Dynamic Stability and Commands Response Study Method

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Abstract: - In this paper, the dynamic stability and the commands response study, in Lyapunov sense, is made by solving the non-homogeneous linear system (with constant coefficients) of the induced deviations equations, which are obtained through the linearized equations of motion. Even if, in the all most cases, the non-homogeneous linear system represents an evident idealization, this model is not catalogued like an inadequate model. But, its validity is no more considerate indisputable or absolute. The analyses' results for a partially guided missile represent a good argument in this respect.

Key-Words: - dynamic stability analysis, longitudinal equations of motion

1 Introduction

We investigate the angular missile motion that occurs after an initial perturbation applied to the equilibrium state which is tested. From the properties of the resulted motion we can infer or deny stability.

It if turns out that the perturbed motion consists in oscillations with increasing amplitude, or is a rapidly increasing departure from the equilibrium state, the equilibrium is unstable; otherwise it is stable.

The practicality of this approach depends crucially on the linearization of the motion equations of the perturbation.

By linearizing we can express the perturbation motion as the superposition of complex exponential elementary solutions.

The characteristic exponents of these solutions can be determined through a characteristic value problem or eigenproblem.

2 Technique solution

Consider the non-homogeneous linear system of differential equations with constant coefficients [1]

$$\frac{dx(t)}{dt} = A\,x(t) + p(t) \tag{1}$$

where A is constant n-by-n matrix, $x(t) \in \Re^n$, $p(t) \in \Re^n$ and $x(t_0) = x_0$.

The general solution to the system (1), as we know is

$$x(t) = \overline{x}(t) + \widetilde{x}(t) \tag{2}$$

where $\overline{x}(t)$ is the general solution of homogeneous associated system

$$\frac{dx^*(t)}{dt} = A\,x^*(t) \tag{3}$$

and $\widetilde{x}(t)$ is a particular solution of system (1).

For homogeneous system, the general procedure is therefore:

- find the eigenvalues of the matrix $A\,(\lambda_1, .., \lambda_n)$ by solving the characteristic equation;
- determine the corresponding eigenvectors $u_1, .., u_n$;
- construct a general solution of homogeneous system (3)

$$\overline{x} = \sum_{i=1}^{n} c_i u_i e^{\lambda_i t}; \tag{4}$$

- find the constants $c_1, .., c_n$ from the initial value

$$x^*(t_0) = \sum_{i=1}^{n} c_i u_i = x_0^*. \tag{5}$$

Then a particular solution to the non-homogeneous system is given by

$$\tilde{x} = \sum_{i=1}^{n} c_i(t) u_i e^{\lambda_i t}, \qquad (6)$$

where the $c_i(t)$ are continuous functions which satisfy the equations

$$\sum_{i=1}^{n} \frac{d}{dt}[c_i(t)] u_i e^{\lambda_i t} = p(t). \qquad (7)$$

3 Mathematical model

To exemplify all the theoretical previously presented aspects, we will analyze the longitudinal plane motion of partially guided missile case.

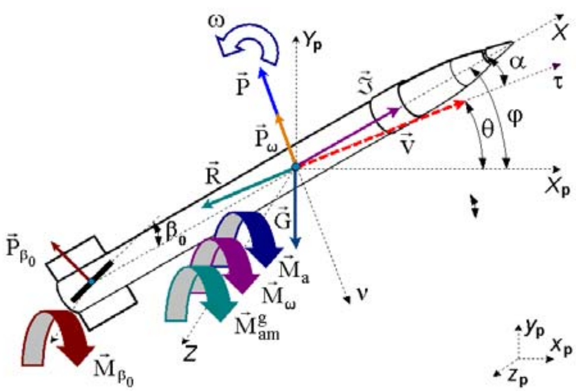

Fig. 1 Longitudinal plane motion of partially guided missile

The longitudinal plane motion is defined by a first order linear system of differential equations, from which canonical form has resulted [2, 3, 4, 5]

$$\begin{aligned}
\frac{dV}{dt} &\approx F_V(V,\theta) - \frac{\rho V^2}{m} S_a C_z^\alpha \beta_0 \sin(\beta_0 + \varphi - \theta) \\
\frac{d\theta}{dt} &\approx F_\theta(V,\theta,\varphi,\omega) + \frac{\rho V^2}{m} S_a C_z^\alpha \beta_0 \cos(\beta_0 + \varphi - \theta) \\
\frac{d\varphi}{dt} &= \omega \equiv F_\varphi(\omega) \qquad (8) \\
\frac{d\omega}{dt} &\approx F_\omega(V,\theta,\varphi,\omega) - \frac{\rho V^2}{J_a} S_a d_c C_z^\alpha \beta_0 \cos\beta_0
\end{aligned}$$

where $F_V(V,\theta) \equiv \frac{\Im}{m} - \frac{\rho V^2}{2m} S C_x - g\sin\theta$,

$F_\theta(V,\theta,\varphi,\omega) \equiv \frac{\Im}{mV}(\varphi - \theta) + \frac{\rho V}{2m} S C_z^\alpha (\varphi - \theta) + $

$+ \frac{\rho}{2m} S l C_z^\alpha \omega - \frac{g\cos\theta}{v}$ and

$F_\omega(V,\theta,\varphi,\omega) \equiv -\frac{\rho V^2}{2J} S l C_m^\alpha (\varphi - \theta) -$

$- \frac{\rho V}{2J} S l^2 C_m^\omega \omega - \frac{\omega Q_e}{J}[(k_{\eta_E}^2 - k_{\eta_P}^2) + (\zeta_E^2 - \zeta_P^2)]$.

It could be supposed that the system induced deviation coefficients (matrix stability derivatives) are proximate constants and equals with their t_i moment value, for sufficient short trajectory arc, or for sufficient short time interval $[t_i, t_{i+1}]$.

Thus, first order linear system of differential equations with constant coefficients of the induced deviations has the below form

$$\frac{d}{dt}\begin{pmatrix} \delta V \\ \delta\theta \\ \delta\varphi \\ \delta\omega \end{pmatrix} \cong \begin{pmatrix} \frac{\partial F_v}{\partial V} & \frac{\partial F_v}{\partial \theta} & \frac{\partial F_v}{\partial \varphi} & \frac{\partial F_v}{\partial \omega} \\ \frac{\partial F_\theta}{\partial V} & \frac{\partial F_\theta}{\partial \theta} & \frac{\partial F_\theta}{\partial \varphi} & \frac{\partial F_\theta}{\partial \omega} \\ \frac{\partial F_\varphi}{\partial V} & \frac{\partial F_\varphi}{\partial \theta} & \frac{\partial F_\varphi}{\partial \varphi} & \frac{\partial F_\varphi}{\partial \omega} \\ \frac{\partial F_\omega}{\partial V} & \frac{\partial F_\omega}{\partial \theta} & \frac{\partial F_\omega}{\partial \varphi} & \frac{\partial F_\omega}{\partial \omega} \end{pmatrix} \begin{pmatrix} \delta V \\ \delta\theta \\ \delta\varphi \\ \delta\omega \end{pmatrix} + \qquad (9)$$

$$+ \begin{pmatrix} -\frac{\rho V^2}{m} S_a C_z^\alpha [\sin(\beta_0 + \varphi - \theta) + \beta_0 \cos(\beta_0 + \varphi - \theta)] \\ \frac{\rho V^2}{m} S_a C_z^\alpha [\cos(\beta_0 + \varphi - \theta) - \beta_0 \sin(\beta_0 + \varphi - \theta)] \\ 0 \\ -\frac{\rho V^2}{J_a} S_a d_c C_z^\alpha (\cos\beta_0 - \beta_0 \sin\beta_0) \end{pmatrix} \delta\beta_0$$

with $\frac{\partial F_V}{\partial V} = -\frac{\rho S}{m}[V C_x + \frac{V^2}{2}\frac{d}{dM}(C_x)\frac{1}{a}]$

$\frac{\partial F_V}{\partial \theta} = -g\cos\theta$

$\frac{\partial F_\theta}{\partial V} = -\frac{1}{V^2}[\frac{\Im}{m}(\varphi - \theta) - g\cos\theta] +$

$+ \frac{\rho S}{2m}(\varphi - \theta)[C_z^\alpha + V\frac{d}{dM}(C_z^\alpha)\frac{1}{a}] + \frac{\rho}{2m} S l \omega \frac{d}{dM}(C_z^\omega)\frac{1}{a}$

$\frac{\partial F_\theta}{\partial \theta} = -\frac{1}{V}[\frac{\Im}{m} - g\sin\theta] - \frac{\rho S C_z^\alpha}{2m} V$

$\frac{\partial F_\theta}{\partial \varphi} = \frac{\Im}{mV} + \frac{\rho S C_z^\alpha}{2m} V$

$\frac{\partial F_\omega}{\partial V} = -\frac{\rho S l}{2J}(\varphi - \theta)[2 V C_m^\alpha + V^2 \frac{d}{dM}(C_m^\alpha)\frac{1}{a}] -$

$- \frac{\rho S l^2}{2J}\omega[C_m^\omega + V\frac{d}{dM}(C_m^\omega)\frac{1}{a}]$

$\frac{\partial F_\omega}{\partial \theta} = \frac{\rho V^2}{2J} S l C_m^\alpha$

$\frac{\partial F_\omega}{\partial \theta} = \frac{\rho V^2}{2J} S l C_m^\alpha$

$\frac{\partial F_\omega}{\partial \omega} = -\frac{\rho V}{2J} S l^2 C_m^\omega - \frac{Q_e[(k_{\eta_E}^2 - k_{\eta_P}^2) + (\zeta_E^2 - \zeta_P^2)]}{J}$

$\frac{\partial F_V}{\partial \varphi} = \frac{\partial F_V}{\partial \omega} = \frac{\partial F_\varphi}{\partial V} = \frac{\partial F_\varphi}{\partial \theta} = \frac{\partial F_\varphi}{\partial \varphi} = 0$ and $\frac{\partial F_\varphi}{\partial \omega} = 1$.

4 Results

In the 2-7 figures are presented the φ and α angles variation; there is also presented the ω angular velocity of the partially guided missile during the burning phase for two study cases (with and without the jet damping moment $M^{g}_{am} = -\frac{\omega Q_e}{J}[(k^2_{\eta_E} - k^2_{\eta_P}) + (\zeta^2_E - \zeta^2_P)])$. The obtained results make obviously the flight stable condition.

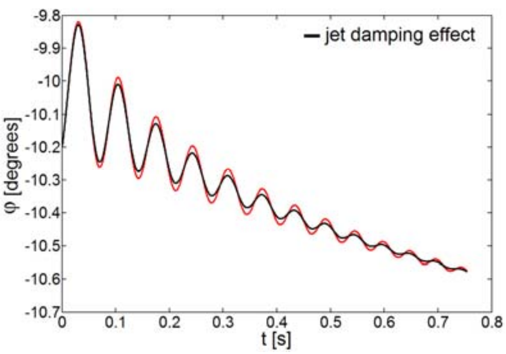

Fig. 2 The φ angle vs. time

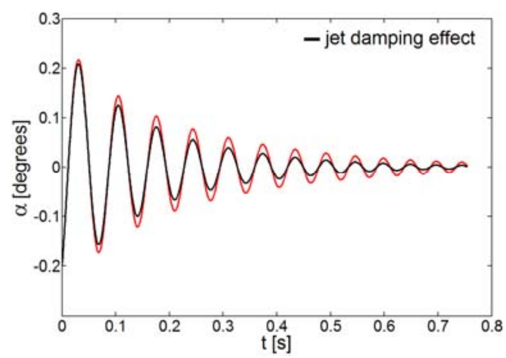

Fig. 3 The α angle vs. time

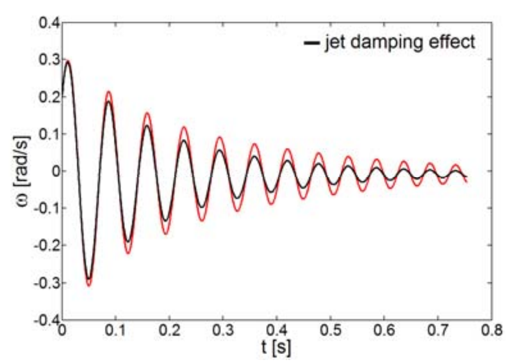

Fig. 4 The ω angular velocity vs. time

The induced deviations for each relevant parameter defining the missile's motion are graphical presented in the 5-9 figures.

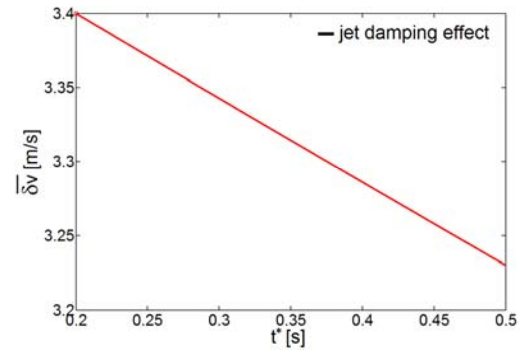

Fig. 5 Induced deviation of the missile speed vs. time

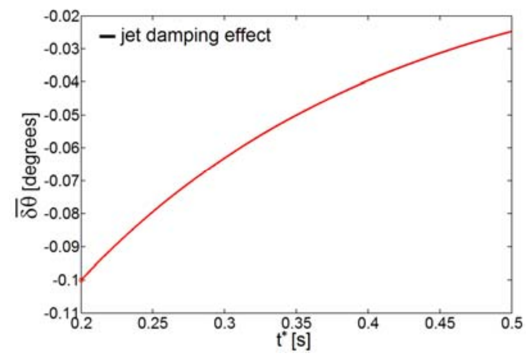

Fig. 6 Induced deviation of the θ angle vs. time

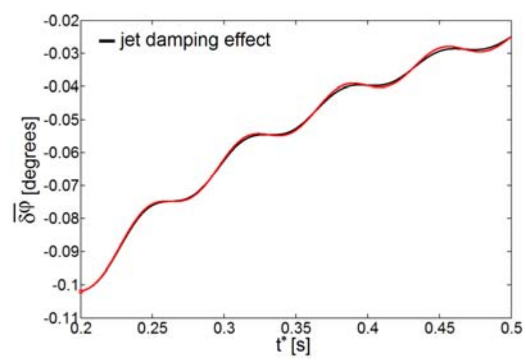

Fig. 7 Induced deviation of the φ angle vs. time

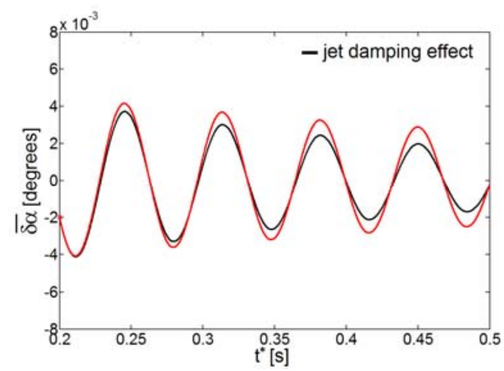

Fig. 8 Induced deviation of the α angle vs. time

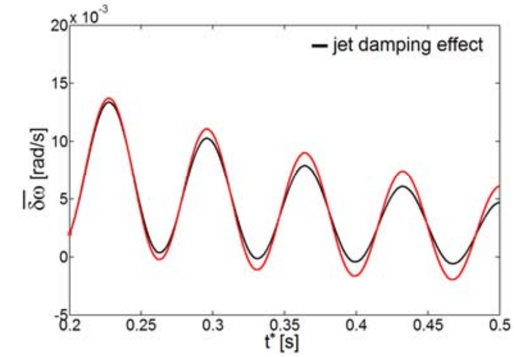

Fig. 9 Induced deviation of the ω angular velocity angle vs. time

The commands response – the particular solution to the non-homogeneous system (9) – is graphical presented in the 10-14 figures.

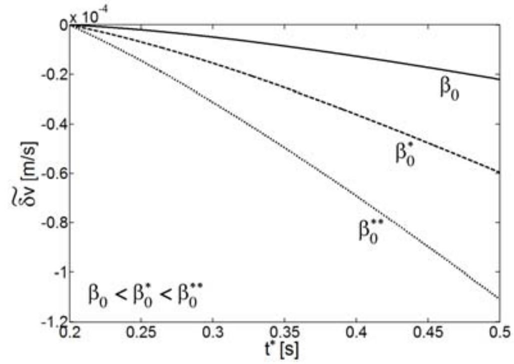

Fig.10 Commands response of the missile speed vs. time

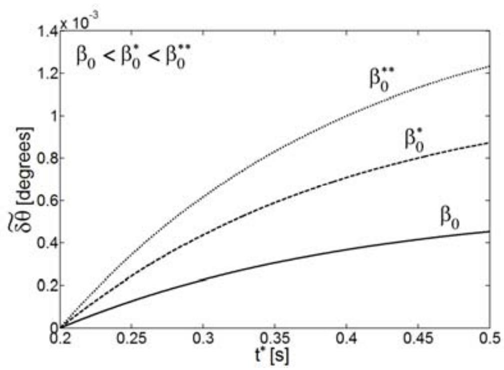

Fig. 11 Commands response of the θ angle vs. time

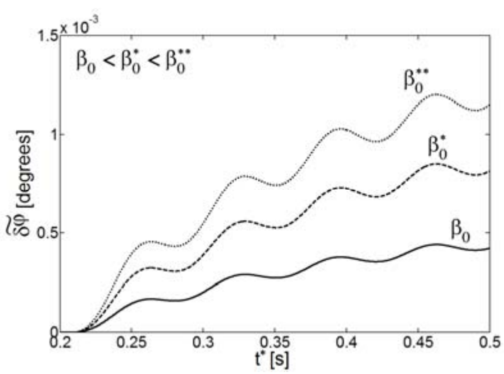

Fig. 12 Commands response of the φ angle vs. time

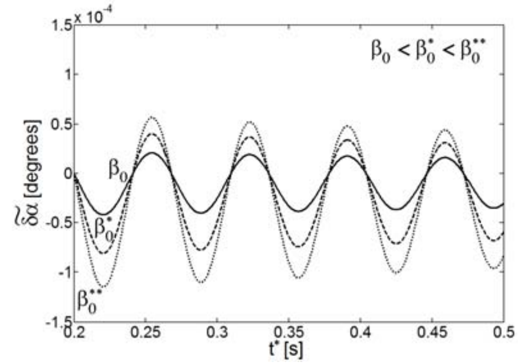

Fig. 13 Commands response of the α angle vs. time

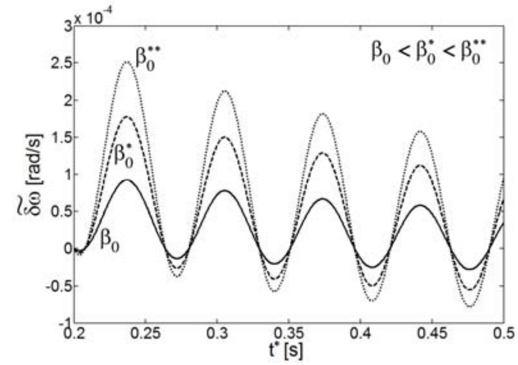

Fig. 14 Commands response of the ω angular velocity vs. time

5 Conclusions

This study has provided very important information about the dynamic stability and the commands response (control) during the burning phase. These are as below:

- the jet damping moment contributes at the acceleration of the induced deviations amortization;
- the commands response is not influenced by the jet damping moment, but it is influenced by the β_0 angle values.

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