

# An Experimental Analysis of an Active Magnetic Bearing System Using PID-Type Fuzzy Controllers with Parameter Adaptive Methods

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**Abstract:** - This paper deals with the experimental control of a rotating active magnetic bearing (AMB) system using PID-type fuzzy controllers (PIDFCs) with parameter adaptive methods. There are three kinds of parameter adaptive methods, including fuzzy tuner, function tuner, and relative rate observer, have been proposed in literatures for tuning the coefficients of PIDFCs. However, only a simulation comparison between these methods for control of a second-order linear system with varying parameters and time delay has been done in literatures. In general, theoretical models need to be confirmed and modified through experimental results. This paper provides experimental verification by applying PIDFCs with self-tuning algorithms for control of a highly nonlinear AMB system.

**Key-Words:** - PID-type fuzzy controllers, parameter adaptive methods, self-tuning scaling factors, active magnetic bearing

## 1 Introduction

AMB systems can support rotors without any contact, provide high rotational speed, no lubrication, low energy consumption, maintenance-free operation, and are useful in special environments such as high temperature or vacuum. Magnetic suspension systems are unstable by nature; so to guarantee stability they need feedback control. In recent years, nonlinear control techniques have been proposed [1]-[3] for AMB systems that include sliding mode, feedback linearization, and hybrid control to improve disturbance rejection properties and their robustness to unmodeled dynamics and parameter uncertainties. In practical systems, however, it is difficult to achieve the fast switching control that is generally required to implement most sliding mode control designs. The drawback of feedback linearization is that it is necessary to know the whole states of a nonlinear system before the controller is designed. Besides, feedback linearization is sensitive to modeling error that results from the fact that an exact model of a nonlinear system is generally not available.

In recent years, there has been growing interest in using fuzzy logic for control of AMB systems. Hung [4] designed a nonlinear controller for a dual-acting

magnetic bearing by using fuzzy reasoning to adjust the output of a linear PID controller. Hong *et al.* [5] proposed a fuzzy logic control scheme for an AMB system subject to harmonic disturbances. Even though these types of FLC applications were successfully used for a number of complex and nonlinear systems, many researchers still attempt to propose more efficient FLCs such as PIDFCs to replace conventional FLCs for most control systems. In general, the tuning parameters of PIDFCs, including proportional gain, integral gain, derivative gain, and scaling factors (SFs), can be calculated during on-line adjustments of the controller to improve the process performance. Of the various tunable parameters, input and output SFs have the highest priority due to their global effect on the control performance [6].

Most of the real processes are nonlinear high-order systems and may have considerable dead-time. Sometimes their parameters may randomly change with time or with changes in the ambient environments. Hence, only static or fixed valued SFs of PIDFCs may not be sufficient to provide optimal performance and robustness against both process disturbances and modeling errors for controlling nonlinear systems. To overcome this, a lot of research works on tuning input

and output SFs of PIDFCs by on-line self-tuning schemes have been reported. Chung *et al.* [7] developed a method for self-tuning both input and output SFs of a PI-type fuzzy controller via a fuzzy tuner that uses only seven tuning rules. Mudi *et al.* [6] proposed a robust self-tuning scheme of the output SF only for fuzzy PI- and PD-type controllers, considering that it is equivalent to the controller gain. Woo *et al.* [8] presented another parameter adaptive method using a function tuner. Güzelkaya *et al.* [9] developed a parameter adaptive method to adjust SFs  $K_D$  and  $\beta$  using a fuzzy inference mechanism in an on-line manner.

As mentioned above, we can summarize the self-tuning PIDFCs within three groups, such as (1) adjusting SFs via fuzzy inference mechanism [6], [7], (2) adjusting SFs via function tuner [8], and (3) adjusting SFs via relative rate observer [9]. In this paper, we focus our attention on the three groups of self-tuning PIDFCs for the control of an AMB system. Furthermore, experimental results of this paper provide comparative evaluation of these self-tuning methods.

## 2 PIDFC Structures

### 2.1 PIDFCs without tuning mechanism

Let us consider the following controller structure that simply connects the PD- and PI-type fuzzy controllers together in parallel as shown in Fig. 1(a). The output of the PIDFC is given by

$$\begin{aligned} u &= u_{PD} + u_{PI} = \alpha U + \beta \int U dt \\ &= \alpha(A + PK_e e + DK_d \dot{e}) + \beta \int (A + PK_e e + DK_d \dot{e}) dt \\ &= \alpha A + \beta A t + (\alpha K_e P + \beta K_d D) e + \beta K_e P \int e dt + \alpha K_d D \dot{e}, \quad (1) \end{aligned}$$

where  $\alpha K_e P + \beta K_d D$ ,  $\beta K_e P$ , and  $\alpha K_d D$  are the equivalent proportional, integral, and derivative gains, respectively. In (1), the relation between the input and output variables of the FLC is given by  $U = A + PE + D\dot{E}$ , where  $E = K_e e$  and  $\dot{E} = K_d \dot{e}$ .

Among various inference methods used in the PIDFC found in [6]-[9], the most widely used ones can be divided into two types: Mamdani type [10] and Takagi-Sugeno type [11]. The MFs for error  $E$  and derivative of error  $\dot{E}$  of the Takagi-Sugeno method are shown in Fig. 1(b) [9]. The rule base for computing  $U$  is shown in Table 1.

### 2.2 PIDFCs with self-tuning mechanisms

Some self-tuning mechanisms have been proposed in literatures for improving the performance of PIDFCs given in the previous section. Three of those methods will be considered in some detail below.

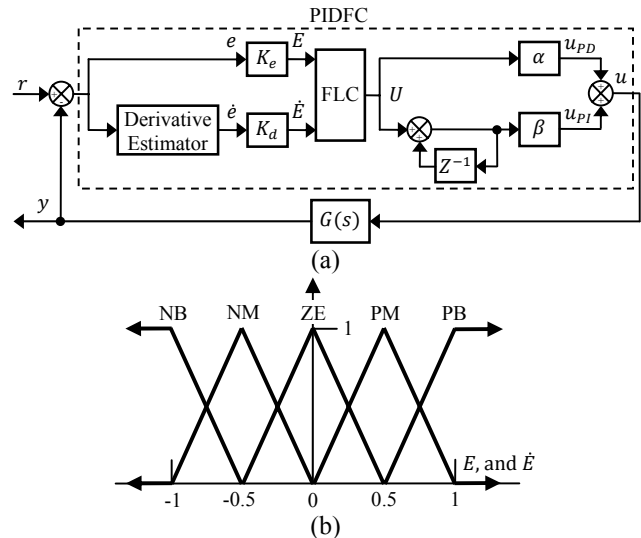


Fig. 1 (a) The standard PIDFC without tuning mechanism. (b) The MFs of  $E$  and  $\dot{E}$ .

Table 1 Fuzzy rule base for computing  $U$

		$E$				
		NB	NM	ZE	PM	PB
$\dot{E}$	NB	-1	-0.7	-0.5	-0.3	0
	NM	-0.7	-0.4	-0.2	0	0.3
	ZE	-0.5	-0.2	0	0.2	0.5
	PM	-0.3	0	0.2	0.4	0.7
	PB	0	0.3	0.5	0.7	1

#### 2.2.1 Fuzzy gain tuning mechanism

Mudi *et al.* [6] proposed a parameter adaptive method for PI- and PD-type FLCs using a fuzzy gain tuning mechanism. Of the various tunable parameters, SFs have the highest priority due to their global effect on the control performance. Hence, they proposed that PI- or PD-type FLC is tuned by modifying the output SF of an existing FLC, which was described to be a self-tuning FLC. Here, the output SF does not remain fixed while the controller is in operation, which is modified in each sampling time by a gain updating factor ( $\gamma$ ), depending on the trend of the controlled process output. The gain updating factor  $\gamma$  was computed on-line using a model independent fuzzy rule base. The block diagram of the self-tuning PIDFC using the fuzzy gain tuning mechanism and the MFs for  $\gamma$  are shown in Fig. 2. The rule base for computing  $\gamma$  is shown in Table 2.

Chung *et al.* [7] developed a method for self-tuning both input and output SFs of a Takagi-Sugeno type fuzzy PI controller via a fuzzy tuner that uses only seven tuning rules. In this paper, as compared with the self-tuning PIDFC using the fuzzy gain tuning mechanism, we consider the PIDFC with the parameter adaptive method proposed by Chung and his associates to tune output SFs only. The structure of the self-tuning PIDFC with such kind of fuzzy tuner is shown in Fig. 3(a). The output SF of the fuzzy tuner is given by

$$G(r, \gamma) = (1 + 1.5\gamma) \cdot r \cdot w, \quad (2)$$

where  $\gamma$  is the output variable of the fuzzy inference system,  $r$  is the set-point, and  $w$  is the convergent coefficient. The MFs for the input variable  $R$  are chosen as triangular functions, as shown in Fig. 3(b), and a crisp output has been used, where  $R = |e/r|$ . Table 3 shows the tuning rules for computation of output variable  $\gamma$ .

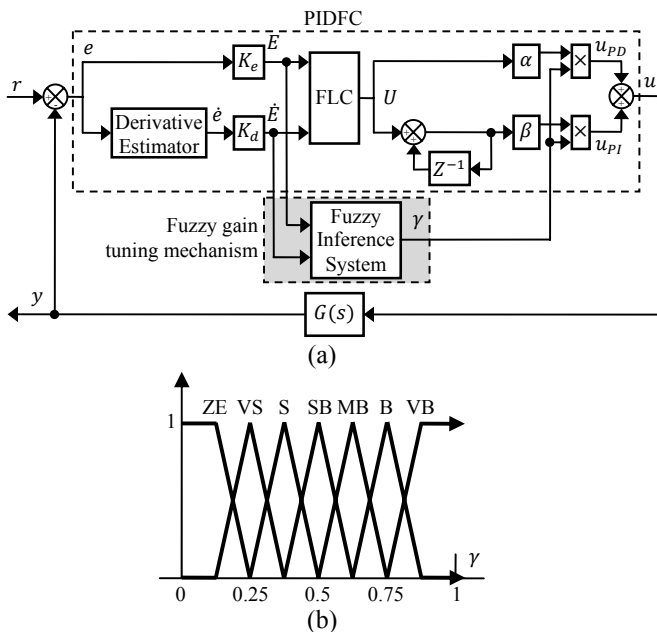


Fig. 2 (a) The self-tuning PIDFC using the fuzzy gain tuning mechanism. (b) The MFs of  $\gamma$ .

Table 2 Fuzzy rule base for computation of  $\gamma$

		$E$						
		NB	NM	NS	ZE	PS	PM	PB
$\dot{E}$	NB	VB	VB	VB	B	SB	S	ZE
	NM	VB	VB	B	B	MB	S	VS
	NS	VB	MB	B	VB	VS	S	VS
	ZE	S	SB	MB	ZE	MB	SB	S
	PS	VS	S	VS	VB	B	MB	VB
	PM	VS	S	MB	B	B	VB	VB
	PB	ZE	S	SB	B	VB	VB	VB

### 2.2.2 Function tuner

Parameter adaptive PIDFC using a function tuner has been proposed by Woo *et al.* [8]. The function tuner tunes the controller parameters  $K_d$  and  $\beta$  simultaneously with time. The algorithm for tuning these parameters is as follows:

$$\beta = \beta_s \cdot f(e), \text{ and} \quad (3)$$

$$K_d = K_{ds} \cdot g(e), \quad (4)$$

where  $\beta_s$  and  $K_{ds}$  are the initial values of  $\beta$  and  $K_d$ , respectively. The empirical functions  $f(e)$  and  $g(e)$  are defined, respectively, by

$$f(e) = a_1 \cdot |e| + a_2, \text{ and} \quad (5)$$

$$g(e) = b_1 \cdot (1 - |e|) + b_2, \quad (6)$$

where  $a_1, a_2, b_1,$  and  $b_2$  are all positive constants. When the error  $e$  decreases, the function  $f(e)$  related to integral factor  $\beta$  decreases and the function  $g(e)$  related to derivative factor  $K_d$  increases. The block diagram of the PIDFC with self-tuning mechanism is shown in Fig. 4.

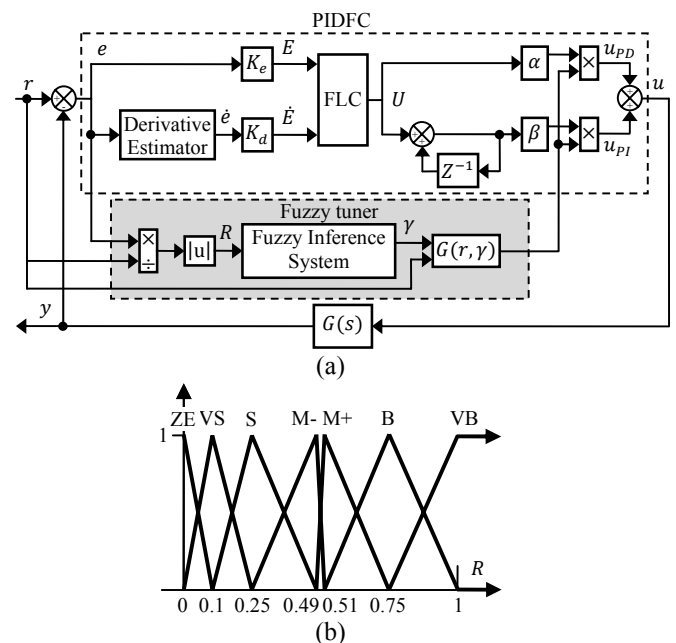


Fig. 3 (a) The self-tuning PIDFC using the fuzzy tuner. (b) The MFs of  $R$ .

Table 3 Fuzzy rule base for computation of  $\gamma$

	$R$						
	<b>ZE</b>	<b>VS</b>	<b>S</b>	<b>M-</b>	<b>M+</b>	<b>B</b>	<b>VB</b>
$\gamma$	0	0	-0.33	-0.66	0	0.66	1

**2.2.3 Relative rate observer (RRO)**

Güzelkaya *et al.* [9] proposed a parameter adaptive method to adjust  $K_d$  and  $\beta$  of the PIDFC using a fuzzy parameter regulator (FPR). The fuzzy parameter regulator has two inputs: one of which is the absolute value of error  $|e|$  and the other one is normalized acceleration  $r_v$ . The output variable of the fuzzy parameter regulator is designated as  $\gamma$ . The normalized acceleration  $r_v(k)$  is defined as

$$r_v(k) = K_{rv} \frac{de(k) - de(k-1)}{de(\cdot)} = K_{rv} \frac{dde(k)}{de(\cdot)}, \quad (7)$$

where  $de(k)$  is the incremental change in error given by  $de(k) = e(k) - e(k - 1)$ ,  $dde(k)$  is the acceleration in error given by  $dde(k) = de(k) - de(k - 1)$ , and  $K_{rv}$  is the SF for  $r_v(k)$ . In (7),  $de(\cdot)$  is the maximum change of  $de(k)$  and the previous value  $de(k - 1)$  designated as follows:

$$de(\cdot) = \begin{cases} de(k), & |de(k)| \geq |de(k - 1)| \\ de(k - 1), & |de(k)| < |de(k - 1)| \end{cases} \quad (8)$$

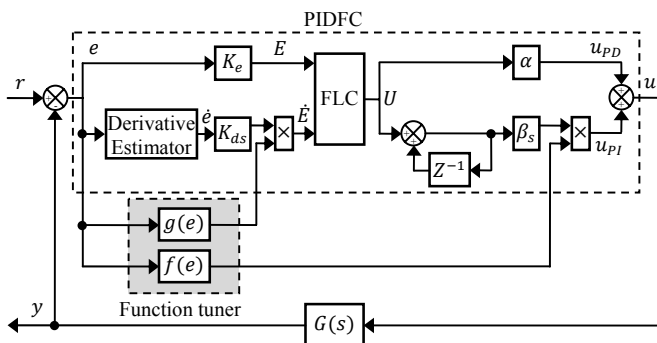


Fig. 4 Block diagram of the self-tuning PIDFC using the function tuner.

The block diagram of the controller structure is shown in Fig. 5(a). Here, the input and output scaling factors  $K_d$  and  $\beta$  for the FLC are adjusted by multiplying and dividing its predetermined value by  $\gamma$ , respectively, as given below:

$$K_d = K_{ds} \cdot K_{fd} \cdot K_f \cdot \gamma, \text{ and} \quad (9)$$

$$\beta = \frac{\beta_s}{K_f \cdot \gamma}, \quad (10)$$

where  $K_{ds}$  and  $\beta_s$  are the initial values of  $K_d$  and  $\beta$ , respectively,  $K_f$  is the output SF for the fuzzy parameter regulator, and  $K_{fd}$  is the additional parameter that affects only the input SF  $K_d$  corresponding to the derivative of error  $\dot{e}$  for the FLC.

The MFs for the input and output variables  $r_v$ ,  $|e|$ , and  $\gamma$  are shown in Fig. 5(b) and (c). Table 4 shows the tuning rules for computation of output variable  $\gamma$ .

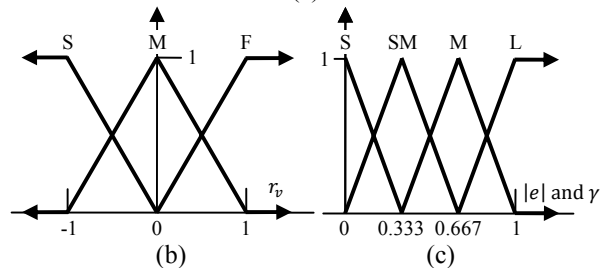
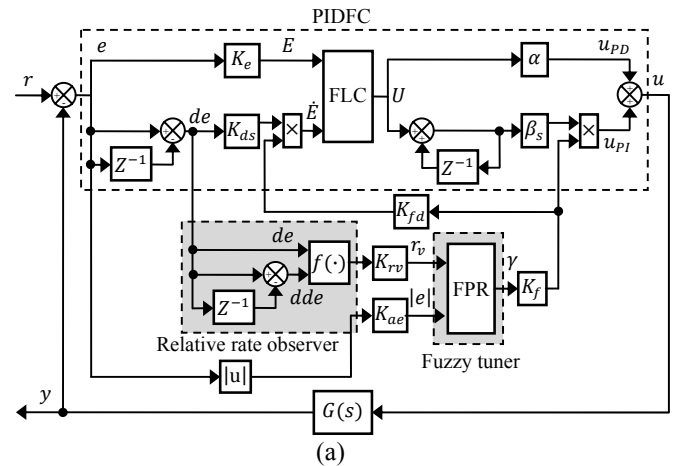


Fig. 5 (a) Block diagram of the self-tuning PIDFC using the relative rate observer. (b) The MFs of  $r_v$ . (c) The MFs of  $|e|$  and  $\gamma$ .

Table 4 Fuzzy rule base for computation of  $\gamma$

	$r_v$		
	<b>S</b>	<b>M</b>	<b>F</b>
$ e $	<b>S</b>	<b>M</b>	<b>L</b>
	<b>SM</b>	<b>M</b>	<b>L</b>
	<b>M</b>	<b>S</b>	<b>SM</b>
	<b>L</b>	<b>S</b>	<b>SM</b>

**3 Magnetic Bearing System**

The experimental setup used in this paper is a two-axis controlled horizontal shaft magnetic bearing with symmetric structure, as shown in Fig. 6. The magnetic bearing has four identical electromagnets equally spaced radially around a rotor disk which is made of laminated stainless steel. Each electromagnet consists

of a coil and a laminated core which is made of silicon steel [12]. The magnetic forces  $f_x$  and  $f_y$  due to the electromagnets in the  $x$ -axis (horizontal) and the  $y$ -axis (vertical) can be modeled by the following equations, respectively [2],

$$f_x = k \left[ \left( \frac{i_0 + i_x}{g - x} \right)^2 - \left( \frac{i_0 - i_x}{g + x} \right)^2 \right], \text{ and} \quad (11)$$

$$f_y = k \left[ \left( \frac{i_0 + i_y}{g - y} \right)^2 - \left( \frac{i_0 - i_y}{g + y} \right)^2 \right], \quad (12)$$

where  $k$  is the electromagnet constant,  $i_0$  is the bias current in the coils,  $g$  is the nominal air gap,  $i_x$  and  $i_y$  are the control current, and  $x$  and  $y$  are the displacements in the  $x$ - and  $y$ -axes, respectively. In equations (11) and (12), the magnetic forces  $f_x$  and  $f_y$  are proportional to the square of current and inversely proportional to the square of the air gap displacement. A photograph of the magnetic bearing system is shown in Fig. 6.

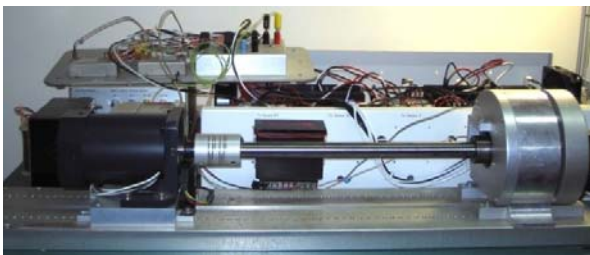


Fig. 6 The experimental setup of the AMB system.

#### 4 Experimental Results

As discussed in Section 2, the two most widely used FISs are the Mamdani and the Takagi-Sugeno type, and the three types of parameter adaptive methods are fuzzy tuner, function tuner, and RRO. Therefore we construct six experiment schemes of self-tuning FPIDCs for the AMB system. The results of six experiments are shown in Figs. 7-9. As shown in Fig. 7, (a1), (b1) to (f1) show the position error of the rotor center in  $y$ -direction when the rotor is at 0 Hz, and (a2), (b2) to (f2) show the trajectories of the rotor center when the rotor is at 0 Hz. As shown in Fig. 8, (a1), (b1) to (f1) show the position error of the rotor center in  $y$ -direction when the rotor is at its highest rotation frequency, and (a2), (b2) to (f2) show the trajectories of the rotor center when the rotor is at its highest rotation frequency.

There are some phenomena obtained from observing the experimental results. First, the levitation

performance of the AMB system, especially in rotation, using the PIDFC construct with Mamdani type FIS is worse than using the Takagi-Sugeno type PIDFC. By observing the difference between Figs. 7 and 8, in general, the process of defuzzification via the Mamdani type FIS will reduce the computation efficiency, so the AMB system performance is worse. Secondly, as observed in Fig. 9, the first mode resonant frequency of the AMB system in rotation is changed with using different controller structure. The first mode resonant frequencies for using Takagi-Sugeno type self-tuning PIDFC are at around 30, 60, and 70 Hz, respectively. Also, we can observe that if the rotation frequency of the AMB system can pass the first mode resonant frequency successfully, the position error will decrease as the rotation frequency grows high. Namely, in three parameter adaptive methods, the control performance via RRO method is better than the other two methods because the first mode resonant frequency occurs at around 30 to 70 Hz as the frequency increases. Before the second mode resonant frequency occurs, the rotation frequency of the AMB system will reach a higher value than those obtained from the other parameter adaptive methods.

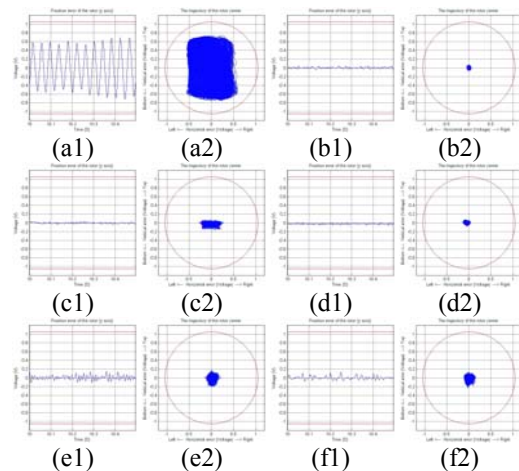


Fig. 7 Position error in  $y$ -axis and orbit of rotor center of six experiments at 0 Hz. (a) No. 1. (b) No. 2. (c) No. 3. (d) No. 4. (e) No. 5. (f) No. 6.

#### 5 Discussions and Conclusions

In this paper, we use two standard PIDFCs, constructed by two major types of fuzzy inference systems: the Mamdani and the Takagi-Sugeno type, to integrate three kinds of parameter adaptive methods proposed in the literature, including fuzzy tuner, function tuner, and

RRO for control of the nonlinear magnetic bearing system. In addition, we design a series of experiments for comparing the control performance of these methods. There are two main conclusions obtained by observing the experimental results. First, in two standard PIDFCs, the Takagi-Sugeno type FIS is better than the Mamdani type FIS in both system performance and computation efficiency in rotation. Second, in three kinds of parameter adaptive methods, the RRO can provide the highest rotor rotation frequency of the AMB system and the smallest average position errors of the rotor center than those provided by the other two methods.

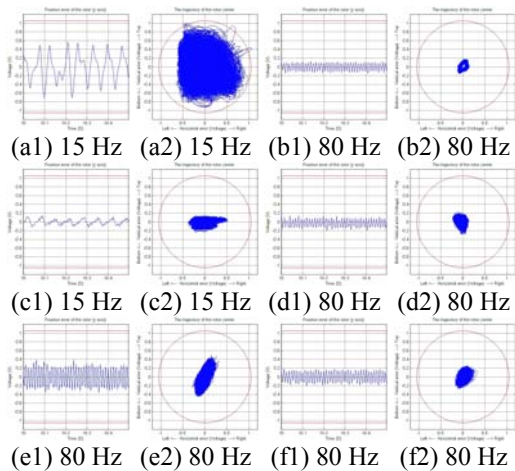


Fig. 8 Position error in  $y$ -axis and orbit of rotor center of six experiments at its highest rotation frequency. (a) No. 1. (b) No. 2. (c) No. 3. (d) No. 4. (e) No. 5. (f) No. 6.

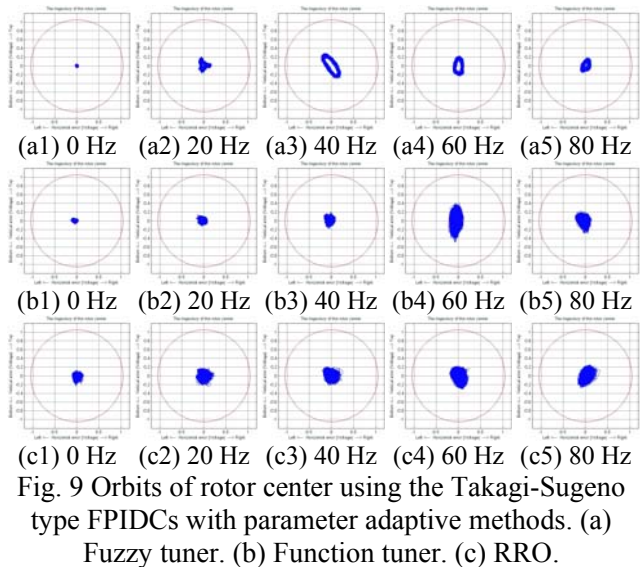


Fig. 9 Orbits of rotor center using the Takagi-Sugeno type FPIDCs with parameter adaptive methods. (a) Fuzzy tuner. (b) Function tuner. (c) RRO.

#### Acknowledgement

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