Modeling and Control of a Neutral Time Delay Test Case Central Heating System

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Abstract: - In the present paper, the mathematical model of a test case central heating system is developed in the form of a nonlinear neutral time delay model which in turn is simplified to a neutral time delay model with constant time delay. In the process it is shown that the influence of the delay is significant, thus its incorporation to the model is of high importance. A PI controller is derived to control the temperature of a room which is modeled as a first order differential equation.

Key-Words: central heating, modeling, pipe network, boiler, radiator, neutral time delay system, PI controller

1 Introduction

The problem modeling and of central heating systems has attracted significant attention during the last years (see f.e. [1]-[13]). In particular, significant attention has been given to the modeling, construction and optimization of core components of central heating system, such as pipe networks and piping elements (see f.e. [4]-[6] and the references therein), radiators and other heating systems (see f.e. [7]-[8]), boilers (see [9]-[10]) etc. Furthermore, different control techniques have been applied to such systems in order to regulate temperature in heated areas (see f.e. [11]-[13]).

In the present paper, the mathematical model of a test case central heating system will be developed in the form of a nonlinear neutral time delay model. In particular, separate models will presented for the core components of the system, i.e. pipe network, radiator and boiler. The separate models will be combined to a nonlinear neutral time delay system (with time varying delay) which will be simplified to a neutral time delay model with constant time delay. It will be shown that the influence of the delay is significant, thus its incorporation to the model is of high importance. Then, a PI controller will be derived to control the temperature of a room, which will be modeled as a first order differential equation.

2 Dynamic Model of a Test Case Central Heating System

In what follows, the general dynamic model of a test case central heating system will be produced. The system consists of the piping network, a radiator and a boiler (see Figure 1). The radiator heats up a room, thus the performance output of the system is power emitted by the radiator which is directly related to the temperature of the ambient air.

2.1 Radiator Modeling

The dynamic model of a radiator is presented in the form of the partial differential equation, [8]

$$C_l \frac{\partial T}{\partial t} = C_p \rho q \frac{\partial T}{\partial x} - \Phi_{0,l} \left(\frac{T - T_a}{\Delta T_{ma,0}} \right)^{n_1} \tag{2.1}$$

where T is the water temperature, T_a is the ambient air temperature, C_l is the heat capacity of water, C_p is the thermal capacity of the water, $\Phi_{0,l}$ denotes the nominal power of the radiator (per length), $\Delta T_{ma,0}$ is the arithmetic mean temperature difference at standard conditions, n_1 is an exponent in the range 1.2 to 1.4 and q is volumetric water flow rate in the radiator.

According to [8] and applying elementary computations, the partial differential equation (2.1) can be rewritten as a set on ODEs in the form of a nonlinear state space model as follows

$$\dot{x}_r(t) = f_r\left(x_r(t), u_r(t), \xi_r(t)\right) \tag{2.2a}$$

while the performance of the radiator is considered to be the emitted thermal power

$$y_r(t) = \frac{\Phi_0}{2} \sum_{j=1}^{N} \left(\frac{x_{j,r}(t) - \xi_r(t)}{60} \right)^{n_1} \tag{2.2b}$$

where

$$x_r(t) = \left[x_{1,r}(t) \quad \cdots \quad x_{N,r}(t) \right]^{\mathrm{T}} =$$

$$= \left[T_1(t) \quad \cdots \quad T_N(t) \right]^{\mathrm{T}}$$

$$u_r(t) = \left[u_{1,r}(t) \quad u_{2,r}(t) \right]^{\mathrm{T}} = \left[q(t) \quad T_i(t) \right]^{\mathrm{T}}$$

$$\xi_r(t) = T_a(t)$$

$$f_r\left(x_r(t), u_r(t)\right) = \left[f_{1,r}\left(x_r(t), u_r(t)\right) \quad \cdots \quad f_{N,r}\left(x_r(t), u_r(t)\right) \right]^{\mathrm{T}}$$

$$f_{1,r}\left(x_r(t), u_r(t), \xi_r(t)\right) = \frac{NH_q u_{1,r}(t)}{C} \times \left[u_{2,r}(t) - (1-\varphi) x_{1,r}(t) - \varphi x_{2,r}(t) \right] - \frac{\Phi_0}{C} \left(\frac{x_{1,r}(t) - \xi_r(t)}{60} \right)^{n_1}$$

$$f_{j,r}\left(x_r(t), u_r(t), \xi_r(t)\right) = \frac{NH_q u_{1,r}(t)}{C} \times \left[(1-\varphi) x_{j-1,r}(t) + (2\varphi - 1) x_{j,r}(t) - \varphi x_{j+1,r}(t) \right] - \frac{\Phi_0}{C} \left(\frac{x_{j,r}(t) - \xi_r(t)}{60} \right)^{n_1}, \ j = 2, \ldots, N-1$$

$$f_{N,r}\left(x_r(t), u_r(t), \xi_r(t)\right) = \frac{NH_q u_{1,r}(t)}{C} \left(x_{N-1,r}(t) - x_{N,r}(t) \right) - \frac{\Phi_0}{C} \left(\frac{x_{N,r}(t) - \xi_r(t)}{60} \right)^{n_1}$$

Note that $H_q = C_p \rho$ and that $f_{N,r}(\cdot,\cdot,\cdot)$ has been produced from $f_{N-1,r}(\cdot,\cdot,\cdot)$ setting $\varphi = 0$.

2.2 Boiler Modeling

The dynamic model of the boiler can be described as a first order differential equation of the form (2.3), [9]-[10]

$$C_b \frac{dT_w(t)}{dt} = n_{com}(T_w) Q_{burner}(t) - \rho C_w q(t) \left(T_{w_s}(t) - T_{w_r}(t) \right) - a_j \left(T_w(t) - T_e(t) \right) \quad (2.3)$$

where C_b is the thermal capacity of the boiler, C_w is the thermal capacity rate of the water, a_j is the rate of heat loss from the boiler jacket to the environment (i.e. boiler room), T_e is the temperature of the boiler room and n_{com} denotes the combustion efficiency of the boiler. The combustion efficiency is given by a polynomial of the form

$$n_{com}(T_w) = \sum_{j=1}^{\infty} a_j \left(T_w / T_{w,\max} \right)^{j-1} \quad (2.4)$$

where the coefficients can be obtained from experimental data. In the present paper it will be assumed that $a_i = 0$ for $i \geq 3$.

Using relation (2.4), the simplified model (2.3) can be rewritten in state space form as

$$\frac{dx_b(t)}{dt} = f\left(x_b(t), u_b(t)\right) \quad (2.5)$$

where

$$x_b(t) = T_{w_s}(t)$$

$$u_b(t) = \left[u_{1,b}(t) \quad u_{2,b}(t) \quad u_{3,b}(t) \quad u_{4,b}(t) \quad u_{5,b}(t) \right]^{\mathrm{T}} = \left[Q_{burner}(t) \quad q(t) \quad T_{w_r}(t) \quad \dot{T}_{w_r}(t) \quad T_e(t) \right]^{\mathrm{T}}$$

$$f\left(x_b(t), u_b(t)\right) = \frac{1}{C_b(1-a)} n_{com}\left(x_b, u_{3,b}\right) u_{1,b}(t) - \frac{\rho C_w}{C_b(1-a)} u_{2,b}(t) \left(x_b(t) - u_{3,b}(t) \right) - \frac{a_j}{C_b(1-a)} \left[a u_{3,b}(t) + (1-a) x_b(t) - u_{5,b}(t) \right] - \frac{a}{1-a} u_{4,b}(t)$$

2.3 Pipe Network Modeling

Given a straight circular pipe with length l, diameter d and roughness e, the dynamic model of one-dimensional incompressible flow q of a fluid of density ρ, driven by the pressure $\Delta p = p_{in} - p_{out}$ across the pipe, can be expressed as (see [8])

$$\frac{dq(t)}{dt} = f_{j,1}(q) \left[p_{in}(t) - p_{out}(t) \right] - f_{j,1}(q) f_{j,2}(q) \quad (2.6)$$

It holds that

$$f_{j,1}(q) = \pi d^2 / 4 \rho l \quad (2.7)$$

while

$$f_{j,2}(q) = 2 \lambda(q,d) q(t)^2 / \pi d^3 \quad (2.8)$$

where $\lambda(q)$ is the friction factor. Clearly, $\lambda(q)$ depends upon the conditions of the flow inside the pipe, i.e. weather the flow is laminar or turbulent. For the laminar region, i.e. if $\mathrm{Re}(q,d) \leq 2300$, it holds that

$$\lambda(q,d) = 64 / \mathrm{Re}(q,d) \quad (2.9)$$

while for the turbulent region, i.e. if $\mathrm{Re}\left(q,d\right) > 3000$, it holds that the friction factor is the solution of relation (2.10) with respect to λ

$$\frac{1}{\sqrt{\lambda\left(q,d\right)}} = -2\log_{10}\left[\frac{2.51}{\mathrm{Re}\left(q,d\right)\sqrt{\lambda\left(q,d\right)}} + \frac{e}{3.71d}\right] \quad (2.10)$$

where e is the pipe roughness. With respect to the transient region, i.e. for $2300 < \mathrm{Re}\left(q,d\right) \leq 3000$, the friction factor can be approximated linearly, [5], as long as the linear function satisfies (2.9) and (2.10) for $\mathrm{Re}\left(q,d\right) = 2300$ and $\mathrm{Re}\left(q,d\right) = 3000$ respectively. The turbulent region, can be divided into smaller regions. For example when the Reynolds number is less than 10^5 and the pipe is smooth, i.e. the pipe roughness is small, the friction factor is described by the Blasius equation

$$\lambda\left(q,d\right) = 0.316 \Big/ \mathrm{Re}\left(q,d\right)^{0.25} \quad (2.11)$$

Approximation (2.11) is valid for typical flow and geometry conditions for single family houses. Hence, assuming that the pipes used are smooth relation (2.10) can be substituted by (2.11). Consequently, the linear approximation for $2300 < \mathrm{Re}\left(q,d\right) \leq 3000$ takes on the form

$$\lambda\left(q,d\right) = 4\cdot 10^{-5}\left[0.533\,\mathrm{Re}\left(q,d\right) - 530.4027\right]$$

where $\mathrm{Re}\left(q,d\right) = \frac{4\rho q}{\pi d\mu}$. Fitting pressure losses are sometimes presented in terms of the equivalent length of straight pipe that would have the same pressure loss as the fitting. If a fitting is to be replaced by an equivalent length L_{eq} of pipe, then it must hold that $L_{eq} = K_f d \big/ \lambda\left(q,d\right)$, where K_f denotes the pressure loss coefficient for fittings. This relationship shows the fundamental shortcoming with the equivalent-length approach. It must be noted that even though K_f and d are constant for a given pipe under various flow conditions, the friction function is not.

According to the above presented formulae the nonlinear model of the piping network can be written in state space form as

$$\frac{dx_p}{dt} = \left[\frac{1}{f_{11}\left(x_p\right)} + \frac{1}{f_{21}\left(x_p\right)} + \frac{1}{f_{31}\left(x_p\right)}\right]^{-1} \times \left[u_p\left(t\right) + \delta P_t\left(x_p\right) + \frac{f_{12}\left(x_p\right)}{f_{11}\left(x_p\right)} + \frac{f_{22}\left(x_p\right)}{f_{21}\left(x_p\right)} + \frac{f_{32}\left(x_p\right)}{f_{31}\left(x_p\right)}\right] \quad (2.12)$$

where

$$x_p\left(t\right) = q_r\left(t\right),\ u_p\left(t\right) = \Delta P\left(t\right)$$

$$\delta P_t\left(x_p\right) = -K_t x_p^2,\ f_{11}\left(x_p\right) = \frac{\pi d^2}{4\rho L},$$

$$f_{21}\left(x_p\right) = \frac{\pi d_r^2}{4\rho L_r}\ f_{31}\left(x_p\right) = \frac{\pi d^2}{4\rho L},$$

$$f_{1,2}\left(x_p\right) = \frac{2\lambda\left(x_p,d\right)}{\pi d^3}x_p^2$$

$$f_{2,2}\left(x_p\right) = \frac{2\lambda\left(x_p,d_r\right)}{\pi d_r^3}x_p^2\quad f_{3,2}\left(x_p\right) = \frac{2\lambda\left(x_p,d\right)}{\pi d^3}x_p^2$$

and where d_r and L_r are the hydraulic diameter and length of the first radiator respectively, d and L are the diameter and length of the pipes connecting the boiler to the radiator respectively. The term δP_t stands for the pressure drop caused by possible turbulence to the entrance of the radiator while ΔP the pressure added to the pipe network by the pump (actuatable input).

2.4 Nonlinear Model of the Overall Plant

In order to construct the nonlinear model of the overall plant, it suffices to establish the necessary algebraic equations standing for the "connections" between the different elements of the plant. Define the composite state, input and disturbance vectors, as defined in Sections 2.1 to 2.3

$$x\left(t\right) = \left[x_1\left(t\right) \mid x_2\left(t\right) \ \cdots \ x_{N+1}\left(t\right) \mid x_{N+2}\left(t\right)\right]^T = \left[q_r\left(t\right) \mid T_1\left(t\right) \ \cdots \ T_N\left(t\right) \mid T_{w_s}\left(t\right)\right]^T$$

$$u\left(t\right) = \left[u_1\left(t\right) \ \ u_2\left(t\right)\right]^T = \left[\Delta P\left(t\right) \ \ Q_{burner}\left(t\right)\right]^T$$

$$\xi\left(t\right) = \left[\xi_1\left(t\right) \ \ \xi_2\left(t\right)\right]^T = \left[T_a\left(t\right) \ \ T_e\left(t\right)\right]^T$$

From Figure 1, it can readily be observed that

$$u_{1,r}\left(t\right) = x_p\left(t\right) = x_1\left(t\right) \quad (2.13a)$$

$$u_{2,r}\left(t\right) = x_b\left(t-\tau\left(t\right)\right) = x_{N+2}\left(t-\tau\left(t\right)\right) \quad (2.13b)$$

$$u_{2,b}\left(t\right) = x_p\left(t\right) = x_1\left(t\right) \quad (2.13c)$$

$$u_{3,b}\left(t\right) = x_{N,r}\left(t-\tau\left(t\right)\right) = x_{N+1}\left(t-\tau\left(t\right)\right) \quad (2.13d)$$

$$u_{4,b}\left(t\right)=\frac{du_{3,b}\left(t\right)}{dt} \qquad (2.13e)$$

Applying elementary computations, the nonlinear model of the overall plant, takes on the form

$$\frac{dx\left(t\right)}{dt}+\tilde{E}_1\frac{d}{dt}x\left(t-\tau\left(t\right)\right)=\tilde{f}\left(x\left(t\right),u\left(t\right),x\left(t-\tau\left(t\right)\right),\xi\left(t\right)\right) \qquad (2.14a)$$

$$\int_{t-\tau(t)}^{t}x_1\left(\rho\right)d\rho=\frac{\pi d^2 L}{4} \qquad (2.14b)$$

$$y\left(t\right)=\frac{\Phi_0}{2}\sum_{j=2}^{N+1}\left(\frac{x_j\left(t\right)-\xi_1\left(t\right)}{60}\right)^{n_1} \qquad (2.14c)$$

where

$$\tilde{f}\left(x\left(t\right),u\left(t\right),x\left(t-\tau\left(t\right)\right),\xi\left(t\right)\right)==\begin{bmatrix}\tilde{f}_1\left(x\left(t\right),u\left(t\right),x\left(t-\tau\left(t\right)\right),\xi\left(t\right)\right)\\ \vdots \\ \tilde{f}_{N+2}\left(x\left(t\right),u\left(t\right),x\left(t-\tau\left(t\right)\right),\xi\left(t\right)\right)\end{bmatrix}$$

$$\tilde{f}_1\left(x\left(t\right),u\left(t\right),x\left(t-\tau\left(t\right)\right),\xi\left(t\right)\right)==\left[\frac{1}{f_{11}\left(x_1\right)}+\frac{1}{f_{21}\left(x_1\right)}+\frac{1}{f_{31}\left(x_1\right)}\right]^{-1}\times\left[u_1\left(t\right)+\delta P_t\left(x_1\right)+\frac{f_{12}\left(x_1\right)}{f_{11}\left(x_1\right)}+\frac{f_{22}\left(x_1\right)}{f_{21}\left(x_1\right)}+\frac{f_{32}\left(x_1\right)}{f_{31}\left(x_1\right)}\right]$$

$$\tilde{f}_2\left(x\left(t\right),u\left(t\right),x\left(t-\tau\left(t\right)\right),\xi\left(t\right)\right)=\frac{NH_q x_1\left(t\right)}{C}\left[x_{N+2}\left(t-\tau\left(t\right)\right)-\left(1-\varphi\right)x_{1,r}\left(t\right)-\varphi x_{2,r}\left(t\right)\right]--\frac{\Phi_0}{C}\left(\frac{x_2\left(t\right)-\xi_1\left(t\right)}{60}\right)^{n_1}$$

$$\tilde{f}_j\left(x\left(t\right),u\left(t\right),x\left(t-\tau\left(t\right)\right),\xi\left(t\right)\right)=\frac{NH_q x_1\left(t\right)}{C}\left[\left(1-\varphi\right)x_{j-1}\left(t\right)+\left(2\varphi-1\right)x_j\left(t\right)-\varphi x_{j+1}\left(t\right)\right]--\frac{\Phi_0}{C}\left(\frac{x_j\left(t\right)-\xi_1\left(t\right)}{60}\right)^{n_1} \quad \text{for } j=2,\ldots,N$$

$$\tilde{f}_{N+1}\left(x\left(t\right),u\left(t\right),x\left(t-\tau\left(t\right)\right),\xi\left(t\right)\right)=\frac{NH_q x_1\left(t\right)}{C}\left(x_N\left(t\right)-x_{N+1}\left(t\right)\right)--\frac{\Phi_0}{C}\left(\frac{x_{N+1}\left(t\right)-\xi_1\left(t\right)}{60}\right)^{n_1}$$

$$\tilde{f}_{N+2}\left(x\left(t\right),u\left(t\right),x\left(t-\tau\left(t\right)\right),\xi\left(t\right)\right)=\frac{1}{C_b\left(1-a\right)}n_{com}\left(x_{N+2}\left(t\right),x_{N+1}\left(t-\tau\left(t\right)\right)\right)u_2\left(t\right)-\frac{\rho C_w}{C_b\left(1-a\right)}x_1\left(t\right)\left[x_{N+2}\left(t\right)-x_{N+1}\left(t-\tau\left(t\right)\right)\right]-\frac{a_j}{C_b\left(1-a\right)}\left[ax_{N+1}\left(t-\tau\left(t\right)\right)+\left(1-a\right)x_{N+2}\left(t\right)-\xi_2\left(t\right)\right]$$

$$\tilde{E}_1=\begin{bmatrix}0_{(N+1)\times N} & 0_{(N+1)\times 1} & 0_{(N+1)\times 1}\\ 0_{1\times N} & \frac{a}{1-a} & 0\end{bmatrix}$$

Note that the time delay $\tau\left(t\right)$ stands for the transport delay from the output of the boiler to the input of the radiator and the output of the radiator to the input of the boiler. It must be noted that these delays are in general different. In the present paper, it is assumed that the length and diameter of the pipes from the boiler to the radiator and vice versa are equal hence the respective time delays, let $\tau_1\left(t\right)$ and $\tau_2\left(t\right)$, are also equal between themselves, i.e. $\tau_1\left(t\right)=\tau_2\left(t\right)=\tau\left(t\right)$.

3 Influence of the Delay to the Nonlinear Model of the Overall Plant

In what follows, the influence of the delay to the response of the overall plant will be examined, through computational experiments. Assume that the model (2.4) operates on certain operating conditions, let $\overline{u}_1$, $\overline{u}_2$, $\overline{\xi}_1$ and $\overline{\xi}_2$ for the actuatable inputs and disturbances yielding to the respective nominal values for the state variables, let $\overline{x}_j$ for $j=1,\ldots,N+2$. Without loss of generality, assume that at $t=0$ the actuatable inputs and disturbances become $u_1\left(t\right)=p_{u_1}\overline{u}_1 u_s\left(t\right)$, $u_2\left(t\right)=p_{u_2}\overline{u}_2 u_s\left(t\right)$, $\xi_1\left(t\right)=p_{\xi_1}\overline{\xi}_1 u_s\left(t\right)$ and $\xi_2\left(t\right)=p_{\xi_2}\overline{\xi}_2 u_s\left(t\right)$, where $p_{u_1}\in\left[\left(p_{u_1}\right)_{\min},\left(p_{u_1}\right)_{\max}\right]$, $p_{u_2}\in\left[\left(p_{u_2}\right)_{\min},\left(p_{u_2}\right)_{\max}\right]$, $p_{\xi_1}\in\left[\left(p_{\xi_1}\right)_{\min},\left(p_{\xi_1}\right)_{\max}\right]$ and $p_{\xi_2}\in\left[\left(p_{\xi_2}\right)_{\min},\left(p_{\xi_2}\right)_{\max}\right]$

and where $u_s(t)$ denotes the unit step function, driving the system to new operation conditions, let $\overline{x}_j$ for $j = 1,\ldots,N+2$, through the respective responses, let $x_j(t)$ for $j = 1,\ldots,N+2$. The same experiment can be carried out for the system (2.14) assuming that the time delay $\tau(t)$ is equal to zero, leading to different responses, let $\tilde{x}_j(t)$ for $j = 1,\ldots,N+2$ Note that the starting and ending nominal points are equal to both cases, with or without the presence of the delay, are equal. In order to examine the influence of the time delay to the system a Euclidean norm type of error will be used. Define the normalized composite response vectors

$$w_j(t) = \left[c_{u_1}\frac{u_1(t)}{\overline{u}_1} \quad c_{u_2}\frac{u_2(t)}{\overline{u}_2} \;\vdots\; c_{\xi_1}\frac{\xi_1(t)}{\overline{\xi}_1} \quad c_{\xi_2}\frac{\xi_2(t)}{\overline{\xi}_2} \quad c_{x_j}\frac{x_j(t)}{\overline{x}_j} \right] \quad (3.1a)$$

$$\tilde{w}_j(t) = \left[c_{u_1}\frac{u_1(t)}{\overline{u}_1} \quad c_{u_2}\frac{u_2(t)}{\overline{u}_2} \;\vdots\; c_{\xi_1}\frac{\xi_1(t)}{\overline{\xi}_1} \quad c_{\xi_2}\frac{\xi_2(t)}{\overline{\xi}_2} \quad c_{x_j}\frac{\tilde{x}_j(t)}{\overline{x}_j} \right] \quad (3.1b)$$

where c_{u_1}, c_{u_2}, c_{ξ_1}, c_{ξ_2} and c_{x_j} are appropriate weight factors. The composite vectors defined in (3.1) are sampled from time $t = 0$ to $t = T_{\max}$ using a sampling period Δt, to yield sampled data vectors $(w_j)_v$ and $(\tilde{w}_j)_v$. $T_{\max}$ is chosen large enough for $w_{j,r}(t)$ and $\tilde{w}_{j,r}(t)$ to have reached steady state while Δt is chosen small enough for the sampled signal to accurately represent the original system, i.e. to satisfy Shannon theorem. The Euclidian norm error is defined as

$$p(w_j, \tilde{w}_j) = \frac{\left\| (w_j)_v - (\tilde{w}_j)_v \right\|_2}{\left\| (\overline{w}_j)_v \right\|_2} \times 100\% \quad (3.2)$$

where $(\overline{w}_{j,r})_v$ stands for the sampled data vector of

$$\overline{w}_{j,r}(t) = \left[c_{u_1}\left(\frac{u_1(t)}{\overline{u}_1} - 1\right) \quad c_{u_2}\left(\frac{u_2(t)}{\overline{u}_2} - 1\right) \;\vdots\; c_{\xi_1}\left(\frac{\xi_1(t)}{\overline{\xi}_1} - 1\right) \quad c_{\xi_2}\left(\frac{\xi_2(t)}{\overline{\xi}_2} - 1\right) \quad c_{x_j}\left(\frac{x_j(t)}{\overline{x}_j} - 1\right) \right]$$

Note that since the flow rate response is not influenced by the delay. In both cases, with or without the presence of the delay, it will be equal, (i.e. by definition zero). Furthermore, for the case where $p_{u_1} = 1$, $p_{u_2} = 1$, $p_{\xi_1} = 1$ and $p_{\xi_2} = 1$ both systems remain in the operating point, hence both numerator and denominator become zero. In that case the error is defined as zero. Let $L = 25[\mathrm{m}]$, $d = 0.015[\mathrm{m}]$, $L_r = 2[\mathrm{m}], d_r = 0.0096153[\mathrm{m}], K_t = 0.00001$, $C = 36[\mathrm{KJ/K}]$, $N = 4$, $\varphi = 0, \Phi_0 = 2500[\mathrm{W}]$, $n_1 = 1.25$ $a = 2/9$, $a_1 = 1$, $a_2 = -0.12$, $T_{w,\max} = 100[^\circ\mathrm{C}]$, $a_j = 5.06[\mathrm{W/K}]$, $C_w = 4180[\mathrm{J/K \cdot Kgr}]$, $C_b = 42400[\mathrm{J/K}]$, $\rho = 971.81[\mathrm{Kgr/m^3}]$ $\mu = 0.0003547[\mathrm{Pa \cdot s}]$ be the parameters of the nonlinear model (2.14). Assuming that $\overline{u}_1 = 3000[\mathrm{Pa}]$, $\overline{u}_2 = 2300[\mathrm{W}]$, $\overline{\xi}_1 = 19.111[^\circ\mathrm{C}]$ and $\overline{\xi}_2 = 10[^\circ\mathrm{C}]$ the starting nominal points are evaluated to be $\overline{x}_1 = 3.6287 \cdot 10^{-5}[m^3/\mathrm{s}]$, $\overline{x}_2 = 69.6437[^\circ\mathrm{C}]$, $\overline{x}_3 = 66.4807[^\circ\mathrm{C}]$, $\overline{x}_4 = 63.5596[^\circ\mathrm{C}]$, $\overline{x}_5 = 60.8587[^\circ\mathrm{C}]$, $\overline{x}_6 = 73.0728[^\circ\mathrm{C}]$, while the nominal emitted power by the radiator can be easily evaluated to be $\overline{y} = 1796.1[\mathrm{W}]$. In Figures 2 to 6 contour plots of the criterion (3.2) are presented while in Figure 7 the same criterion is presented for the performance variable for a wide range of p_{u_1} and p_{u_2}, indicatively for $p_{\xi_1} = 105\%$ and $p_{\xi_2} = 95\%$. The weight factors have been chosen to be $c_{u_1} = 0$, $c_{u_2} = 0$, $c_{\xi_1} = 0$, $c_{\xi_2} = 0$ and $c_{x_j} = 1$. It can readily be observed that the influence of the delay to the system is significant, especially to the performance output. Hence, it can be safely stated that the incorporation of the delay to the system makes it more accurate. It must be noted that the Euclidean Norm Error plots have been zoomed to a particular area in order to demonstrate specific details.

3.1 Approximation of the Nonlinear Model of the Plant

In order to simplify the nonlinear model of the plant it will be approximated using constant delays, instead of the time varying ones, i.e. the dynamic model (2.14) becomes

$$\frac{dx(t)}{dt} + \tilde{E}_1 \frac{d\left(\nabla_{\bar{\tau}} x(t)\right)}{dt} = \tilde{f}\left(x(t), u(t), x(t-\bar{\tau}), \xi(t)\right) \quad (3.3a)$$

$$y(t) = \frac{\Phi_0}{2} \sum_{j=2}^{N+1} \left(\frac{x_j(t) - \xi_1(t)}{60} \right)^{n_1} \quad (3.3b)$$

In order to evaluate the constant delay, it will be assumed that the system operates on its nominal points. It can easily be verified that

$$\bar{\tau} = \pi d^2 L \big/ 4\bar{x}_1 \quad (3.4)$$

In order to check the accuracy of the proposed simplification a cost criterion similar to that in (3.2) will be used. Assume that the model (3.3) operates on certain operating conditions, let $\bar{u}_1$, $\bar{u}_2$, $\bar{\xi}_1$ and $\bar{\xi}_2$ for the actuatable inputs and disturbances yielding to the respective nominal values for the state variables, let $\bar{x}_j$ for $j = 1, \ldots, N+2$. Without loss of generality, assume that at $t = 0$ the actuatable inputs and disturbances become $u_1(t) = p_{u_1} \bar{u}_1 u_s(t)$, $u_2(t) = p_{u_2} \bar{u}_2 u_s(t)$, $\xi_1(t) = p_{\xi_1} \bar{\xi}_1 u_s(t)$ and $\xi_2(t) = p_{\xi_2} \bar{\xi}_2 u_s(t)$, where $p_{u_1} \in \left[\left(p_{u_1}\right)_{\min}, \left(p_{u_1}\right)_{\max} \right]$, $p_{u_2} \in \left[\left(p_{u_2}\right)_{\min}, \left(p_{u_2}\right)_{\max} \right]$, $p_{\xi_1} \in \left[\left(p_{\xi_1}\right)_{\min}, \left(p_{\xi_1}\right)_{\max} \right]$, $p_{\xi_2} \in \left[\left(p_{\xi_2}\right)_{\min}, \left(p_{\xi_2}\right)_{\max} \right]$ and where $u_s(t)$ denotes the unit step function, driving the system to new operation conditions, let $\bar{\bar{x}}_j$ for $j = 1, \ldots, N+2$, through the respective responses, let $x_j(t)$ for $j = 1, \ldots, N+2$. The same experiment can be carried out for the system (3.3), leading to different responses, let $\hat{x}_j(t)$ for $j = 1, \ldots, N+2$. Note that the starting and ending nominal points are equal to both cases, with constant or time varying delay. In order to examine the influence of the constant time delay to the system a Euclidean norm type of error will be used, similar to that defined in (3.2). Consider the data presented in Section 2.5. In Figures 8 to 12 contour plots of the cost criterion are presented while in Figure 13 the same criterion is presented for the performance variable for a wide range of p_{u_1} and p_{u_2}, indicatively for $p_{\xi_1} = 105\%$ and $p_{\xi_2} = 95\%$. Note that the weight factors are chosen to be $c_{u_1} = 0$, $c_{u_2} = 0$, $c_{\xi_1} = 0$, $c_{\xi_2} = 0$ and $c_{x_j} = 1$ It can readily be observed that the influence of the time varying delay to the system as compared to the constant delay case is not significant. Hence, the system can safely be simplified using constant delay rather than time varying. It must be noted that the Euclidean Norm Error plots have been zoomed to a particular area in order to demonstrate specific details.

4 Temperature Control

In the section, a PI controller will be designed in order to regulate the temperature of a room in which a central heating system of the form presented in Figure 1 is installed. In particular the room will be modeled as a first order differential equation, [8], under the assumptions that the walls of the room are subject to the same external temperature, there is no influence of the weather (wind or rain) on the thermal resistance of the walls, radiative heat transfer is negligible, there is no ventilation and no influence from the humidity of the air, there are no heat losses from the ceiling and the floor and there are no heat sources in the room besides the radiator. Under the above assumptions the dynamic model of the room takes on the form

$$\frac{dT_r(t)}{dt} = \frac{1}{C_r R_w}\left[T_{out}(t) - T_r(t)\right] + \frac{1}{C_r} P_{rad}(t) \quad (4.1)$$

where T_r is the room temperature, T_{out} is the environment temperature, P_{rad} is the thermal power emitted by the radiator, C_r is the thermal capacity of the room and R_w is the thermal resistance of the outer walls. For simulation purposes it will be assumed that $C_r = 72.1\left[\mathrm{KJ/K}\right]$ and $R_w = 7.85 \cdot 10^{-3}\left[\mathrm{K/W}\right]$. It can readily be observed that the environment temperature acts as a disturbance while the radiator power is the "actuatable" input.

In the overall system, i.e. room plus central heating unit, it will be assumed that the only measurable variable is the room temperature while the actuatable input is fuel, in the form of power, applied to the boiler. The controller is chosen of the form [14] $u_2(t) = f_p e(t) + f_i \int_0^t e(\tau)\, d\tau$, where

$e(t) = T_r(t) - r(t)$ and r is the room set point temperature. It can be observed that the first actuatable input of the overall system, i.e. the pump pressure, is kept to its nominal value. In order to choose the controller parameters, two techniques have been applied. First, a classical Ziegler-Nichols approach has been used, see f.e. [14], but the results were totally inadequate. Second, a heuristic approach, similar to that presented in [15], has been applied. In order to demonstrate the proposed control scheme, consider the data presented in Sections 3 and 4. Note that the environment nominal temperature has been chosen to be $5[°\mathrm{C}]$ producing the same nominal temperature $\bar{\xi}_1$ for the room. Using the data presented in previous sections an assuming that r is chosen to be of the form $r(t) = \bar{\xi}_1 + 3u_s(t)$ where $u_s(t)$ is the unit step function, the heuristic algorithm produces $f_p = 760$ and $f_i = 0.24$. In Figure 14 the performance of the closed loop system is presented, together with the open loop response (i.e. for the case where the fuel provided to the boiler is equal to $2826.816[\mathrm{W}]$. In Figure 15, the supplied power to the boiler is presented. In both figures, for the closed loop case, it is observed that the variables remain within acceptable levels. With respect to the performance variable, i.e. the room temperature, it can be observed that the response of the closed loop system is satisfactory, with rise and settling time $20.2[\mathrm{min}]$ and $66.4[\mathrm{min}]$ respectively, while the overshoot is 9.15%. For the open loop case, it can be observed that the response does not reach rise time within simulation time limits.

5 Conclusions

In the present paper, the mathematical model of a test case central heating system has been developed in the form of a nonlinear neutral time delay model which in turn has been simplified to a neutral time delay model with constant time delay. In the process it has been shown that the influence of the delay is significant, thus its incorporation to the model is of high importance. A PI controller has been derived to control the temperature of a room which has been modeled as a first order differential equation. An other contribution of the paper is that it provides the analytic description o a nonlinear neutral time delay model being useful for testing of various control design techniques.

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Figures

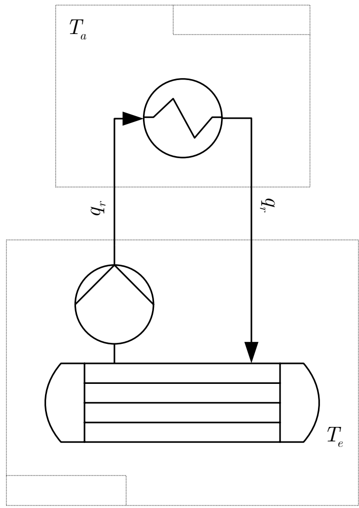

Fig. 1: Layout of the Test Case Central Heating System

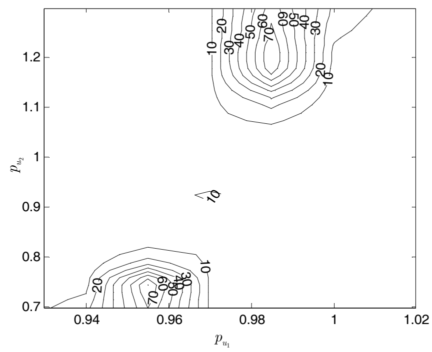

Fig. 2: Euclidian Norm Error $p\left(w_2,\tilde{w}_2\right)$ for $p_{\xi_1}=105\%$ and $p_{\xi_2}=95\%$

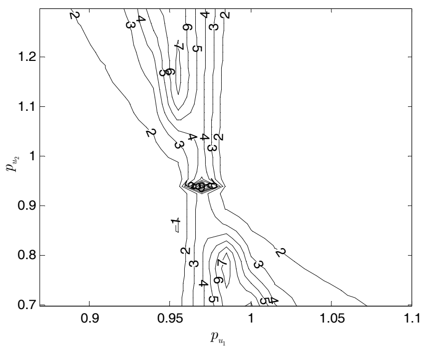

Fig. 4: Euclidian Norm Error $p\left(w_4,\tilde{w}_4\right)$ for $p_{\xi_1}=105\%$ and $p_{\xi_2}=95\%$

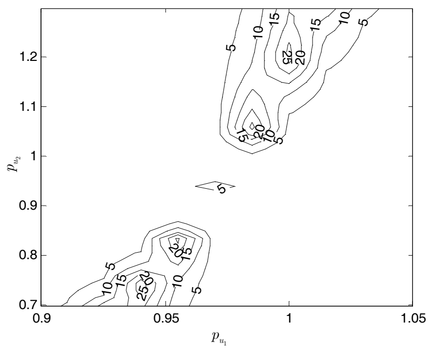

Fig. 6: Euclidian Norm Error $p\left(w_6,\tilde{w}_6\right)$ for $p_{\xi_1}=105\%$ and $p_{\xi_2}=95\%$

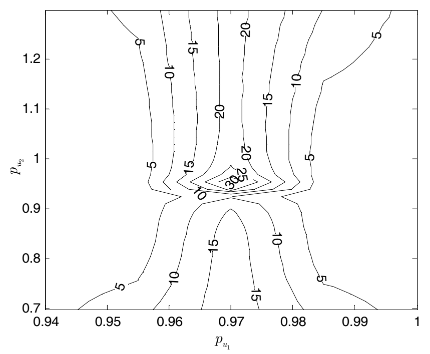

Fig. 3: Euclidian Norm Error $p\left(w_3,\tilde{w}_3\right)$ for $p_{\xi_1}=105\%$ and $p_{\xi_2}=95\%$

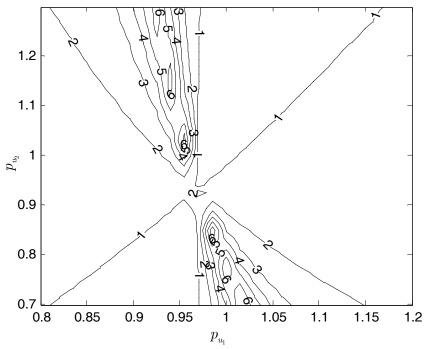

Fig. 5: Euclidian Norm Error $p\left(w_5,\tilde{w}_5\right)$ for $p_{\xi_1}=105\%$ and $p_{\xi_2}=95\%$

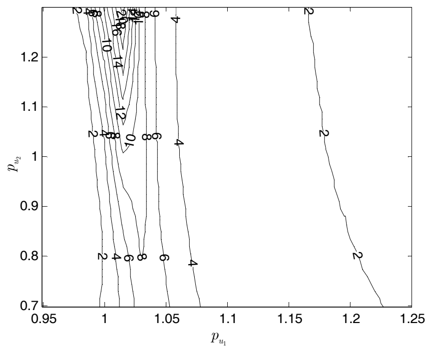

Fig. 7: Euclidian Norm Error $p\left(y,\tilde{y}\right)$ for $p_{\xi_1}=105\%$ and $p_{\xi_2}=95\%$

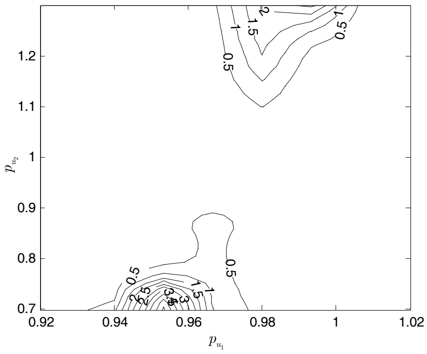

Fig. 8: Euclidian Norm Error $p\left(w_2, \widehat{w}_2\right)$ for $p_{\xi_1} = 105\%$ and $p_{\xi_2} = 95\%$

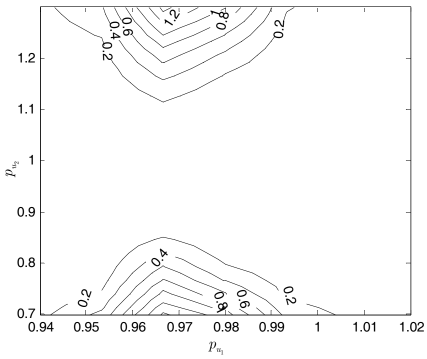

Fig. 9: Euclidian Norm Error $p\left(w_3, \widehat{w}_3\right)$ for $p_{\xi_1} = 105\%$ and $p_{\xi_2} = 95\%$

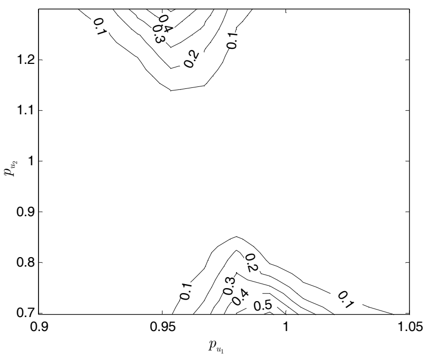

Fig. 10: Euclidian Norm Error $p\left(w_4, \widehat{w}_4\right)$ for $p_{\xi_1} = 105\%$ and $p_{\xi_2} = 95\%$

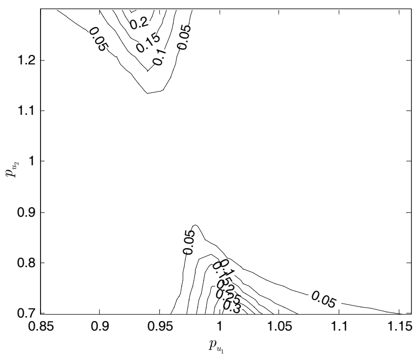

Fig. 11: Euclidian Norm Error $p\left(w_5, \widehat{w}_5\right)$ for $p_{\xi_1} = 105\%$ and $p_{\xi_2} = 95\%$

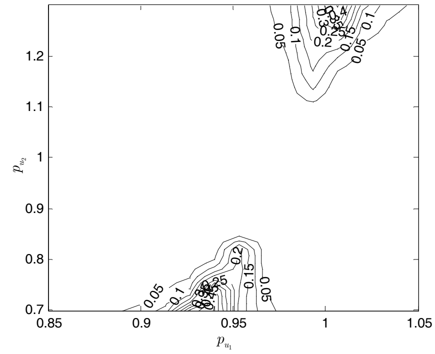

Fig. 12: Euclidian Norm Error $p\left(w_6, \widehat{w}_6\right)$ for $p_{\xi_1} = 105\%$ and $p_{\xi_2} = 95\%$

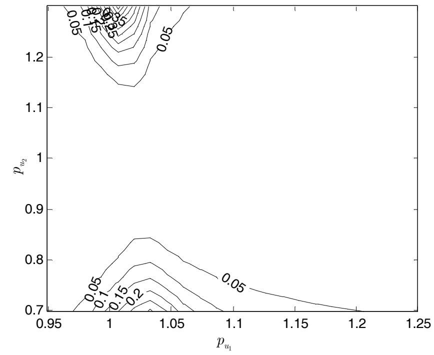

Fig. 13: Euclidian Norm Error $p\left(y, \widehat{y}\right)$ for $p_{\xi_1} = 105\%$ and $p_{\xi_2} = 95\%$

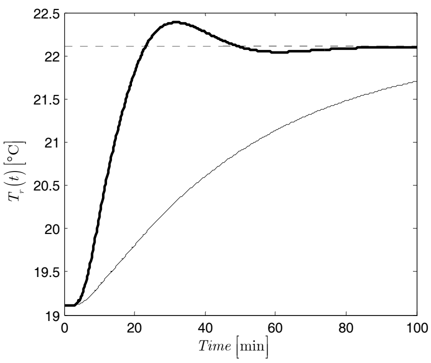

Fig. 14: Room Temperature (dashed – reference, thin – open loop , thick – closed loop)

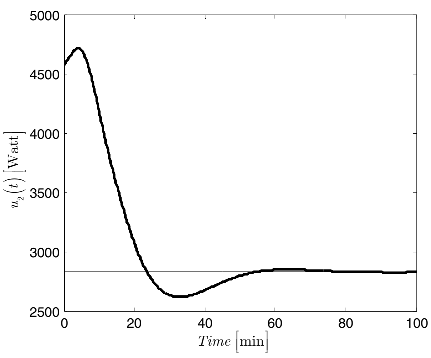

Fig. 15: Power Supply to the Boiler (thin – open loop, thick – closed loop)