Design and Modeling of Integral Control State-feedback Controller for Implementation on Servomotor Control

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Abstract: - This paper presents the design and modeling for servomotor position control using state-space technique. The aim of this paper is to compare the control performance between State-feedback controller with integral control and State-feedback controller without integral control. A mathematical model of the system is derived and verified by SIMULINK/MATLAB. Full-state feedback controller with integral control and full-state feedback controller without integral control are proposed for the controller structures. The performances between these two structures are analyzed. Simulation results are given for performance verification. The paper is organized as follows. Section 1 provides brief introduction on the project. In section 2, a mathematical model of servomotor is discussed where the system is considered as a second order system. In section 3, state-space modeling for continuous and discrete time system is presented. Subsequently in section 4, full-state feedback controller using pole-placement design is derived. Case studies are provided in section 5 where simulation analyses are thoroughly discussed. Conclusions are given in section 6. It is expected that the State-feedback controller with integral control gives better performance as compared to State-feedback controller without integral control and can further be implemented using DDC via GUI.

Key-words: Full-state feedback control, Bass and Gura's approach, Graphical user interface, State-space, Servomotor, Direct digital control, Integral control.

1 Introduction

Direct digital control (DDC) is one form of the automatic control where DDC is termed as using a digital computer to interface directly with a plant or system as the control device [1]. The disparity between DDC and indirect digital control (supervisory control) is it does not require for any additional hardware to implement the controller. Everything from controller's algorithms or structures in terms of codes and scripts can be manipulated and constructed inside the computer by the help of software.

The overall objectives of this project is to create and develop a graphical user interface (GUI) using Microsoft Visual Basic 6.0 that implements DDC and state-space technique in controlling servomotor shaft position. In designing the controller structures, full-state feedback with and without integral control [2] where pole-placement design via Bass and Gura's approach [3] is proposed. The full-state feedback controller via pole-placement is chosen since it has the best performance compared to other controllers in terms of oscillation and settling time [4]. Moreover, the pole-placement design could also handle systems with time-varying state space representation [5], or systems with multiple operating conditions [6], as well as systems with multi-input-multi-output (MIMO) signals requirement [7]. Normally, controller design for linear time-varying differential systems is generally a difficult problem, because of the fundamental problems related to the analysis of such systems [5]. For simplicity, the servomotor system will be analyzed as a linear time-invariant (LTI) system, with only single-input and single output condition. This is due to the method has the properties of the flexibility of shaping the dynamics of the closed-loop system to meet the desired specifications [8].

2 Modeling of Servomotor System

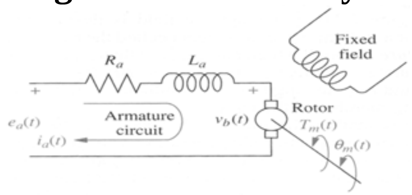


Fig.1: Schematic of servomotor system

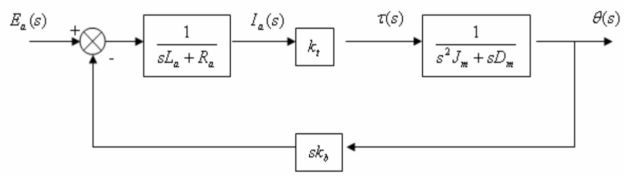


Fig.2: Time domain block-diagram representation of servomotor system.

Fig.1 above shows the schematic of the servomotor, meanwhile Fig.2 is the frequency-domain open loop block diagram constructed based on this schematic. Based on the block diagram of Fig.2, by ignoring the armature inductance of the system, the open loop transfer function for a second order armature controlled servomotor system is given as follows:

$$\frac{\theta_m(s)}{E_a(s)} = \frac{\left(\frac{k_t}{R_a J_m}\right)}{s^2 + s\frac{1}{J_m}\left(D_m + \frac{k_t k_b}{R_a}\right)} \tag{1}$$

where

J_m = equivalent inertia by the motor = 30 x 10^{-6} kgm^2
k_b = back-emf constant = 60 x 10^{-3} $Vsrad^{-1}$
k_t = motor torque constant = 17 x 10^{-3} NmA^{-1}
R_a = armature resistance = 3.2 Ω
D_m = equivalent viscous density by the motor =small (cannot be quoted)

The armature inductance is ignored for simplicity of equation and controller derivation so that we'll only be dealing with second order system.By substituting all the parameters into the equation (1), the open loop transfer function for the second order armature controlled servomotor system could be obtained as follows:

$$G(s) = \frac{\theta_m(s)}{E_a(s)} = \frac{177.0833}{s^2 + 10.625s} \tag{2}$$

3 State-Space Modeling

3.1 Modeling in continuous time

Based on the block diagram of the servomotor as shown in Fig.2, the state variables for second order system are defined as:

$x_1(t) = \theta_m(t)$ = angular displacement of the motor shaft

$x_2(t) = \frac{d\theta_m(t)}{dt}$ = angular velocity of the motor shaft

Meanwhile, the state input and state output for the second order system are defined as:

$u(t) = e_a(t)$ = input signal into the servomotor

$y(t) = x_1(t)$ = output signal from the servomotor

Let $\dot{x}_1(t) = \frac{dx_1(t)}{dt} = \frac{d\theta_m(t)}{dt} = x_2(t)$ (3)

Now let $\dot{x}_2(t) = \frac{dx_2(t)}{dt} = \frac{d^2\theta_m(t)}{dt^2}$ (4)

By deriving equation (2), it could be obtained that

$$\dot{x}_2(t) = -\left[\frac{1}{J_m}\left(D_m + \frac{k_t k_b}{R_a}\right)\right]x_2(t) + \frac{k_t}{R_a J_m}e_a(t) \tag{5}$$

Therefore, the state-space representation of servomotor in space matrix could be expressed in this form:

$$\begin{bmatrix}\dot{x}_1(t)\\ \dot{x}_2(t)\end{bmatrix} = \begin{bmatrix}0 & 1\\ 0 & -\frac{1}{J_m}\left(D_m + \frac{k_t k_b}{R_a}\right)\end{bmatrix}\begin{bmatrix}x_1(t)\\ x_2(t)\end{bmatrix} + \begin{bmatrix}0\\ \frac{k_t}{R_a J_m}\end{bmatrix}e_a(t) \tag{6}$$

and
$$y(t) = \begin{bmatrix}1 & 0\end{bmatrix}\begin{bmatrix}x_1(t)\\ x_2(t)\end{bmatrix} \tag{7}$$

3.2 Modeling in discrete time

From equation (6), it could be re-written into

$$\begin{bmatrix}\dot{x}_1(t)\\ \dot{x}_2(t)\end{bmatrix} = \begin{bmatrix}0 & 1\\ 0 & -M\end{bmatrix}\begin{bmatrix}x_1(t)\\ x_2(t)\end{bmatrix} + \begin{bmatrix}0\\ N\end{bmatrix}e_a(t) \tag{8}$$

where M and N are defined as:

$$M = \frac{D_m R_a + k_b k_t}{J_m R_a} \tag{9}$$

$$N = \frac{k_t}{J_m R_a} \tag{10}$$

Lets the dynamic of the linear continuous-time servomotor system be represented by the following state and output equations respectively:

$$\dot{x}(t) = Fx(t) + Gr(t) \tag{11}$$

$$y(t) = Cx(t) + Dr(t) \tag{12}$$

where $F = \begin{bmatrix} 0 & 1 \\ 0 & -M \end{bmatrix}, G = \begin{bmatrix} 0 \\ N \end{bmatrix}$ and $C = \begin{bmatrix} 1 & 0 \end{bmatrix}$, D=0

The system matrix $A(T)$ and output matrix $B(T)$ for discrete time servomotor system can be easily determined as follows [9]:

$$A(T) = e^{Ft}\,\big|_{t=T} = \left| L^{-1}\left(SI - F\right)^{-1} \right|_{t=T} \qquad (13)$$

$$B(T) = \int_0^T e^{F\tau} G\, d\tau = \int_0^T \left| L^{-1}\left(SI - F\right)^{-1} \right|_{t=\tau} G\, d\tau$$

$$= \int_0^T A(\tau) G\, d\tau \qquad (14)$$

Therefore, the state-space representation of servomotor in discrete time space matrix could be expressed in this form:

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} = \begin{bmatrix} 1 & \frac{\left(1-e^{(-MT)}\right)}{M} \\ 0 & e^{(-MT)} \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + \begin{bmatrix} \left(\frac{N}{M}\right)T - \frac{N}{M^2}\left(1+e^{(-MT)}\right) \\ \frac{N}{M}\left(1-e^{(-MT)}\right) \end{bmatrix} e_a(k) \qquad (15)$$

and $$y(k) = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} \qquad (16)$$

4 Full-state Feedback Controller Design

4.1 State feedback without integral control

The concept of feed-backing all the state variables back to the input of the system through a suitable feedback matrix in the control strategy is known as the full-state variable feedback control technique. Using this approach, the desired location of the closed-loop eigenvalues (poles) of the system will be specified. Thus, the aim is to design a feedback controller that will move some or all of the open-loop poles of the measured system to the desired closed-loop pole location as specified. Hence, this approach is often known as the *pole-placement control* design. In this paper, pole-placement technique via Bass and Gura's formula is proposed.

In order to perform the pole-placement design technique, the system must be a "completely state controllable" system. In other words, it is possible to move all the of system's open-loop poles by state variable feedback, to any arbitrary closed-loop locations. Therefore, before designing the controller, a test has to be performed on the system matrix where by checking its rank; it shall be equal to the number of the column vector. Then it can be concluded that the system is completely state controllable. Otherwise, another controller design has to be performed.

Fig.3 below shows detailed block diagram of system with state feedback control.

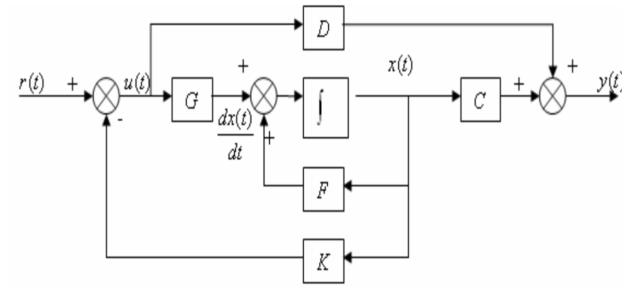


Fig.3: Detailed block diagram of system with state feedback control

The general state space equation for the block diagram in the Figure 4.1 above is derived as:

$$\dot{x}(t) = \left[F - GK\right]x(t) + Gr(t) \qquad (17)$$

Bass and Gura's formula to determine the state feedback gain matrix is given as follows [3]:

$$K^T = \left[W^T S^T\right]^{-1}\left(a - \tilde{a}\right) \qquad (18)$$

where

K =State feedback gain matrix

$S = \begin{bmatrix} G & FG & \cdots & F_{n-1}G \end{bmatrix}$=Controllability matrix

$a = \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{bmatrix}$=Coefficients of desired characteristic equation.

$\tilde{a} = \begin{bmatrix} \tilde{a}_1 \\ \tilde{a}_2 \\ \vdots \\ \tilde{a}_n \end{bmatrix}$=Coefficients of system characteristic equation

$$W = \begin{bmatrix} 1 & \tilde{a}_1 & \cdots & \tilde{a}_{n-1} \\ 0 & 1 & \cdots & \tilde{a}_{n-2} \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \cdots & 1 \end{bmatrix}$$

4.2 State feedback with integral control

Normally, designing the state feedback controller by using only the pole-placement design will give one major disadvantage where large steady-state error will be introduced. Therefore, in order to compensate for this problem, an integral control is added where it will eliminate the steady-state error in the response to the step input.

Fig.4 shows the block diagram of the system with the integral control added into it. In the dashed box is the state feedback controller which is designed before. A feedback path from the output has been added to the error, e, which is fed forward to the controlled via an integrator. The main function of adding an integrator is to increase the system type thus reduces the previous finite steady-state error to zero. Therefore, a design for zero steady state error for a step input can be obtained.

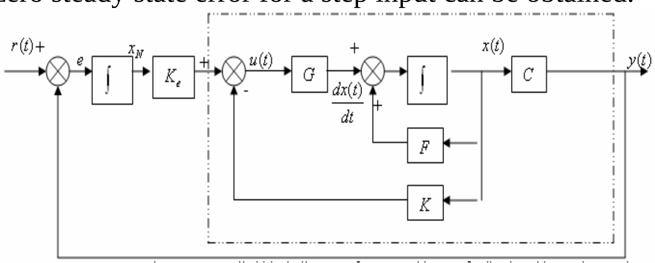


Fig.4: Detailed block diagram of system with state feedback and integral control.

From the block diagram above, let the system matrix with integral control be given as [2]:

$$\begin{bmatrix} \dot{x}(t) \\ \dot{x}_N(t) \end{bmatrix} = \begin{bmatrix} F & 0 \\ -C & 0 \end{bmatrix} \begin{bmatrix} x(t) \\ x_N(t) \end{bmatrix} + \begin{bmatrix} G \\ 0 \end{bmatrix} u(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} r(t) \quad (19)$$

and $y(t) = \begin{bmatrix} C & 0 \end{bmatrix} \begin{bmatrix} x \\ x_N \end{bmatrix}$ (20)

However, from Fig.4, realize that

$$u(t) = -Kx(t) + K_e x_N = -\begin{bmatrix} K & -K_e \end{bmatrix} \begin{bmatrix} x \\ x_N \end{bmatrix} \quad (21)$$

Therefore, the final derivation for the system matrix with the integral control is as follows:

$$\begin{bmatrix} \dot{x}(t) \\ \dot{x}_N(t) \end{bmatrix} = \begin{bmatrix} (F-GK) & GK_e \\ -C & 0 \end{bmatrix} \begin{bmatrix} x(t) \\ x_N(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} r(t) \quad (22)$$

and $y(t) = \begin{bmatrix} C & 0 \end{bmatrix} \begin{bmatrix} x \\ x_N \end{bmatrix}$ (23)

In order to implement the Bass and Gura's formula to find the state feedback gain matrix for the state-space system with integral control, some equation modification has to be performed.

Let $F' = \begin{bmatrix} F & 0 \\ -C & 0 \end{bmatrix}$ and $G' = \begin{bmatrix} G \\ 0 \end{bmatrix}$ (24) & (25)

The controllability matrix, S', can be obtained using the F' and G' above. Now, the state feedback gain matrix can be obtained by rewriting equation (18) as follows:

$$K'^T = \left[W'^T S'^T \right]^{-1} \left(a' - \tilde{a}' \right) \quad (26)$$

Therefore, $K = \begin{bmatrix} K'_1 & K'_2 & \cdots & K'_{n-1} \end{bmatrix}$ (27)

and the gain integral can obtained is given as:

$$K_e = -K'_n \quad (28)$$

5 Simulation Analysis

Fig.5 shows the continuous time hardware realization for the servomotor position control system with the: (a) state feedback controller without integral control (b) state feedback controller with integral control. Simulations are performed for both controllers' structure where a unit step input (5-volts step) signal is used as the reference signal. To accomplish one of the design requirements, it is desired that the output signals to follow the given reference signals. Through simulation, the mathematical modeling for the servomotor is verified and the performances for both controllers' structures are analyzed.

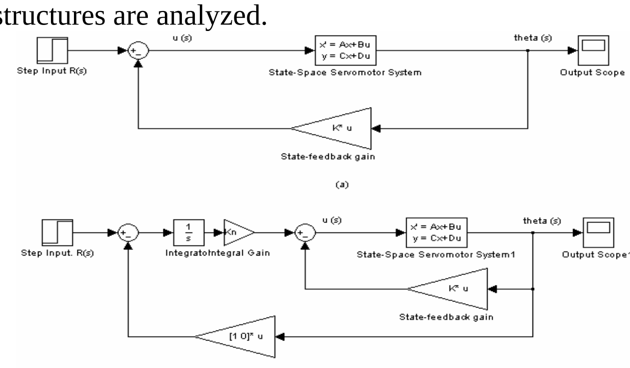


Fig.5: State feedback controller (a) without integral control (b) with integral control

5.1 Simulation results

In order to analyze the performances of the proposed controllers, the system is simulated using MATLAB/SIMULINK. For this paper, simulation results for two cases study are presented for discussion.

5.1.1 State feedback controller with integral control

Case study 1: To obtain output response with 10% maximum percentage overshoot, 2 seconds of settling time and 0 volt initial condition where state feedback controller with integral control is used..

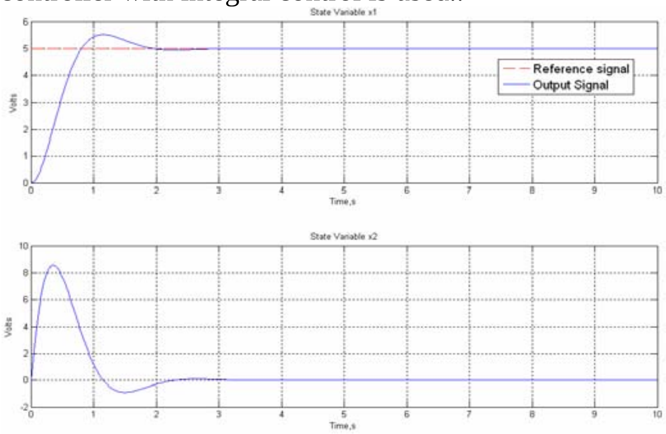


Fig.6: Simulation result for case study 1

The simulation result for case study 1 is shown in Fig.6, meanwhile Table 1 shows for its performance. From Table 1, it can be seen that the output response for simulation result of case study 1 is obtained with absolute discrepancy of 0.44% for percentage maximum overshoot and 1s for settling time.

Table 1: Performance result for case study 1

Performance	Desired	Actual	|Discrepancy|
% maximum overshoot	10	10.44	0.44
Settling time (s)	2	3	1
%Steady-state error	0	0	0

5.1.2 State feedback controller without integral control

Case study 2: To obtain output response with 10% maximum percentage overshoot, 2 seconds of settling time and 0 volt initial condition where state feedback controller without integral control is used.

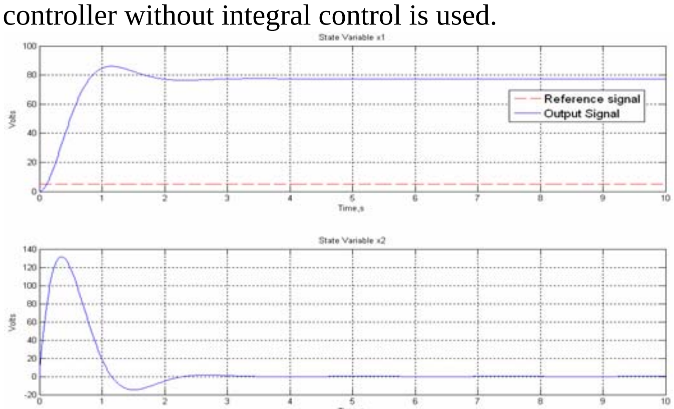


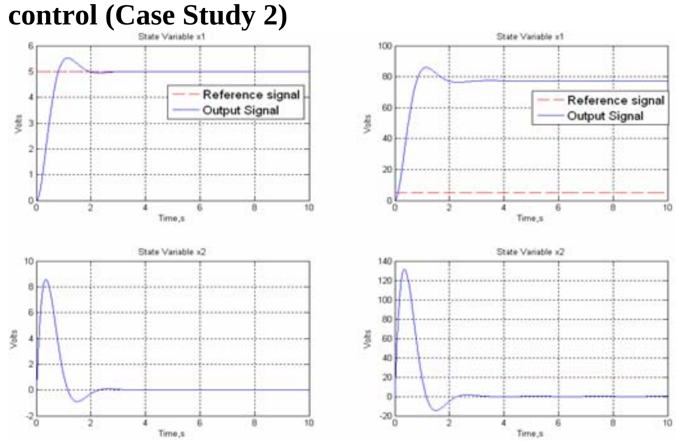
Fig.7: Simulation result for case study 2

The simulation result for case study 2 is shown in Fig.7, meanwhile Table 2 shows for its performance. From Table 2, it can be seen that the output response for simulation result of case study 2 is obtained with absolute discrepancy of 1.1% for percentage maximum overshoot and also 1s for settling time. However, the output response of case study 2 exhibits very large percentage steady state error.

Table 2: Performance result for case study 2

Performance	Desired	Actual	|Discrepancy|
% maximum overshoot	10	11.1	1.1
Settling time (s)	2	3	1
%Steady-state error	0	1447.2	1447.2

5.1.3 Comparison of State feedback controller with integral control (Case Study 1) and without integral control (Case Study 2)



(a) (b)

Fig.8: Comparison of simulation result for state feedback controller (a) with integral control (b) without integral control.

Fig.8 shows the comparison of simulation results for case study 1 and 2. Meanwhile Fig.9 shows the comparison only on the output response. The performance comparison between these cases study is shown in Table 3 below:

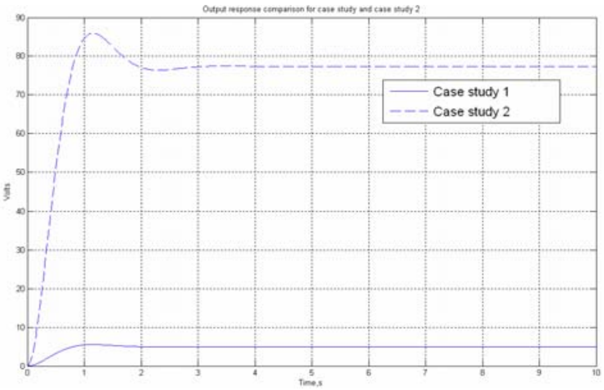


Fig.9: Output response comparison for case study 1 and case study 2

Table 3: Performance comparison of simulation results between case study 1 and case study 2.

Performance	Case Study 1	Case Study 2	|Discrepancy|
% maximum overshoot	10.44	11.1	0.66
Settling time (s)	3	3	0
%Steady-state error	0	1447.2	1447.2

From Table 3, it can be concluded that the state feedback controller with integral control gives better performance in terms of percentage steady-state error. This is due to the fact that the state feedback controller without integral control produce very large percentage steady state error as can be observed in Fig.9.

5.2 Discussion

Two cases study are presented to study the performances of state-feedback controller with integral control and state feedback controller without integral control. Based on the simulation result, both give comparatively equal performance in terms of percentage maximum overshoot and settling time. However, if we were to compare in terms of percentage steady state error, based on Fig.9 and Table 3 it can be seen that the state feedback controller with integral control exhibits better performance compared to state feedback controller without the integral control.

6 Conclusion

The performances of State feedback controller with integral control and State feedback controller without integral control are studied in two cases study. For both cases, it is desired that the output response to obtain a certain percentage maximum overshoot with specified settling time and minimum percentage steady state error. Based on these criteria, it is analyzed that both state feedback controller with integral control and state feedback controller without integral control give equal performances in terms of maximum percentage overshoot and settling time but state feedback controller with integral control exhibits better performance in terms of percentage overshoot. As a conclusion, the state feedback controller with integral control is better than the state feedback controller without the integral control, thus state feedback controller with integral controller structures can be further be implemented using DDC via GUI.

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