Nonlinear Vibration System with Nonlinear Inertia for Force and Influence of Vibration on Rub and Wears

BANGCHUN WEN*, YIMING ZHANG*, ZHAOHUI REN*,
NAHUI SONG**, LILI XIN**
*Professor, **Assistant Professor, School of Mechanical engineering and Automation,
Northeastern University, Shenyang, Liaoning, P R China, 110004

Abstract: - Utilization of vibration and wave is a type of the most valued applicable technology developed during the later part of the 20^{th} century and still in rapid development now. Since the technology is closely associated with industrial and agricultural production, can create huge social and economical benefits for our society and provide excellent service for human beings, it becomes a necessary mechanism in human production and life. The theory of the vibrating machines has been developed and used widely in this field. In this paper following problems have been studied.(1)non-linear vibration of the vibrating body with sliding material; (2) the combination coefficient and damping coefficient of material under sliding condition; (3) nonlinear vibration of a vibrating body with throwing materials; (4) the combination coefficient and damping coefficient of material under throwing condition. The conclusion of our research is that vibration can decrease rub and wears between vibrating body and materials and it is very important to prolong the working life of machines and their parts.

Key-Words: - Nonlinear vibration system, vibration utilization engineering, vibrating machinery, rub, wear

1 Introduction

In some vibrating machines, depending on the acceleration of the vibrating body, the material on the body will move in various motion forms (such as sliding and throwing etc.). In the process of motion, the material will invoke various kinds of nonlinear force, namely, impact force, dry friction force, and the sectional inertial force. In order to analyze the vibrating systems with the nonlinear force, and investigate the influence of material on the motion of a vibrating body, the expressions of various nonlinear forces must be given preliminarily, then using the successive approximation method [1] in nonlinear theory, the approximation solution of the system will be found. On the basis of the above, we may present the calculation method of the combination coefficient and damping coefficient of the material [2--6]. We will also discuss the higher-harmonic vibration and the possible sub-harmonic vibration under the action of the nonlinear force.

2 Nonlinear vibration systems with sliding material on vibration body

Fig.1 shows the mechanical model of the cooling machine with the elastic connecting rod. Since the selected acceleration is less, the material only slips, so the material always keeps contact with the vibrating body in direction y. Hence the equation of motion of the vibrating body may be expressed as follows:

$$\begin{aligned} & m_p\ddot{s} + f_s\dot{s}\sin^2\delta \\ & +\varepsilon F_m(\ddot{x}, x, x)\cos\delta + ks = k_0(r\sin vt - s) \end{aligned} \qquad (1)$$

where m_p is the mass of the vibrating body, m_m is the mass of the material, f_s is the damping coefficient, other signs are shown in Fig.1. $\varepsilon F_m(\ddot{x}, x, x)$ is the nonlinear force by which the material acts on the vibrating body [3]:

$$F_m(\ddot{x}, x, x) = \begin{cases} m_m\ddot{s}\cos\delta, \text{when}\Phi_e - 2\pi \le \Phi \le \Phi_k, \Phi_m \le \Phi \le \Phi_Q, \\ -m_m f(g + \ddot{s}\sin\delta), \text{when } \Phi_k \le \Phi \le \Phi_m, \\ m_m f(g + \ddot{s}\sin\delta), \text{when } \Phi_Q \le \Phi \le \Phi_e, \end{cases} \qquad (2)$$

where f is the sliding friction coefficient between the material and vibrating body, Φ_k (or Φ_Q) and Φ_m (or Φ_e) are the phase angles of the beginning and ending of the material slip in the positive or negative direction.

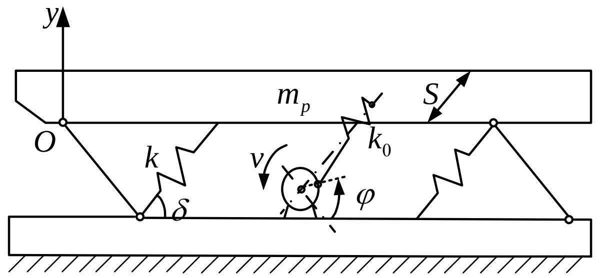

Fig. 1 The mechanical model of vibrating cooling machine with elastic connecting rod

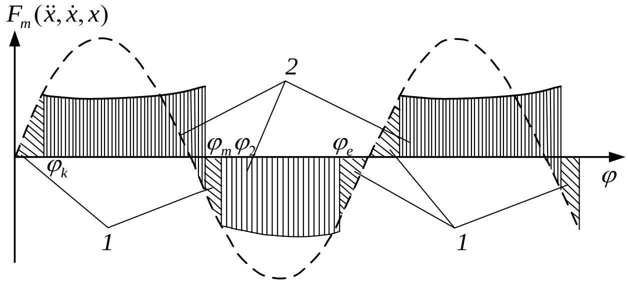

Fig. 2 Sectional inertial force and the sectional friction force
1. The sectional inertial force; 2. The sectional friction force

The vibrating machine, shown as Fig.2, works under the near resonance generally, hence we will find the approximation solution in the case of the main resonance, the first approximation solution may be shown as the following [1]:

$$s = a\cos(vt+\theta) = a\cos\psi$$
$$f_0(a,\psi) = \varepsilon F_m(-a\omega^2\cos\psi, -a\omega\sin\psi, a\cos\psi)\cos\delta$$
$$\frac{\mathrm{d}a}{\mathrm{d}t} = -\frac{f\,a}{2m_p} - \frac{\varepsilon}{2\pi m_p\omega}\int_0^{2\pi} f_0(a,\psi)\sin\psi\,\mathrm{d}\psi - \frac{\varepsilon k_o r\cos\theta}{m_p(\omega+v)a}$$
$$\frac{\mathrm{d}\theta}{\mathrm{d}t} = \omega - v - \frac{\varepsilon}{2\pi m_p a\omega}\int_0^{2\pi} f_0(a,\psi)\cos\psi\,\mathrm{d}\psi - \frac{\varepsilon k_o r\sin\theta}{m_p(\omega+v)a} \qquad (3)$$

When $\mathrm{d}a/\mathrm{d}t$=0, $\mathrm{d}\theta/\mathrm{d}t - 0$, we may find the amplitude a =const, and the phase $\theta =$const.

3 The Combination Coefficient and Damping Coefficient of Material under Sliding Condition

The combination coefficient and damping coefficient of material can be found from the following equations:

$$K_m = \sin^2\delta - \frac{b_1\cos\delta}{m_m v^2 a}, \quad f_m = \frac{a_1\cos\delta}{va} \qquad (4)$$

Where

$$a_1 = \frac{m_m v^2 a}{\pi}\{-\frac{1}{2}\cos\delta\sin^2\Phi(\Big|_{\Phi_e-2\pi}^{\Phi_k} - \Big|_{\Phi_m}^{\Phi_Q}) - f(\frac{g}{v^2 a}\sin\Phi - \frac{1}{2}\sin^2\Phi\sin\delta)(\Big|_{\Phi_k}^{\Phi_m} - \Big|_{\Phi_q}^{\Phi_e})$$
$$b_1 = \frac{m_m v^2 a}{\pi}\{-\frac{1}{2}\cos\delta(\Phi - \frac{1}{2}\sin 2\Phi)(\Big|_{\Phi_e-2\pi}^{\Phi_k} + \Big|_{\Phi_m}^{\Phi_q}) - f[\frac{g}{v^2 a}\cos\Phi - \frac{1}{2}\sin\delta(\Phi - \frac{1}{2}\sin 2\Phi)](\Big|_{\Phi_k}^{\Phi_m} - \Big|_{\Phi_q}^{\Phi_e}) \qquad (5)$$

After expanding the expression of nonlinear force $f_m(\ddot{x},x,x)\cos\delta$ into Fourier series expression, besides the term of first order of the harmonic wave, there are also higher harmonic wave terms, which cause the system to vibrate in higher harmonic waves.

We calculate the values of Φ_k, Φ_m, Φ_Q, $\Phi_e, a_1, b_1, K_m, f_m$, when a=13, 14.5, 16, 17.5mm, n=33 r/min, and $\delta = 22^\circ$ m the slope angle $\mu = 43^\circ 40'$. The combination coefficient of material will be from 0.4 to 0.7 and less than 1, and the damping coefficient of material will be from 0.25 m_m v to 0.33 m_m v and less than 1 m_m v.

Due to vibration the combination coefficient and damping coefficient of material under sliding condition will be decreased. Then the rub and wears of the machine's parts will be also reduced. It is very important to prolong the working life of the machines and their parts.

4 Nonlinear Vibration System with Throwing Material on Vibrating Body

Fig.3 shows the mechanical model of self - synchronous vibrating machines usually moves in the forma of throwing. Hence, when we build up the vibrating equations, these nonlinear forces of material should be considered. They consist of sectional inertial force, impact force and sectional friction force etc. So the vibrating equations in the direction y and x of the vibrating body are expressed as follows:

$$m_p\ddot{y} + f_y\dot{y} + k_y y + \varepsilon F_m(\ddot{y},y,y) = F_o\sin\delta\sin\Phi,$$
$$m_p\ddot{x} + f_x\dot{x} + k_x x + \varepsilon F_m(\ddot{x},x,x) = F_o\cos\delta\sin\Phi,$$

where $\varepsilon F_m(\ddot{y},y,y)$ and $\varepsilon F_m(\ddot{x},x,x)$ are nonlinear force in the direction y and x of the material. It may be expressed as follows:

$$\varepsilon F_m(\ddot{y},\dot{y},y)=\begin{cases} 0, & \text{when } \Phi_d<\Phi<\Phi_z, \\ m_m(\ddot{y}+g), & \text{when } \Phi_z-2\pi+\Delta\Phi<\Phi<\Phi_d \\ m_m(\dot{y}_m-\dot{y}_2)/\Delta t, & \text{when } \Phi_z\le\Phi\le\Phi_z+\Delta\Phi \end{cases}$$

$$\varepsilon F_m(\ddot{x},\dot{x},x)=\begin{cases} 0, & \text{when } \Phi_d<\Phi<\Phi_z \\ m_m\ddot{x}, \text{when} & \Phi'_{ij}\le\Phi\le\Phi''_{1j},\ j=1,2,3,\cdots \\ \pm f(g+-\ddot{y}\ )m_m, \text{when} & \Phi'_{2j}\le\Phi\le\Phi''_{2j} \\ -fm_m(\dot{y}_m-y_2)/\Delta t, \text{when} & \Phi_z\le\Phi\le\Phi_z+\Delta\Phi \end{cases} \tag{6}$$

Φ_d and Φ_z are the angles of jumping beginning and jumping ending, respectively. They are phase angles of the vibrating body, when the material begins to leave the vibrating body and drops on it again, y_m and y_z are, respectively, the instantaneous vertical velocities of the material on the vibrating body, when the material is dropping on the vibrating body, Δt is the impact time of the dropping material, $\Delta\Phi=v\Delta t, \Delta t$ is very short, so $\Delta\Phi\to 0$.

$$\begin{aligned} y&=a_y\cos\psi_y+\lambda_y\sin(vt-\alpha_y)+\varepsilon u_{1y}(a_y,\psi_y,vt)+\cdots, \\ x&=a_x\cos\psi_x+\lambda_x\sin(vt-\alpha_x)+\varepsilon u_{1x}(a_x,\psi_x,vt)+\cdots, \end{aligned} \tag{7}$$

where a_x, and ψ_y may be defined from the following equations:

$$\begin{aligned} \frac{\mathrm{d}a_y}{\mathrm{d}t}&=\frac{-f_y a_y}{\lambda_y m_p}+\varepsilon u_{1y}(a_y)+\cdots, \\ \frac{\mathrm{d}\psi_y}{\mathrm{d}t}&=\omega_y+\varepsilon B_{1y}(a_y)+\cdots, \\ \omega_y^2&=\frac{k_y}{m_p},\quad \lambda_y=\frac{F_o\sin\delta_o\cos\alpha_y}{k_y-(\frac{\varepsilon a_{y1}}{\lambda_y v^2}+m_p v^2)} \\ \alpha_y&=\mathrm{arctg}\frac{f_y+\varepsilon b_{y1}(\lambda_y v)}{k_y-(\frac{\varepsilon a_{y1}}{\lambda_y v^2}+m_p v^2)} \end{aligned} \tag{8}$$

A_{1y} and B_{1y} in Eq. 8 may be found form the following equations:

$$\begin{aligned} A_{1y}(a_y)&=\frac{-1}{4\pi^2\omega}\int_0^{2\pi}\int_0^{2\pi}F_m(a_y,\psi_y,\Phi_y)\sin\psi_y\mathrm{d}\Phi\mathrm{d}\psi_y \\ B_{1y}(a_y)&=\frac{-1}{4\pi^2\omega}\int_0^{2\pi}\int_0^{2\pi}F_m(a_y,\psi_y,\Phi_y)\cos\psi_y\mathrm{d}\Phi\mathrm{d}\psi_y \end{aligned} \tag{9}$$

a_{1y} and b_{y1} may be calculated from the following formulas:

$$\begin{aligned} b_{y1}&=\frac{1}{\pi}\int_0^{2\pi}\varepsilon F_m(\ddot{y},\dot{y})\sin\Phi\,\mathrm{d}\Phi=\frac{m_m v^2 a_y}{\pi}\{[-\frac{g}{v^2 a_y}\cos\Phi--\frac{1}{2}(\Phi-\frac{1}{2}\sin 2\Phi)]\Big|_{\Phi_z-2\pi}^{\Phi_d} \\ &+(\theta\sin\Phi_d-\cos\Phi_z+\cos\Phi_z)\sin\Phi_z\},\quad \theta=\Phi_z-\Phi_d \end{aligned} \tag{10}$$

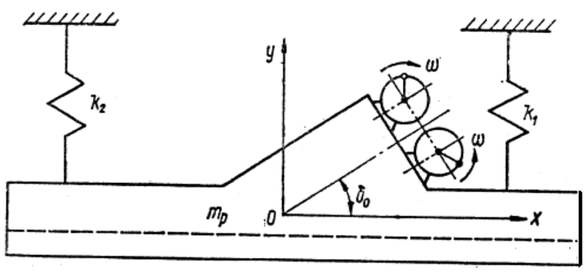

Fig. 3 The mechanical model of vibrating screen

Now we assume that the friction coefficient in direction x is much larger, after the material is dropped on the vibrations body; its velocity in direction x equals the velocity of vibrating body, the material will not slip. At that time the impact force in direction x of the material may be calculated by the principle of the impact, the Fourier coefficient will be:

$$a_1=\frac{1}{\pi}\int_0^{2\pi}\varepsilon F_m(\ddot{y},\dot{y})\sin\Phi\mathrm{d}\Phi=\frac{m_m v^2 c_x}{\pi}[-\frac{1}{2}\sin^2\Phi\Big|_{\Phi_z-2\pi}^{\Phi_d}+(-\cos\Phi_d+\cos\Phi_z)\cos\Phi_z]$$

$$b_{x1}=\frac{1}{\pi}\int_0^{2\pi}\varepsilon F_m(\ddot{x},\dot{x})\sin\Phi\,\mathrm{d}\Phi=\frac{m_m v^2\lambda_x}{\pi}\cdot[-\frac{1}{2}(\Phi-\sin 2\Phi)\Big|_{\Phi_z-2\pi}^{\Phi_d}+(-\cos\Phi_d+\cos\Phi_z)\sin\Phi_z]$$

5 The Combination Coefficient and Damping Coefficient of Material under Throwing Condition

The combination coefficient K_m and the damping coefficient f_m of the material can be calculated from the following equations:

$$K_m = K_{my}\sin^2\delta + K_{mx}\cos^2\delta$$

$$f_m = f_{my}\sin^2\delta + f_{mx}\cos^2\delta$$

$$K_{mx} = \frac{a_{x1}}{m_m v^2 \lambda_x}, \quad K_{my} = \frac{a_{y1}}{m_m v^2 \lambda_y} \tag{11}$$

$$f_{my} = \frac{\varepsilon b_{y1}}{v\lambda_y}, \quad f_{mx} = \frac{\varepsilon b_{x1}}{v\lambda_x}$$

Now taking the throwing exponent D=2.25, 2.5, 2.75, 3.00 for example, we only consider that the friction is much larger, and the slip may be omitted. Thus, according to [3] and the formulas obtained above, we may calculate $\Phi_d, \Phi_z, a_{y1}, a_{x1}, b_{x1}, K_m, f_m$. When D=2.25, K_m=0.14<<1， f_m=0.165 $m_m v$， when D=2.5, K_m=0.09<<1， f_m=0.18 $m_m v$， when D=2.75, K_m=0.041<<1， f_m=0.177 $m_m v$， when D=3.00, K_m=0<<1， f_m=0.165 $m_m v$.

Due to vibration the combination coefficient and damping coefficient of material under throwing condition will be decreased obviously. Then the rub and wears of the machine's parts will be also reduced. It is very important to prolong the working life of the machines and their parts.

6 Conclusions

The combination coefficient will be decreased with the increasing of the amplitude, so the vibrating equation considering the nonlinear force of material has nonlinear character.

The combination coefficient under sliding condition The combination coefficient K_m of material will be from 0.7 to 0.4<1, and the damping coefficient f_m of material will be from 0.33 $m_m v$ to 0.25$m_m v$

The combination coefficient under throwing condition K_m<0.3<1 （D<3） and K_m<0<<1 （D=3～3.3）and the damping coefficient of the materials f_m=0.177 $m_m v$～165 $m_m v$.

Due to vibration the combination coefficient and damping coefficient of material under sliding and throwing condition will be decreased obviously. Then the rub and wears of the machine's parts will be also reduced. It is very important to prolong the working life of the machines and their parts.

Acknowledgments

This research is supported by Chinese National Natural Science Foundation (Grant No.50535010, 10572036).

References:

[1] Bogolyuboo N., Mitropolsky Yu. A. Asymptotic Methods in Theory of Nonlinear Oscillation. –Fizmatgiz. Moscow, 1958, (in Russian).\-408. p.

[2] Wen B.C, Li Y.N. Zhang Y. M., Song J.W. Vibration Utilization Engineering. Science Press, 2005 (in Chinese)

[3] Wen B. C., Liu S.Y., He Q., Theory and Dynamic Design Methods of Vibrating Machinery, Mechanical Industry Press, 2001 (in Chinese)

[4] Wen B.C., Liu F.Q. Theory of Vibrating Machinery and Its Application, Mechanical Industry Press, 1982 (in Chinese)

[5] Wen B. C., Liu F.Q.,Liu Ji. Design and Adjustment of Vibrating Machinery, Chemical industry Press, 1989 (in Chinese)

[6] Wen B. C., LI Y. N., Xu P. M., Han Q. K. Engineering Non-linear Vibration. Science Press, 2007 (in Chinese)

[7] Inoue J., Mechanical Dynamics, Scientific and Technological Press, 1982.(in Japanese)

[8] Hoormann W. Ober den Einflub des Fordergutes auf das Betriebsver Halten von Schwingriner durch Dampfung und Massenankopplung,《Fordern und Heben》18 Jahrgang April 1968.