Estimation of Flow Rates in Naturally Ventilated Buildings Using Simplified Method

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Abstract: A simplified design method has been derived for use in the estimation of the flow rates in naturally ventilated buildings. The method is based on a one-dimensional "loop analysis" in which the buoyancy forces are balanced by the pressure drops to due friction. Multiple zone, multiple floor buildings are catered for with each zone characterised by its internal gains and its discharge coefficients. Wind effects at the entrance and exits are also taken into account. The procedure yields the zone mass flow rates and temperatures directly by the solution of a simple cubic equation for each loop. Interaction between loops requires a few iterations. The procedure is straight forward and simple enough to be put on a Spreadsheet. This methodology allows the architect to explore various building configurations at little expense and hence to focus on those designs which warrant further detailed analysis using a full building simulation package. In this paper, the fundamental theory behind the loop analysis is described together with the first results at its validation using an emulation of the simple single stack PV cladding arrangement.

Keywords: loop analysis, natural ventilation, multiple zone, single stack PV cladding

1 Introduction

The drive for low energy consumption in buildings serves to motivate architects, and building services engineers to seek new techniques both for retrofit and new construction. The replacement of mechanical ventilation with natural ventilation is one such important technique made difficult by the dependency on climatic conditions, building use, and lack of simplified design guidelines and rules of thumb.

A procedure is presented below in which the mass flow rate within each zone of a building can be computed without recourse to a complicated and time consuming CFD code. It is based on a "loop analysis" around which the buoyancy force balances the friction. The building is represented as a closed pipe or duct and pressure losses are calculated in a similar fashion to those in a pipe circuit. Pressure losses across ventilation openings and exits are easily calculated using conventional pressure loss fitting factors, Cd, called discharge coefficients herein. But friction factors for real, occupied rooms are more difficult to estimate as the room structure constitutes a poor representation of one-dimensional flow assumed for conventional pipework and ducts. Still, friction can be represented by a Reynolds number dependency with a coefficient, Z, whose value can be established by experiment and used for estimation of the building performance. Wind pressure coefficients, CP1 and CP2, and hence effects, too, can be calculated by conventional methods thus revealing the benefits of wind pressure applied normal to the inlet and tangential to the exit.

2. Single Storey, Single Loop Analysis

Consider the single storey building shown in Fig. 1. In steady state, the mass flow rate, **m**, is constant everywhere in the loop flow path denoted by the thin line. Around the loop, the momentum equation states that the sum of the pressure drop around the loop due to friction must equal the buoyancy force developed due to a change in density, hence temperature. The buoyancy force is generated due to the temperature rise as a result of internal gains due to solar gains, heating, lighting, equipment, and occupancy minus any fabric heat loss. In the following, Qg denotes the **net** internal heat gain. This is often easily estimated in the design stage using various building design guidelines or from energy audits on existing buildings.

which by conservation is constant around the loop, the pressure drop around the loop must equal the buoyancy force, i.e.

$$gh(\rho_o - \rho_2) = \tfrac{1}{2} C_{d1} \frac{\rho_o m^2}{\rho_o{}^2 A_1{}^2} + \tfrac{1}{2} C_{d2} \frac{\rho_2 m^2}{\rho_2{}^2 A_2{}^2} + \tfrac{1}{2} f \frac{L}{D} \frac{\rho_o m^2}{\rho_o^2 A_c^2} \tag{1}$$

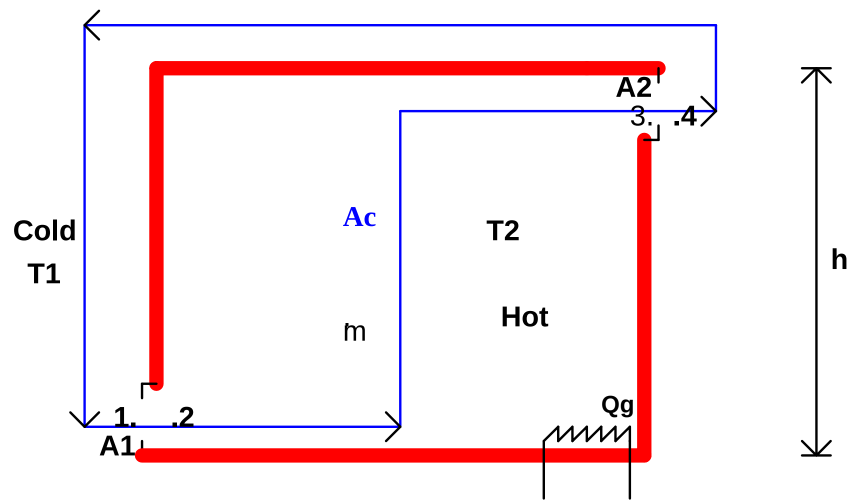

Figure 1 A simple loop

The building of height h, is characterised by the cross sectional area A_1 at inlet and A_2 at outlet. Inside, the air is heated to temperature T_2 such that it has an average density ρ_2 The external air has a density ρ_o at an average temperature T_1. From the Fig. 1 the pressure balance around the loop reads $\Sigma \Delta P = (P_1-P_2) + (P_2 - P_3) + (P_3-P_4) + (P_4-P_1) = 0$ where (P_1-P_2) and (P_3-P_4) denotes the sum of all the internal friction and flow resistance pressure caused by the stack on the air flowing through the control volume, of cross sectional area Ac. The Bernoulli equations read, $P_2 = P_3 + \rho_2 gh + F$ and $P_1 = P_4 + \rho_o gh$ where F denotes the head loss due to internal friction along the flow path, with $F=f(L/D_h)\rho_o U^2/2$. Hence $(P_2 - P_3) + (P_4-P_1) = gh(\rho_2 - \rho_o) + F$ or rearranged, $(P_1-P_2) + (P_3-P_4) = gh(\rho_o-\rho_2) - F$. The pressure drops (P_1-P_2) and (P_3-P_4) can be expressed in terms of a pipe fitting pressure drop loss coefficient, C_d , $\Delta P=\rho\, U^2/2$ where **U** = local mean velocity across an opening. Since **U**=**m**/Aρ , and **m** is the loop mass flow rate,

The flow is basically incompressible so that in the pressure drop terms $\rho_2 \approx \rho_o$ is assumed and the Boussinesq approximation is made in the buoyancy term so that

$$(\rho_o-\rho_2) = \rho_o\beta(T_2-T_1) \tag{2}$$

with $\beta=1/T$ where $T=(T_1+T_2)/2$ and T is the absolute temperature in Kelvin.

From the energy equation, see Fig. 1, $\mathbf{m}C_p(T_2-T_1)=Q_g$ so that

$$gh\beta(T_2-T_1) = gh\beta Q_g/(mC_p) \tag{3}$$

and therefore, the final result becomes

$$m^3 = \frac{2\rho_o^2 gh\beta Q_g}{C_p\left(\frac{C_{d1}}{A_1^2} + \frac{C_{d2}}{A_2^2} + \frac{f(L/D_h)}{A_c^2}\right)} \tag{4}$$

Equation 4 is the basic fundamental "loop equation" used as the starting point for all subsequent derivations. Note that in the absence of friction, F=0, equation 4 predicts that the mass flow rate is proportional to the net internal gain, $(Qg)^{1/3}$.

3 Formulation For Single Storey With Stack And Wind Effects

Since the pressure created by the wind and the buoyancy forces act at the same time, it is necessary to estimate the resultant air flow. The total pressures at the inlet and exit are re-written $P1'=P1 \pm CP1\rho_o U^2/2$ and $P4'=P4 \pm CP2\rho_o U^2/2$. Following through the derivation leads to,

$$gh\rho_o\beta\frac{Q_g}{mC_p} \pm \tfrac{1}{2}\rho_o CP1W_{in}^2 = \tfrac{1}{2}C_{d1}\frac{\rho_o m^2}{A_1^2\rho_o^2} + \tfrac{1}{2}C_{d2}\frac{\rho_o m^2}{A_2^2\rho_o^2} \mp \tfrac{1}{2}\rho_o CP2W_{out}^2 \tag{5}$$

A positive wind pressure coefficient, +CP1, at the inlet and a negative (suction) wind pressure coefficient,-CP2, at the exit will enhance the mass flow rate. Rearranging yields the fundamental single loop equation with wind effect which can be solved for **m**

$$m^3\left[\frac{C_{d1}}{A_1^2} + \frac{C_{d2}}{A_2^2}\right] - \rho_o^2 m\left[\pm CP2W_{out}^2 \pm CP1W_{in}^2\right] - \frac{2\rho_o^2 gh\beta Q_g}{C_p} = 0 \tag{6}$$

Equation 6 is a cubic equation which predicts that for very strong wind pressures, in at the inlet and tangential to the exit, the mass flow rate is linearly proportional to the net internal gain, **m** $\propto$**Qg,** and **m** $\propto$**W** for constant Qg, see Figs. 2 and 3.

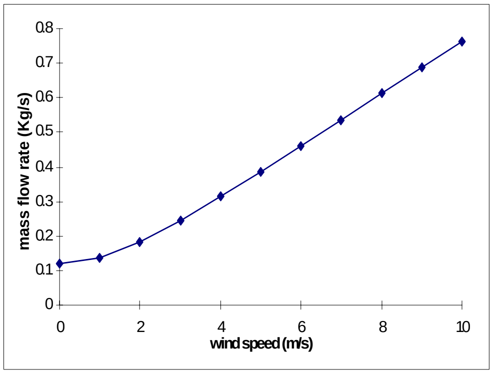

Figure 2: Mass flow rate in the single storey

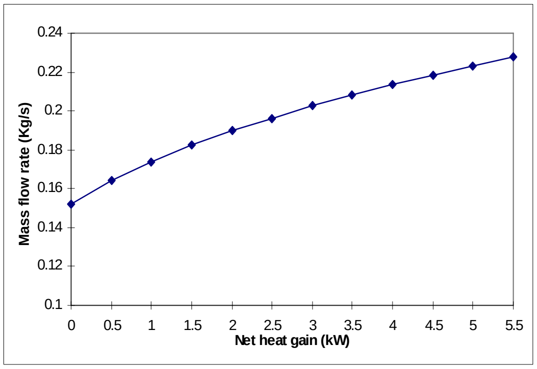

Figure 3: Mass flow rate for single storey building for variation of the wind speed versus net heat gain at 2m/s wind speed

4. Formulation For Multi Storey Buildings

The single loop analysis can be extended to include a multiple storey building with different internal gains for each room zone (denoted with "i") and its adjacent stack zone (denoted with "j") as depicted in Fig. 4. Again, for reasons of brevity only the main features of the derivation are included.

Each loop must be considered individually. But each loop mass flow rate is different and the pressure drops at each inlet and exit depend on the local mass flow rate through that opening. Likewise the internal heat gains in each loop and the stack (Loop4) depend on the flow rates through each zone. It is convenient, therefore, to introduce an "effective" heat gain or area based on the ratio of the local mass flow rate to the total mass flow rate at the entrance to the building, see. Fig. 4. By denoting of Q_{si} as the equivalent heat gain, $Q'_{si}=Q_{si}(m_{ri}/m)$ and A'_{si} as the effective area, $A'_{si}=A_{si}(m_{ri}/m)$ the generalisation to a multi-loop system including friction can be expressed as;

$$m_1^3\left[\frac{C_{ds1}}{A_{s1}^{'2}}+\frac{C_{dr1,1}}{A_{r1,1}^2}+\frac{C_{dr1,2}}{A_{r1,2}^2}+\sum_{k=1}^{k=j}\frac{f_{1k}L/D}{A_{1k}^2}\right]-\rho_o^2 m_1\left[\pm CP_{out}W_{out}^2 \pm CP_{in}W_{in}^2\right]-\frac{2h\rho_o^2 g\beta\left[Q_{s1}^{'}+Q_{r1}\right]}{C_p}=0 \qquad (8)$$

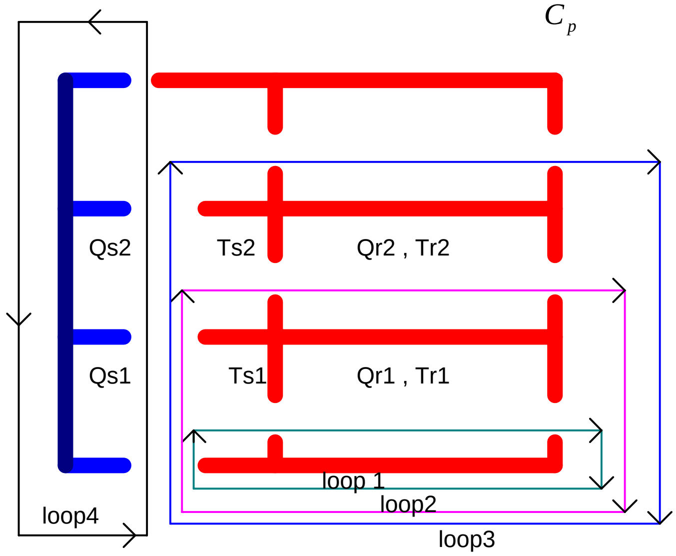

Figure 4: 2D model multiloop solution.

$$m_i^3=\frac{2\rho_o^2 gh\beta\left[\left(\sum_{k=1}^{k=j}Q_{sk}^{'}\right)+Q_{ri}\right]}{C_p\left[\left(\sum_{k=1}^{k=j}\frac{C_{dsk}}{A_{sk}^{'2}}\right)+\left(\sum_{k=1}^{k=j}\frac{f_{ik}\frac{L}{D}}{A_{ik}^2}\right)+\frac{C_{dri,1}}{A_{ri,1}^2}+\frac{C_{dri,2}}{A_{ri,2}^2}\right]} \qquad (7)$$

where sj denotes the stack at floor j, r denotes the room number and i denotes the loop number. From this equation the direct solution for the mass flow rate into the room can be found once the total mass flow rate, **m**, is known.

5. Solution For Multi Storey With Wind Effects And Friction

The governing equation 7 can be extended as for the single loop case to include wind pressure effects at the building inlet and the flow loop exit , see Fig. 4. As the notation becomes somewhat unwieldy we illustrate the application to Loop 1 which becomes,

Equation 8 is of the form $Am_1^3 + Bm_1 - C = 0$, i.e. a cubic equation. These equations are then solved for each loop in turn using the Newton Raphson technique. The corrected mass flow rates into the rooms are then found by iteration of equation 8, since the effective area and equivalent heat gain depend on the ratio of the room mass flow rate for loop *i* to the total stack mass flow rate, **m.**

The model for a multi storey building was run which assumed that the net heat gain in a room of $15m^2$ floor area is 1.5 kW, with entrance and exit areas of $2.5m^2$ and 0.2 m^2 respectively. The opening to every floor through a stairwell is taken as 3.75 m^2. The direction of the wind speed on the building is taken as normal to the entrance (positive pressure effect) and tangential to the exit (beneficial suction pressure effect). The results of the model can be seen in Fig. 5 depending on the wind speed. The mass flow rate for the first floor is slightly higher than floors above it because the path through the first floor to the room has less resistance in its loop and therefore more air flows through this path.

6. Friction

6.1 Textbook

Within each building zone the through flow **velocity** can be expected to be low and therefore the pressure drop can be dominated by the zone entrance and exit vent resistances, the Cd's. In sharp contrast, the flow through a PV ventilated stack with its narrow gap and tall structure can be expected to be dominated by the internal flow resistance, F, and the nature of the flow structure. In both areas the real flow structure will be 3 dimensional, and characterised by entrance lengths, and transition to turbulence. We can retain the simplicity of the 1 dimensional duct flow analogy in three ways;

Firstly, one can use a textbook analogy whereby $F= f\ (L/D)\ \rho\ U^2/2$ with the friction factor, f, determined by the simplest fully developed laminar flow Reynolds number relationship, e.g. f=64/Re or extended to a general turbulent flow correlation of the form $Re=CRe^{-m}$. The Reynolds number in turn can be re-written as

$Re=UD/\nu = \{\dot{\underline{\mathbf{m}}}/\rho\ Ac)\}D_h/\nu$ so that for the simplest analogy

$$F= [64/\{\dot{\underline{\mathbf{m}}}/\rho\ Ac)\}D/\nu\ ]\ (L/D_h)\ \rho\ U^2/2 \qquad (9)$$

This, when inserted in the simple loop equation 4, leads to a quadratic equation for **m** The dependency of **m** on the net internal gain is therefore $\mathbf{m}\propto(Qg)^{0.5}$ for negligible duct entrance/exit effects.

6.2 Hollands correlations

For the PV ventilated stack the entrance lengths, transition to turbulence, turbulence, and aspect ratio must be included. Hollands (1981) has determined a number of correlations which characterise air collectors operating under a fixed 60 Pa, pressure drop and forced convection. These take the form whose coefficients are summarised in Table 1, $f=f_o + f_1/(L/D_h) + f_2/Re^m$

Table 1. Summary of the Hollands (1981) friction coefficients

	f_o	f_1	f_2	m
Laminar	0	0.9	24	1
Transition	0.0094	$2.92Re^m$	0	-0.15
Turbulent	0	0.73	0.059	0.2

These correlations can be implemented provided the appropriate characteristic hydraulic diameter, D_h, is used, 4*perimeter/area.

6.3 Emprical Z*f

Since a real building or a real PV ventilated stack does not resemble perfectly the ideal one dimensional duct flow and within the duct there are entrance effects, laminar flow, transition to turbulence and turbulent flow we can introduce an empirical constant, Z, to lump all these effects together so that $f'=Z*f_{\ theoretical}$. The factor Z becomes an empirical constant for each individual case to be determined by experiment and parameter identification. Z can be expected to depend on the net heat gain, Q_g, hence the flow regime, hence Reynolds number, the duct aspect ratio, L/D_h,, the entrance lengths and inlet/exit vent design, hence flow resistances, and on the wind effects. Further study is warranted to establish a simple form of this dependency. In the following results, seen in Fig. 6, a simple relationship $\mathbf{Z\approx0.8L/D_h}$ was found suitable. Here, the experimental results of Sandberg and Moshfeg (1995) are shown for a simple, well controlled experiment of a simulated ventilated PV stack of height 6.5m and aspect ratio about 25 without wind effects and for a uniform heat flux of up to 300 W/m^2 (i.e. the net internal gain). Both the laminar flow textbook constant 64 and the Hollands constants lead to theoretical mass flow rates of 2.1-2.4 the measured values. This is expected due to the entrance lengths and flow regimes encountered for which the only recourse would be to model the flow using a CFD package.

But, by applying the empirical factor Z=14.5 to the simplest laminar flow analogy, Z*64/Re, or Z=11 as a multiplier of the Holland's formulation good agreement between the empirical and the experimental results are obtained. There is no need for a CFD code. As seen, Fig. 6, there is about a 25% error in the predicted mass flow rate at the lowest heat gain, Q_g=50 W/m^2 but much better as Qg increases. Also, the simplest Re analogy and the Hollands's coefficients lead to the same results throughout. Just a simple Z factor correlation could be proposed, $Z=z_0+z_1*Q_g$. Such correlations are left to future work.

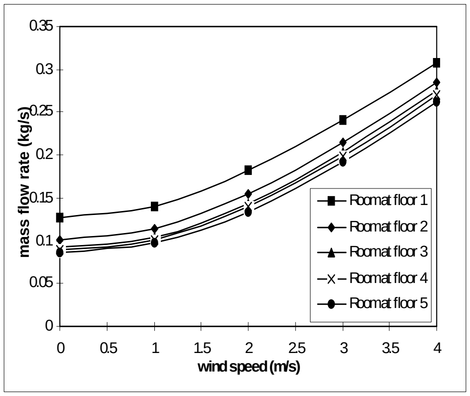

Figure 5 : Mass flow rate of the room in each heat gain

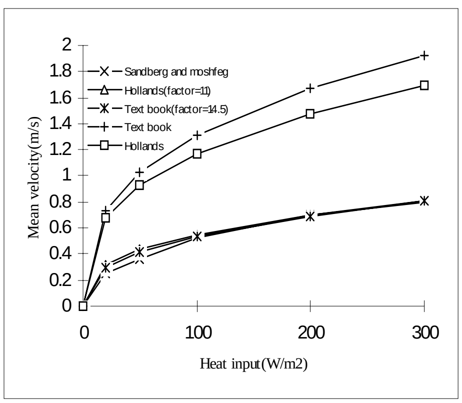

Figure 6: Mean velocity in air gap versus thefloor against wind speed

7. Conclusions

A direct solution for the mass flow rate in a multi storey naturally building with wind pressure effects was developed The theory predicts the zone mass flow rates (and hence temperatures) given the net internal gain to each zone. It was shown that the ventilation mass flow rates depend on the gain to the power 1 for wind dominated conditions, 0.5 for internal friction dominated conditions, and 0.33 for entrance/exit duct domination over internal friction.

The loop solution can be implemented easily as a Spreadsheet programme and used to assess viable designs for new or retrofit applications without recourse to detailed CFD codes. The methodology could also be implemented in a building simulation code as a beginning-of-hour calculation to establish the ventilation heat loss/gain.

The stack effect and wind effect can be used to reduce temperatures of building integrated PV panels to increase their efficiency.Comparison with existing experimental work shows good agreement for the predicted mass flow rates using existing textbook and forced convection internal friction calculations provided that an empirical constant, Z, can be established which makes the analogy to a simple one-dimensional flow in a duct a viable model possible.

8. References

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