a liquid container carried by a manipulator. A simplified time varying model of the pouring process during the outpouring phase is derived. The proposed controllers are designed ignoring the time variation of the pendulum’s parameters due to the loss of liquid’s mass and considering the mass and the equivalent length of the pendulum to remain equal to initial values they had before the initiation of the outpouring phase. The PID controller is tuned based on the linearization of the corresponding time-invariant process model around the operating point corresponding to the initial critical angle, where liquid outpouring initiates.

Section 2 presents the time-varying modeling equations of the automatic pouring system during the outpouring phase and determines the design goals that have to be satisfied by the control scheme. Section 3 presents the proposed time-invariant control design approach. Finally, Section 4 presents simulation results from the application of the proposed controller to the automatic pouring system.

2 Problem Formulation

2.1 Time-Varying Pendulum-Type Model

As already mentioned, the liquid’s motion within a container may be approximated by a pendulum-type sloshing model [1]. The mass \( m_p(t) \) of the pendulum is equal to the mass of the liquid within the container at each instant of time, which can be computed by the relation:

\[
m_p(t) = \frac{m_{p,0}}{\eta(t)} h(t)
\]

where \( h(t) \) the liquid level within the container at each instant of time, while \( m_{p,0} \) and \( \eta_{t,0} \) denote the mass and the level, respectively, of the liquid within the tank before initiation of the outpouring phase.

The equivalent length \( l_p(t) \) of the pendulum is determined based on the natural frequency given by the perfect fluid theory [2], [4]. Assuming the dimension of the sloshing mode to be equal to one, the natural frequency \( f_s(t) \) is related to the liquid level \( \eta_{i}(t) \) according to the relation [2], [4]

\[
f_s(t) = \frac{1}{2\pi} \sqrt{\frac{g \pi}{R} \tan \left( \frac{\pi \eta_{i}(t)}{R} \right)}
\]

where \( R \) the distance between the walls of the liquid’s container. Then the length \( l_p(t) \) is given by

\[
l_p(t) = \frac{g}{4\pi^2 f_s^2(t)}
\]

The torque applied to the pendulum due to the viscosity of the liquid and the friction between the liquid and the walls of the tank is equal to

\[-c l_p(t) \cos^2(\eta(t)) \eta(t), \]

where \( c \) the equivalent coefficient of viscosity [1], \( \eta(t) \) the angle between the liquid’s free surface and the horizontal axis, \( \eta(t) = \eta_{i}(t) - \eta(t) \) the angle between the liquid’s surface and the bottom of the tank and \( \eta(t) \) the angle of the tank’s rotation with respect to the perpendicular axis.

Let \( \eta_{t}(t) \) denote at each instant of time, the critical value of the angle \( \eta(t) \) at which liquid outpouring initiates, that is the value of \( \eta(t) \) at which the liquid’s surface reaches the edge of the container. This critical value depends on the liquid’s level \( \eta_{i}(t) \) according to the relation

\[
\eta_{t}(t) = \tan^{-1} \left( \frac{2(\eta(t) - \eta_{t}(t))}{R} \right)
\]

where \( \eta(t) \) the height of the tank. Relation (4) implies that at each instant of the angle \( \eta(t) \) is such that the liquid’s surface reaches the edge of the container ( \( \eta(t) = \eta_{t}(t) \)), the liquid’s level \( \eta_{i}(t) \) can be determined by the relation

\[
\eta_{i}(t) = \eta(t) - \frac{R}{2} \tan(\eta(t) - \eta(t))
\]

Equation (5) may be used to express the loss of liquid’s mass during the outpouring phase. More specifically, consider that the outpouring phase initiates at \( t = 0 \), which implies that

\[
\eta(0) = \eta_{t}(0) = \eta_{t,0} = \tan^{-1} \left( \frac{2(\eta(t) - \eta_{t,0})}{R} \right).
\]

Then, using (5) it follows that the rate of variation of the
liquid’s level $h_i(t)$ can be determined by

$$
\dot{h}_i(t) = \begin{cases} 
\frac{R}{2} \eta^2(t) \dot{\eta}(t), & \eta(t) = \eta_i(t) \text{ and } \dot{\eta}(t) > 0 \\
0, & \eta(t) < \eta_i(t) \text{ or } \dot{\eta}(t) \leq 0 
\end{cases}
$$

(6)

Note that the determination of the equivalent length $l_p(t)$ depending on liquid’s level $h_i(t)$ is valid for relatively small values of the angle $\eta(t)$. When $\eta(t)$ gets large enough the distribution of the liquid’s mass within the tank changes significantly, and the natural frequency $f_i(t)$ should be computed from (2) using smaller values of $h_i(t)$ than that corresponding to the equilibrium state. However, it is significant to note that the function at the right hand side of (2) saturates for sufficiently large values of $h_i(t)$. Thus, the natural frequency $f_i(t)$ and consequently the length $l_p(t)$ remain practically invariant provided that $h_i(t)$ remains sufficiently large.

2.2 Modeling Equations

Consider an automatic pouring system, consisting by an articulated robotic manipulator carrying the liquid container (Figure 1). Liquid pouring is accomplished by appropriately tilting the container, through an actuatable revolute joint that connects the tank with the robot’s end effector. The tank is firmly grasped by the robotic manipulator’s end effector. Liquid sloshing is neglected in directions that do not lie on the structure’s plane of motion. The liquid container is represented by a single link, whose mass $m_l$ and inertia $I_l$ are equal to the corresponding parameters of the tank (Figure 2). This link is modeled as an additional third link of the robotic manipulator, considering the size, the mass and the moment of inertia of the robot’s end-effector to be neglectable. Liquid sloshing is modeled by the pendulum-type sloshing model introduced in the previous Subsection (Figure 2).

It is obvious from the above discussion that the dynamic equations of the robotic structure illustrated in Figure 1 may be derived using Euler-Langrange modeling equations for a planar robotic manipulator, whose last two links represent the tank and the pendulum (see also [7] and [8]). The last two joints of the manipulator are revolute joints. The control torque of the last joint is set identically equal to zero since the pendulum’s motion is not actuatable. Finally, an additional torque equal to $-c_l^2(t)\cos^2(q_i)(\dot{q}_i - \ddot{q}_i)$ is considered to act on the last joint due to viscosity and friction. Note that these modeling steps may be used not only for the specific manipulator of Figure 1, but for any robotic manipulator that may be used to carry the tank.

According to the above, the pouring system is described by the following simplified nonlinear dynamic model:

$$
D(q(t),h_i(t))\ddot{q}(t) + C(q(t),\dot{q}(t),h_i(t))\dot{q}(t) + G(q(t),h_i(t)) = u(t)
$$

(7a)

Condition 1: $\eta(t) = \eta_i(t)$ and $\dot{\eta}(t) \geq 0$, $\forall t \in [t_s,t_e]$

Modeling equation 1:

$$
\dot{h}_i(t) = \frac{R}{2} \left( \frac{1}{\eta^2(t)} \dot{\eta}(t) - \frac{2}{\eta(t)} \dot{\eta^2}(t) \right),
$$

$\forall t \in [t_s,t_e]$(7b)
Condition 2: \( \eta(t) < \eta_a(t) \text{ or } \dot{\eta}(t) < 0, \forall t \in (t_a, t_b) \)

Modeling equation 2:
\[
\ddot{h}_i(t) = \dot{h}_i(t) = 0, \forall t \in [t_a, t_b]
\]
(7c)

where \( q = [q_1, q_2, q_1, q_2] \) denote the generalized coordinates of the structure, with \( q_1 \) and \( q_2 \) being the generalized variables of the robotic manipulator, \( q_1 \) the angle of the tank’s rotation with respect to the perpendicular axis and \( q_2 \) the angle between the liquid’s free surface and the horizontal axis, \( \eta = q_1 - q_2 \) and \( \eta_a \) as defined by (4). Moreover, \( u(t) = [u_1(t) \ u_2(t) \ u_3(t) \ 0]^T \), where \( u_1(t), u_2(t) \) and \( u_3(t) \) denote the control torques that actuate the first manipulator joint, the second manipulator joint and the joint actuating the tank’s motion, respectively.

The matrices \( D(q(t), \dot{h}_i(t)), C(q(t), \dot{q}(t), \ddot{h}_i(t)) \) and \( G(q(t), \dot{h}_i(t), \ddot{h}_i(t), \dddot{h}_i(t)) \), with \( D(q(t), \dot{h}_i(t)) \) being symmetric positive definite, are derived by the Euler-Lagrange modeling equations for robotic structures, taking into account the aforementioned simplifying considerations. The dependence of these matrices on \( h_i \) and its first and second order derivatives \( (\dot{h}_i, \ddot{h}_i) \) is derived by replacing in the Euler-Lagrange modeling equations the time varying parameters \( m_p(t), I_p(t) \), as well as their first and second order derivatives \( \dot{m}_p(t), \ddot{m}_p(t), \dot{I}_p(t) \) and \( \ddot{I}_p(t) \) using relations (1)-(3).

The switching differential equations (7b) and (7c) are derived by differentiating equation (6). Equation (7b) describes the reduction of liquid’s height due to liquid’s outpouring. This equation is valid at those instants of time at which the liquid’s surface reaches the edge of the tank (\( \eta(t) = \eta_a(t) \)), while tank’s tilting and/or liquid sloshing cause outpouring (\( \dot{\eta}(t) \geq 0 \)). Equation (7c) becomes valid at those instants of time at which either the liquid’s surface is away from the edge of the tank (\( \eta(t) < \eta_a(t) \)) or it is on the edge of the tank, however tank’s tilting and/or liquid sloshing prevent outpouring (\( \dot{\eta}(t) < 0 \)). Equation (7b) and (7c) constitute a switching model that describes the variation of the liquid’s height.

The parameters of model (7) are defined in Table 1.

### Table 1. Parameters of the liquid transfer structure

<table>
<thead>
<tr>
<th>Parameter Value</th>
<th>Physical Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.68 kg</td>
<td>Tank’s mass</td>
</tr>
<tr>
<td>10 kg</td>
<td>Mass of the 1st link</td>
</tr>
<tr>
<td>0.033 m</td>
<td>Distance between the 1st joint and the CM</td>
</tr>
<tr>
<td>0.113 m</td>
<td>Distance of the 1st link’s center of mass from the 1st joint</td>
</tr>
<tr>
<td>7.193 x 10^-3 kg m^2</td>
<td>Tank’s moment of inertia</td>
</tr>
<tr>
<td>0.14 m</td>
<td>Distance between the walls of the tank</td>
</tr>
<tr>
<td>0.17 m</td>
<td>Height of the tank</td>
</tr>
<tr>
<td>0.14 m</td>
<td>Initial liquid level</td>
</tr>
<tr>
<td>2.744 kg</td>
<td>Initial pendulum’s mass</td>
</tr>
<tr>
<td>0.0442 m</td>
<td>Initial pendulum’s length</td>
</tr>
<tr>
<td>1.88 N·sec/m</td>
<td>Liquid coefficient of viscosity</td>
</tr>
<tr>
<td>9.81 m/sec^2</td>
<td>Gravity acceleration</td>
</tr>
</tbody>
</table>

2.3 Design Requirements

The system described in the previous section may be used for automatic liquid transfer and pouring in several industrial applications as for example metal casting. The objectives of any control system designed for the outpouring phase of such applications are the following:

1) To achieve sufficiently fast execution of the outpouring task in order to avoid metal cooling and to increase productivity.
II) To control the amount of liquid that outpours into the mold.

III) To avoid liquid sloshing during the outpouring phase, since this may cause outpouring of excessive amounts of liquid or even liquid loss outside the mold.

In the following sections, the design of an automatic control system aiming to achieve these design goals is studied.

3 Control Design

In [7] and [8] a control design scheme comprising a partial inverse dynamics controller with a heuristically tuned PID controller has been proposed to suppress liquid sloshing during liquid transfer, as well as during the forward and backward tilting phase of the pouring process.

In the following, a design scheme with similar structure is proposed, which is specifically oriented to serve the design requirements I, II and III, presented in Subsection 2.3, during the outpouring phase of the pouring process. The design scheme exploits measurements of the manipulator links and the container position and velocity variables, but it does not use measurements of the pendulum’s position and velocity, which as already mentioned are not considered to be measurable.

The proposed partial inverse dynamics controller is described by the equation (see also [7] and [8])

\[
\begin{bmatrix}
    u_1 \\
    u_2 \\
    u_3
\end{bmatrix} = \overline{C}(\bar{q}, \ddot{\bar{q}}) \ddot{\bar{q}} + \overline{G}(\overline{q}) + \overline{D}(\overline{q}) \begin{bmatrix}
    \dot{q}_{d,1} \\
    \dot{q}_{d,2} \\
    \dot{q}_{d,3}
\end{bmatrix}
\]

(8)

where \( \bar{q} = [q_1, q_2, q_3]^T \), \( w_3(t) \) is an auxiliary input variable,

\[
\overline{D}(\overline{q}) = [d_{i,j}, ([\overline{q}^T \ 0 \ h_{i,0}]^T)], \ i, j = 1, 2, 3
\]

\[
\overline{C}(\bar{q}, \ddot{\bar{q}}) = [c_{i,j}, ([\bar{q}^T \ 0]^T, [\ddot{\bar{q}}^T \ 0]^T, h_{i,0}^T)], \ i, j = 1, 2, 3
\]

\[
\overline{G}(\overline{q}) = [G_i, ([\overline{q}^T \ 0 \ h_{i,0} \ 0 \ 0]^T)], \ i = 1, 2, 3
\]

while \( q_{d,1}(t) \) and \( q_{d,2}(t) \) denote the desired trajectories for the first and the second manipulator’s joint variables, respectively. Note that \( q_{d,1}(t) \) and \( q_{d,2}(t) \) are determined using inverse kinematics, so as to achieve the desired motion of the tank.

The partial inverse dynamics controller (8) would achieve linearization and input/output decoupling of the liquid’s transfer structure dynamics in the ideal case where the liquid’s surface would remain always horizontal (\( q_0 = 0 \)) and no liquid outpouring takes place (\( h_i = 0 \)). The controller parameters \( k_i \) and \( k_2 \) are appropriately selected so that they would achieve closed-loop stability and sufficiently small settling time in the aforementioned ideal case.

The inverse dynamics controller parameters \( k_i \) and \( k_2 \) are selected equal to 11 and 30, respectively [8]. These values correspond to roots of the polynomial \( s^2 + k_1 s + k_2 = 0 \) equal to \(-5\) and \(-6\), which can achieve sufficiently fast asymptotic tracking for the generalized variables \( q_i \) and \( q_2 \).

Consider now an additional PID controller that drives the auxiliary input variable \( w_3(t) \):

\[
w_3(t) - \eta_{i,0} = f_i e_i(t) + \dot{f}_i \int e_i(\tau) d\tau \quad (9)
\]

where \( e_i(t) = q_{d,i}(t) - q_i(t) \) denotes the tracking error between the desired trajectory \( q_{d,i} \) for the tank’s rotation and the corresponding variable \( q_i \),

\[
\eta_{i,0} = \tan^{-1} \left( \frac{2(h_i - h_{i,0})}{R} \right)
\]

is the value of the critical angle at which the outpouring phase initiates, and \( f_i, i = 1, 2, 3 \) are the controller parameters to be determined.

The parameters \( f_i, i = 1, 2, 3 \) of the PID controller (9) are determined using a metaheuristic search algorithm introduced in [9], which is appropriately adjusted to meet the design requirements of the pouring task. The metaheuristic algorithm performs repeatedly random search within an appropriate search area, which has the form of a hyperrectangle in the controller parameters space. At each repetition of the search, simulation is performed for the linearization of the closed-loop system produced by the application of controllers (8) and (9) to system (1). Linearization is performed around the operating point corresponding to a tilting angle for the container equal to \( \eta_{i,0} \). The metaheuristic search algorithm utilizes the simulation results to solve numerically an optimization under constraints problem, that has been determined in [8].

The application of the metaheuristic optimization algorithm presented in [8] with the above parameters, resulted in the determination of a PID controller of the form (9) with \( f_1 = 1.8566 \).
\[ f_2 = 2.4191, \quad f_1 = 0.5634. \]

4 Simulation Results

In the following, the performance of the proposed control scheme is illustrated through simulation results derived from its application to the nonlinear time-varying system (7).

The initial conditions for the state variables of system (7), are considered to be:

\[ q_1(0-)=\frac{\pi}{2}[\text{rad}], \quad q_2(0-)=-\frac{\pi}{2}[\text{rad}], \]
\[ q_3(0-)=-0.405[\text{rad}], \quad q_\rho(0-)=0[\text{rad}], \]
\[ \dot{q}_1(0-)=0[\text{rad/second}], \quad \dot{q}_2(0-)=0[\text{rad/second}], \]
\[ \dot{q}_3(0-)=-\frac{\pi}{12}[\text{rad/second}], \quad \dot{q}_\rho(0-)=-\frac{\pi}{16}[\text{rad/second}], \]
\[ h_1(0-)=h_{1,0}=0.14[\text{m}], \quad \dot{h}_1(0-)=\frac{R - \frac{\pi}{2}}{48\eta_{1,0}} = -0.0054[\text{rad/sec}] \]

These initial conditions imply that the liquid’s surface is at the edge of the tank, while the initial velocities of the tank and the pendulum tend to initiate liquid’s outpouring. Thus the modeling equation (7b) is valid.

The external commands that drive the joints of the robotic manipulator aim to keep the robot’s end-effector at its current position, thus \( q_{d,1}(t) = \frac{\pi}{2}[\text{rad}] \) and \( q_{d,2}(t) = -\frac{\pi}{2}[\text{rad}] \) for \( t \geq 0 \). The external command for the tank’s rotation is \( q_{d,\rho}(t) = 1.5\eta_{1,0} = -0.6075[\text{rad}] \). Ignoring liquid sloshing, this corresponds to reduction of the liquid height at the final value \( h_{1,f} = 0.1214[\text{m}] \) and corresponding reduction of the liquid’s mass to the final value \( m_{p,f} = 2.3785[\text{kgr}] \).

The simulation results are presented in Figures 3-12. Figures 3 and 4 present the closed-loop trajectories for the two first joints. It is obvious from Figures 3 and 4 that the motion of the tank and the liquid, as well as the reduction of liquid’s mass practically has no effect on the positioning of the manipulator’s end-effector. Figures 5 and 6 present the closed-loop trajectories for the rotations of the tank and the pendulum, while Figures 7 and 8 illustrate the variables \( \eta(t) \) and \( \dot{\eta}(t) \). It is obvious from Figure 9, that \( \dot{\eta}(t) \) remains strictly positive for all \( t \), which implies that liquid sloshing remains sufficiently small (see also Figure 6), so as to not to cause interruption of liquid outpouring. Figures 10-12 illustrate the variations of the liquid’s height \( h_1(t) \), the liquid’s mass \( m_\rho(t) \) and the pendulum’s equivalent length \( l_\rho(t) \), respectively. As it follows from Figure 10 and 11, the reduction of the liquid’s level and equivalently the reduction of the liquid’s mass are equal to the desired ones. Hence, liquid sloshing has not resulted in excessive outpouring. Moreover, it is noticed from Figure 12, that the variation of the pendulum’s equivalent length \( l_\rho(t) \) is very small, which implies that the values of \( h_1(t) \) remains within the saturation area of the function that determines the natural frequency and consequently the equivalent length of the pendulum (see equations (2) and (3)).

5 Conclusions

In the present work, a simplified nonlinear time-varying model for the outpouring phase of the pouring process has been derived, for the case of a liquid container carried by a manipulator. The performance of a two stage control design scheme on the control of liquid outpouring and the suppression of liquid sloshing during the outpouring phase has been studied. The proposed scheme, that does not require measurements of the liquid’s motion within the tank, combines a partial inverse dynamics controller with a PID controller, tuned with the use of a “metaheuristic” search algorithm. Both controllers are designed ignoring the loss of liquid’s mass due to outpouring. The controller performance is studied using simulation results, where the liquid’s motion is modeled using a pendulum-type model whose parameters vary with time accordingly with the loss of liquid’s mass.

Acknowledgment

The present work is co-financed by the Hellenic Ministry of Education and Religious Affairs’ and the ESF of the European Union within the framework of the “Operational Programme for Education and Initial Vocational Training” (Operation “Archimedes-II”).

Aknowledgment

The present work is co-financed by the Hellenic Ministry of Education and Religious Affairs’ and the ESF of the European Union within the framework of the “Operational Programme for Education and Initial Vocational Training” (Operation “Archimedes-II”).
REFERENCES


