# MODIFIED EXPLICIT DECOUPLED GROUP METHOD IN THE SOLUTION OF 2-D ELLIPTIC PDES

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*Abstract:* - In this paper, the formulation of a new explicit group method in solving the two-dimensional Poisson equation is presented. The method is derived from a skewed (rotated) five-point finite difference discretisation which results in a reduced system with lower computational complexity compared to schemes derived from the standard five-point difference approximation. The details of the algorithm will be discussed. Numerical experimentations will be conducted and comparison with the common existing schemes is reported.

*Key-Words:* - finite difference method, group explicit method, Poisson equation, partial differential equation, five-point operator

## 1 Introduction

Fast computational methods for solving partial differential equations using finite difference schemes derived from skewed (rotated) difference operators have been extensively investigated over the years ([1], [2], [3], [4], [5], [10], [11], [12]). The methods have several advantages such as they are explicit in nature so that parallelism is favorable and require lesser execution timings than the schemes derived from the standard five-point difference operators without jeorpadising the order of accuracies. The use of this skewed difference formula [8] leads to schemes with lower computational complexities since the iterative procedure need only involve nodes on half of the total grid points in the solution domain and thus a reduced system of linear equations is attained. In [1] in particular, a group iterative scheme, the Explicit Decoupled Group (EDG) scheme, was developed by Abdullah (1991) as a more efficient Poisson solver on rotated grids by using small fixed size group strategy which was shown to be more economical computationally than the Explicit Group (EG) scheme due to Yousif and Evans [9]. In 2000, Othman and Abdullah [7] modified the formulation of the EG method by altering the ordering of grid points taken in the iterative process to come up with the modified four-point EG where this method (MEG) was shown to be more superior in timings than both the original EG and EDG methods. In this paper, we extend the idea of the modification to the EDG method to investigate whether this strategy is able to produce a more improved scheme in solving the two-dimensional Poisson equation.

Section 2 gives an overview of the explicit group methods under study. Section 3 describes the formulation of the modified EDG method followed by the computational complexity in Section 4. The numerical experiments and results are presented in Section 5. The conclusion is given in Section 6.

## 2 The Group Iterative Methods

Consider the two dimensional Poisson equation

$$u_{xx} + u_{yy} = f(x,y), \quad (x,y) \in \Omega, \tag{1}$$

with a Dirichlet boundary condition. on $\partial\Omega$, where $\partial\Omega$ is the boundary of the unit square solution domain [$0 \le x,y \le 1$]. Let $\Omega$ be discretized uniformly in both x and y directions with a mesh size $h = 1/n$, where $n$ is an arbitrary positive integer. The solutions of $(n\text{-}1)^2$ internal mesh points $(x,y)$ can be approximated by various finite difference schemes. Two such approximations are the standard five-point difference formula

$$4u_{i,j} - u_{i+1,j} - u_{i-1,j} - u_{i,j+1} - u_{i,j-1} = -h^2 f_{i,j} \tag{2}$$

and the rotated five-point difference formula

$$4u_{i,j} - u_{i+1,j+1} - u_{i-1,j-1} - u_{i-1,j+1} - u_{i+1,j-1} = -2h^2 f_{i,j} \tag{3}$$

Eq. (3) is obtained by rotating the i-plane axis and the j-plane axis clockwise by 45° [1]. Applying Eq. (2) to groups of four points will result in the following (4x4) system

$$\begin{bmatrix} 4 & -1 & 0 & -1 \\ -1 & 4 & -1 & 0 \\ 0 & -1 & 4 & -1 \\ -1 & 0 & -1 & 4 \end{bmatrix} \begin{bmatrix} u_{i,j} \\ u_{i+1,j} \\ u_{i+1,j+1} \\ u_{i,j+1} \end{bmatrix} = \begin{bmatrix} u_{i-1,j} + u_{i,j-1} - h^2 f_{i,j} \\ u_{i+2,j} + u_{i+1,j-1} - h^2 f_{i+1,j} \\ u_{i+2,j+1} + u_{i+1,j+2} - h^2 f_{i+1,j+1} \\ u_{i-1,j+1} + u_{i,j+2} - h^2 f_{i,j+1} \end{bmatrix} \quad (4)$$

which can be easily inverted to produce a four-point EG equation [9]

$$\begin{bmatrix} u_{ij} \\ u_{i+1,j} \\ u_{i+1,j+1} \\ u_{i,j+1} \end{bmatrix} = \frac{1}{24} \begin{bmatrix} 7 & 2 & 1 & 2 \\ 2 & 7 & 2 & 1 \\ 1 & 2 & 7 & 2 \\ 2 & 1 & 2 & 7 \end{bmatrix} \begin{bmatrix} u_{i-1,j} + u_{i,j-1} - h^2 f_{ij} \\ u_{i+2,j} + u_{i+1,j-1} - h^2 f_{i+1,j} \\ u_{i+2,j+1} + u_{i+1,j+2} - h^2 f_{i+1,j+1} \\ u_{i-1,j+1} + u_{i,j+2} - h^2 f_{i,j+1} \end{bmatrix} \quad (5)$$

Similarly, applying Eq. (3) to groups of four points of the solution domain will result in a (4x4) system of equations

$$\begin{bmatrix} 4 & -1 & 0 & 0 \\ -1 & 4 & 0 & 0 \\ 0 & 0 & 4 & -1 \\ 0 & 0 & -1 & 4 \end{bmatrix} \begin{bmatrix} u_{i,j} \\ u_{i+1,j+1} \\ u_{i+1,j} \\ u_{i,j+1} \end{bmatrix} = \begin{bmatrix} u_{i-1,j-1} + u_{i+1,j-1} + u_{i-1,j+1} - 2h^2 f_{i,j} \\ u_{i,j+2} + u_{i+2,j} + u_{i+2,j+2} - 2h^2 f_{i+1,j+1} \\ u_{i,j-1} + u_{i+2,j-1} + u_{i+2,j+1} - 2h^2 f_{i+1,j} \\ u_{i-1,j+2} + u_{i-1,j} + u_{i+1,j+2} - 2h^2 f_{i,j+1} \end{bmatrix} \quad (6)$$

which can be written in a decoupled system of (2x2) equations whose explicit forms [1] are given by

$$\begin{bmatrix} u_{i,j} \\ u_{i+1,j+1} \end{bmatrix} = \frac{1}{15} \begin{bmatrix} 4 & 1 \\ 1 & 4 \end{bmatrix} \begin{bmatrix} u_{i-1,j-1} + u_{i+1,j-1} + u_{i-1,j+1} - 2h^2 f_{i,j} \\ u_{i,j+2} + u_{i+2,j} + u_{i+2,j+2} - 2h^2 f_{i+1,j+1} \end{bmatrix} \quad (7)$$

and

$$\begin{bmatrix} u_{i+1,j} \\ u_{i,j+1} \end{bmatrix} = \frac{1}{15} \begin{bmatrix} 4 & 1 \\ 1 & 4 \end{bmatrix} \begin{bmatrix} u_{i,j-1} + u_{i+2,j-1} + u_{i-2,j+1} - 2h^2 f_{i+1,j} \\ u_{i-1,j+2} + u_{i+1,j} + u_{i+1,j+2} - 2h^2 f_{i,j+1} \end{bmatrix} \quad (8)$$

From Fig. 1, it may be observed that the evaluation of Eq. (7) involves points of type ● (including the ungrouped ○ points) only, while Eq. (8) can be evaluated involving points of type ▲ only. Thus, the calculations of Eq. (7) and (8) can be carried out independently which may save the execution time by nearly half if the iteration is carried out on only one type of points; either the type ● or ▲. Fig. 1 shows the various types of points involved if iteration is carried out on ● points (grouped) using Eq. (7) and on ○ points (ungrouped) using Eq. (3) for $n$=14. After convergence is achieved, the solutions at the other remaining half of the points (▲) are computed directly once using the standard five-point formula of Eq. (2). By this way, the execution time of this EDG method will be reduced to half of the execution time of EG method.

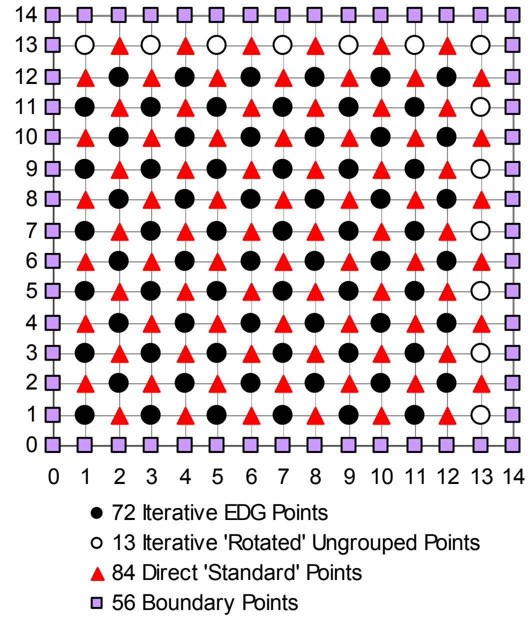


Fig. 1. Type of points in EDG method for $n$=14

## 3 Modified EDG Method

We now modify the EDG method described in the previous section by considering points at grid size of $2h = 2/n$. Applying centered difference equation on these $2h$ spaced points results in the following standard five-point formula (with spacing $2h$)

$$4u_{i,j} - u_{i+2,j} - u_{i-2,j} - u_{i,j+2} - u_{i,j-2} = -4h^2 f_{i,j} \quad (9)$$

Rotating the x-y axis clockwise 45° and applying centered difference equation on these points will result in the following rotated five-point difference formula (with spacing $2h$)

$$4u_{i,j} - u_{i+2,j+2} - u_{i-2,j-2} - u_{i-2,j+2} - u_{i+2,j-2} = -8h^2 f_{i,j} \quad (10)$$

Now we apply Eq. (10) to groups of four points as shown in Fig. 2 and produce the following (4x4) system of equations

$$\begin{bmatrix} 4 & -1 & 0 & 0 \\ -1 & 4 & 0 & 0 \\ 0 & 0 & 4 & -1 \\ 0 & 0 & -1 & 4 \end{bmatrix} \begin{bmatrix} u_{i,j} \\ u_{i+2,j+2} \\ u_{i+2,j} \\ u_{i,j+2} \end{bmatrix} = \begin{bmatrix} u_{i-2,j-2} + u_{i+2,j-2} + u_{i-2,j+2} - 8h^2 f_{i,j} \\ u_{i,j+4} + u_{i+4,j} + u_{i+4,j+4} - 8h^2 f_{i+2,j+2} \\ u_{i,j-2} + u_{i+4,j-2} + u_{i+4,j+2} - 8h^2 f_{i+2,j} \\ u_{i-2,j+4} + u_{i-2,j} + u_{i+2,j+4} - 8h^2 f_{i,j+2} \end{bmatrix} \quad (11)$$

which can be inverted and rewritten in explicit forms of a decoupled system of (2x2) equations as

$$\begin{bmatrix} u_{i,j} \\ u_{i+2,j+2} \end{bmatrix} = \frac{1}{15} \begin{bmatrix} 4 & 1 \\ 1 & 4 \end{bmatrix} \begin{bmatrix} u_{i-2,j-2} + u_{i+2,j-2} + u_{i-2,j+2} - 8h^2 f_{i,j} \\ u_{i,j+4} + u_{i+4,j} + u_{i+4,j+4} - 8h^2 f_{i+2,j+2} \end{bmatrix} \quad (12)$$

and

$$\begin{bmatrix} u_{i+2,j} \\ u_{i,j+2} \end{bmatrix} = \frac{1}{15}\begin{bmatrix} 4 & 1 \\ 1 & 4 \end{bmatrix}\begin{bmatrix} u_{i,j\text{-}2} + u_{i+4,j\text{-}2} + u_{i\text{-}4,j+2} - 8h^2 f_{i+2,j} \\ u_{i\text{-}2,j+4} + u_{i+2,j} + u_{i+2,j+4} - 8h^2 f_{i,j+2} \end{bmatrix} \quad (13)$$

Similar to the original EDG method, the evaluation of Eq. (12) and Eq. (13) can be performed independently. Fig. 3 shows the discretization points of a unit square domain with $n$=14 and the various types of points involved. It is obvious that the evaluation of Eq. (12) involves only points of type ● and Eq. (13) only points of type ▲. In this paper we solve points of type ● iteratively using Eq. (12) until convergence after which the points of type ▲ is computed directly once using the standard 2$h$ spaced five-point formula of Eq. (9). The remaining in-between points of type □ are also computed directly once using the rotated five-point difference formula of Eq. (3), and followed by points of type ◇ using the standard five-point difference formula of Eq. (2).

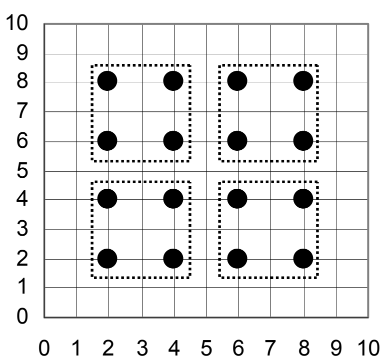


Fig. 2. Group of four points with 2$h$ spacing

We define the four point MEDG method with successive over-relaxation (SOR) iterative scheme as follows.

1. Divide the solution domain into five types of points as shown in Fig. 3 (for the case $n$=14).
2. Group all the 2$h$ spaced ● and ▲ points into four-point groups.
3. Iterate the intermediate solution of points ● in each group using
$$\begin{bmatrix} \hat{u}^{(k+1)}_{i,j} \\ \hat{u}^{(k+1)}_{i+2,j+2} \end{bmatrix} = \frac{1}{15}\begin{bmatrix} 4 & 1 \\ 1 & 4 \end{bmatrix}\begin{bmatrix} u^{(k+1)}_{i\text{-}2,j\text{-}2} + u^{(k+1)}_{i+2,j-2} + u^{(k)}_{i-2,j+2} - 8h^2 f_{i,j} \\ u^{(k)}_{i,j+4} + u^{(k)}_{i+4,j} + u^{(k)}_{i+4,j+4} - 8h^2 f_{i+2,j+2} \end{bmatrix}$$
and implement the relaxation scheme
$$\begin{bmatrix} u^{(k+1)}_{i,j} \\ u^{(k+1)}_{i+2,j+2} \end{bmatrix} = \begin{bmatrix} u^{(k)}_{i,j} \\ u^{(k)}_{i+2,j+2} \end{bmatrix} + \omega\left(\begin{bmatrix} \hat{u}^{(k+1)}_{i,j} \\ \hat{u}^{(k+1)}_{i+2,j+2} \end{bmatrix} - \begin{bmatrix} u^{(k)}_{i,j} \\ u^{(k)}_{i+2,j+2} \end{bmatrix}\right)$$
4. If solution converged, go to step 5. Otherwise, repeat the iteration step 3.
5. Evaluate the remaining points in this sequence:
   a. points of type ▲
   $$u_{i,j} = \frac{1}{4}\left(u_{i+2,j} + u_{i-2,j} + u_{i,j+2} + u_{i,j-2} - 4h^2 f_{i,j}\right)$$
   b. points of type □
   $$u_{i,j} = \frac{1}{4}\left(u_{i+1,j+1} + u_{i-1,j-1} + u_{i-1,j+1} + u_{i+1,j-1} - 2h^2 f_{i,j}\right)$$
   c. points of type ◇
   $$u_{i,j} = \frac{1}{4}\left(u_{i+1,j} + u_{i-1,j} + u_{i,j+1} + u_{i,j-1} - h^2 f_{i,j}\right)$$

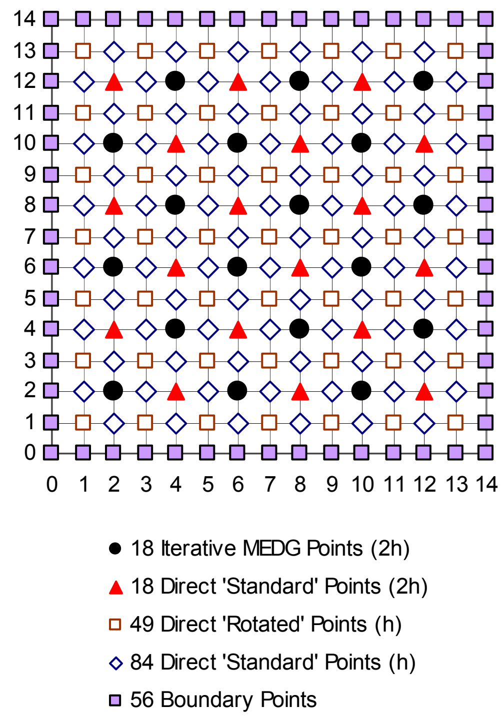


Fig. 3. Type of points in MEDG methods for $n$=14

## 4 Computational Complexity

We develop here the computational complexity for the MEDG method based on the algorithm defined in the previous section. Likewise, the computational complexity for other explicit group methods (EG, EDG and MEG) are also derived based on the similar iterative scheme. First we need to derive the number of various points involved in the whole solution process. Assume the solution domain is discretized with even integer $n$, then the number of internal mesh points is given by $m^2$ where $m$=$n$-1. There are two main type of internal mesh points namely, iterative points which are points involved in the iteration process, and direct points which solutions are computed directly from a specific difference formula once after the iteration process converged. Table 1 lists the number of points for the various internal mesh points for the proposed MEDG method as well as for the other existing explicit group methods.

Next we estimate the computational complexity of MEDG method in terms of arithmetic operations performed in an iteration (excluding the convergence test). From Eq. (12) we obtain

$$\begin{bmatrix} u_{i,j} \\ u_{i+2,j+2} \end{bmatrix} = \frac{1}{15}\begin{bmatrix} 4 & 1 \\ 1 & 4 \end{bmatrix}\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} \quad (14)$$

where

$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} u_{i-2,j-2} + u_{i+2,j-2} + u_{i-2,j+2} - 8h^2 f_{i,j} \\ u_{i,j+4} + u_{i+4,j} + u_{i+4,j+4} - 8h^2 f_{i+2,j+2} \end{bmatrix} \quad (15)$$

Thus, the updated equation for an individual point, say $u_{ij}$ , is given by

$$\tilde{u}_{i,j} = \frac{1}{15}\left[4b_1 + b_2\right] \quad (16)$$

and the SOR iterative scheme is given by

$$u_{i,j}^{(k+1)} = u_{i,j}^{(k)} + \omega\,(\tilde{u}_{i,j} - u_{i,j}^{(k)}) \quad (17)$$

Since the update is done in groups of two points, the values of $b_1$ and $b_2$ calculated only once before computing those two points of $u$. This results in 6 additions (*adds*) and 3 multiplications (*mults*) for a single point, provided the constants $^1/_{15}$ and $8h^2 f_{ij}$ are computed and stored beforehand. Similarly it can be shown that the computational cost of a single iterative grouped point in EG and MEG methods is 7 *adds*+4 *mults*, and the EDG method 6 *adds*+3 *mults*. Further details of the complexity analysis for the EG and EDG methods can be found in [6].

The cost of computing an iterative ungrouped point in EG and EDG methods using either standard or rotated five-point difference formula is 6 *adds*+2 *mults* while the direct solution after convergence requires 4 *adds*+1 *mult* per point. Hence, the number of arithmetic operations required in an iteration and in the direct solution after convergence for the EG, EDG, MEG and MEDG methods in term of $m$ can be derived and this is given in Table 2.

It is clear from Table 2 that the computational cost of MEDG is the least among the four-point explicit groups methods. For large $m$, it is approximately 10%, 25% and 40% of the computational cost for EG, EDG and MEG methods respectively.

Table 1. Number of various mesh points in the explicit group methods

| Point types | Number of points | | | |
|---|---|---|---|---|
| | EG | EDG | MEG | MEDG |
| Iterative grouped points | $(m-1)^2$ | $(m-1)^2/2$ | $(m-1)^2/4$ | $(m-1)^2/8$ |
| Iterative ungrouped points | $2m-1$ | $m$ | - | - |
| Total iterative points, *ip* | $m^2$ | $(m^2+1)/2$ | $(m-1)^2/4$ | $(m-1)^2/8$ |
| Direct $2h$ spaced 'standard' points | - | - | - | $(m-1)^2/8$ |
| Direct h spaced 'rotated' points | - | - | $(m+1)^2/4$ | $(m+1)^2/4$ |
| Direct h spaced 'standard' points | - | $(m^2-1)/2$ | $(m^2-1)/2$ | $(m^2-1)/2$ |
| Total direct points, *dp* | - | $(m^2-1)/2$ | $(3m^2+2m-1)/4$ | $(7m^2+2m-1)/8$ |
| Total internal points, *ip*+*dp* | $m^2$ | $m^2$ | $m^2$ | $m^2$ |

Table 2. The computing cost for the explicit group methods.

| Method | Per Iteration | | After convergence | |
|---|---|---|---|---|
| | *adds* | *mults* | *adds* | *mults* |
| EG | $7(m-1)^2 + 6(2m-1)$ | $4(m-1)^2 + 2(2m-1)$ | - | - |
| EDG | $3(m-1)^2 + 6m$ | $3(m-1)^2/2 + 2m$ | $2(m^2-1)$ | $(m^2-1)/2$ |
| MEG | $7(m-1)^2/4$ | $(m-1)^2$ | $3m^2+2m-1$ | $(3m^2+2m-1)/4$ |
| MEDG | $3(m-1)^2/4$ | $3(m-1)^2/8$ | $(7m^2+2m-1)/2$ | $(7m^2+2m-1)/8$ |

## 5 Experimental Results

In order to compare the proposed method with the other explicit group methods, experiments were carried out on a PC with Pentium 4 2.80 GHz, 512 MB RAM running Windows XP Pro using Borland C++. All four methods were run to solve the following model problem

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = (x^2 + y^2)e^{xy}, \quad (x, y) \in \Omega = [0,1] \times [0,1] \quad (18)$$

with Dirichlet boundary conditions satisfying the exact solution $u(x, y) = e^{xy}, \quad (x, y) \in \partial\Omega$.

The theoretical optimum relaxation factor $\omega_o$ for implementing the group SOR iterative scheme can be computed from the formula

$$\omega_o = \frac{2}{1 + \sqrt{1 - \rho^2(B)}} \quad (19)$$

where $\rho(B)$ is the spectral radius of the group Jacobian iterative matrix which can be estimated for EG, EDG and MEG methods as $1 - \pi^2 h^2$, $1 - \sqrt{2}\pi^2 h^2$ and $1 - 4\pi^2 h^2$ respectively ([1], [7], [9]). For the MEDG method, the spectral radius can be estimated as

$$\rho(B) \approx 1 - 4\sqrt{2}\pi^2 h^2. \quad (20)$$

The theoretical number of iteration to converge with the error tolerance $\varepsilon$ can then be estimated as

$$\kappa_o \approx \frac{\ln \varepsilon}{\ln(\omega_o - 1)} \quad (21)$$

All methods were run with several mesh sizes of 194, 242 and 482. The experimental values of relaxation factor $\omega_e$ for the various mesh sizes were obtained within ±0.0001 by choosing the ones that resulted in the least number of iterations. The convergence criteria used was the $l_\infty$ norm with error tolerance $\varepsilon \le 10^{-5}$. The results are given in Table 3 together with the computed theoretical values.

It is clearly seen from Table 3 that the MEDG method is the fastest method among the four explicit group methods. The timings obtained as shown in Table 3 and Fig. 4 show that the execution times of MEDG is only about 10% and 15% of those of EG and EDG methods respectively, while MEG is about 14% and 22% of EG and EDG execution times. This shows that MEDG outperforms MEG in terms of computing time saving. Furthermore, MEDG also exhibits better accuracy in all cases observed.

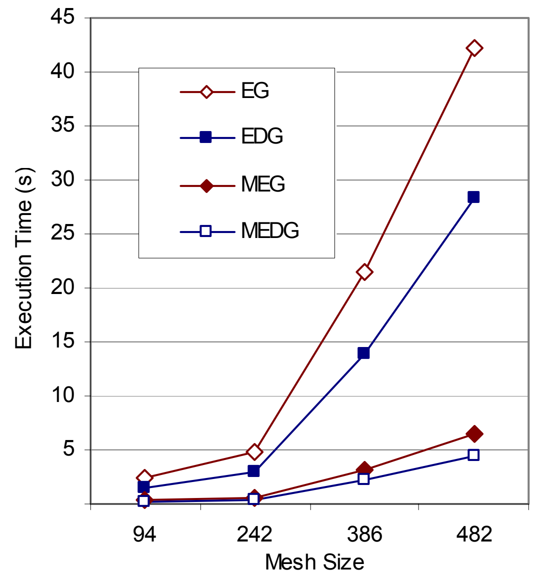


Fig. 4. Execution times (in secs.) on various mesh sizes

## 6 Conclusion

We have presented a preliminary work on the MEDG method as a faster Poisson solver compared to the existing explicit group methods derived from the standard and rotated five-point formulas. The theoretical computational complexity analysis of the MEDG method is found to be in agreement with the experimental execution time obtained. It is also observed that the accuracy of the proposed method is maintained as good as the existing schemes even though the domain grid size for the iterative solution is doubled. While the methods presented here are sequential, the parallel versions are still under investigation and will be reported soon.

Table 3. Theoretical and experimental results of the EG, EDG, MEG and MEDG methods with red-black ordering strategy

| | | Theoretical | | | Experimental | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Sizes | Methods | $\rho$ | $\omega_o$ | $\kappa_o$ | $\omega_e$ | $\kappa_e$ | MaxError | AveError | ExecTime |
| 194 | EG | 0.9997 | 1.9552 | 252 | 1.9553 | 268 | 1.18E-4 | 4.98E-5 | 2.3600 |
| 194 | EDG | 0.9996 | 1.9470 | 212 | 1.9483 | 268 | 1.28E-4 | 4.81E-5 | 1.4060 |
| 194 | MEG | 0.9990 | 1.9124 | 126 | 1.9125 | 139 | 8.11E-5 | 3.47E-5 | 0.3120 |
| 194 | MEDG | 0.9985 | 1.8967 | 106 | 1.9003 | 136 | 6.94E-5 | 2.28E-5 | 0.2190 |
| 242 | EG | 0.9998 | 1.9639 | 314 | 1.9636 | 333 | 2.03E-4 | 8.33E-5 | 4.7960 |
| 242 | EDG | 0.9998 | 1.9573 | 264 | 1.9585 | 328 | 1.42E-4 | 5.39E-5 | 3.0470 |
| 242 | MEG | 0.9993 | 1.9292 | 157 | 1.9286 | 173 | 1.26E-4 | 5.33E-5 | 0.6400 |
| 242 | MEDG | 0.9990 | 1.9163 | 132 | 1.9209 | 165 | 4.67E-5 | 1.52E-5 | 0.4060 |
| 386 | EG | 0.9999 | 1.9772 | 501 | 1.9769 | 522 | 2.97E-4 | 1.19E-4 | 21.4070 |
| 386 | EDG | 0.9999 | 1.9730 | 421 | 1.9735 | 511 | 2.38E-4 | 9.17E-5 | 13.9060 |
| 386 | MEG | 0.9997 | 1.9550 | 251 | 1.9543 | 271 | 2.01E-4 | 8.37E-5 | 3.1720 |
| 386 | MEDG | 0.9996 | 1.9467 | 211 | 1.9488 | 261 | 1.04E-4 | 3.87E-5 | 2.2190 |
| 482 | EG | 1.0000 | 1.9817 | 625 | 1.9812 | 649 | 4.63E-4 | 1.86E-4 | 42.1720 |
| 482 | EDG | 0.9999 | 1.9783 | 526 | 1.9783 | 637 | 3.71E-4 | 1.44E-4 | 28.3750 |
| 482 | MEG | 0.9998 | 1.9638 | 313 | 1.9630 | 335 | 2.77E-4 | 1.13E-4 | 6.4380 |
| 482 | MEDG | 0.9998 | 1.9571 | 263 | 1.9589 | 321 | 1.14E-4 | 4.28E-5 | 4.4530 |

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