# An hybrid model of Mathematical Programming and analytic hierarchy process for the GISMR: The Industrial localization

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*Abstract:* - One of the major questions which arise for the decision makers is the localization choice of places of their establishment related to the constraint of the space, social, economic and policy difference between the places of production and those of consumption. This question is related to the diversity of the criteria integrated in the decision-making, and to the very great number of possible space alternatives. We propose in this article an approach of aid to the industrial localization decision by profiting from the advantages offered by the geographical information systems to multiple representation compounds to the AHP method and from the advantages of the mathematical programming models. The use of a GISMR combined with AHP will help us to reduce the number of the space alternatives and to evaluate them according to real criteria of decision which are quantitative or qualitative. The evaluations obtained will be integrated in a mathematical model to make the final choice of the best alternatives. In fact this model is used to identify the best solution taking into account the criteria fixed by the decision-maker and the data provisions generated by the GISMR.

*Key-Words:* - Geographical Information Systems to Multiple Representation (GISMR), Multi-Criteria Decision Analysis (MCDA), Analytic Hierarchy Process (AHP), Mathematical Programming (MP), Industrial localization.

## 1 Introduction

The industrial localization is one of the most significant strategic decisions because it conditions the long-term operation of the firm, and it often depends on several contradictory factors. To choose an optimal location, the decision-makers must consider some rational criteria such as profit or efficiency and use rigorous and reliable methods.

To make this decision, we must raise the difficulties of the diversity of the criteria and factors of decision, the importance of the number of possible solutions, the heterogeneity of socio-economic space and the two dimensions of localization: inter-regional (level of the great economic space) and intra-regional (level of the regions and localities) [1].

Our work appears in the context of development of models of territorial decision-making aid. Our interest was focused on the power of space analysis of the GISMR, and the decisional capacities of the AHP [2] and the MP.

We will describe our approach of aid to the decision making (section 2). An illustration of our approach is presented in section 3. The conclusion and the perspectives of our work are exposed in the last section.

## 2 Proposed approach

This approach is based on the remarkable potentialities of the integration of AHP and the MP in GISMR. This makes it possible to enrich the decision-making process of industrial localization through the complementarity between these tools, as illustrate in what follows:

– The taking into account of the real criteria of decision which are quantitative or qualitative, the use of a very thorough analysis of all the elements necessary to a good evaluation of the various possible solutions, and the study of the sensitivity of these solutions. It is something lacking in the MP but can be filled by AHP;

– Great capacity to solve the problem by considering a great number of possible solutions subject to a set of constraints. It is a limit of AHP and favours MP;

– GISMR offers an effective visual multi-representation of the possible solutions and is used for the management of a significant volume of the data that is missing with AHP and MP;

– Finally, this integration has the merit to answer various recent requirements which the new function of the GIS imposes: to evolve to really computerized decision-making systems with spatial reference. Indeed, GISMR has the possibility of incorporating all information necessary concerning the decisional problem and in a coherent and structured way. This aggregation of information is of primary importance in order to draw a classification or a choice by integrating AHP and the MP.

However, the various stages, processes, and the relations ships between these elements remain to be specified. This will allow the description of the advantages above to answer specificities of the industrial localization problems.

## 2.1 General approach

If the scale is adopted as the main factors, the definition of the problem will be categorically influenced. Otherwise, the criteria are more or less relevant according to the scale of work since any criterion cannot be considered on any scale [3]. With the scale of a country for example, it is not necessary to take into account the facility of connection to the sewerage system. This criterion is on the other hand of primary importance when it is about only one area.

According to this report, we propose a hierarchical approach of localization (figure 1) by considering that this decision must be done in two levels: level of great space (for example a country) and level of localities (for example an area).

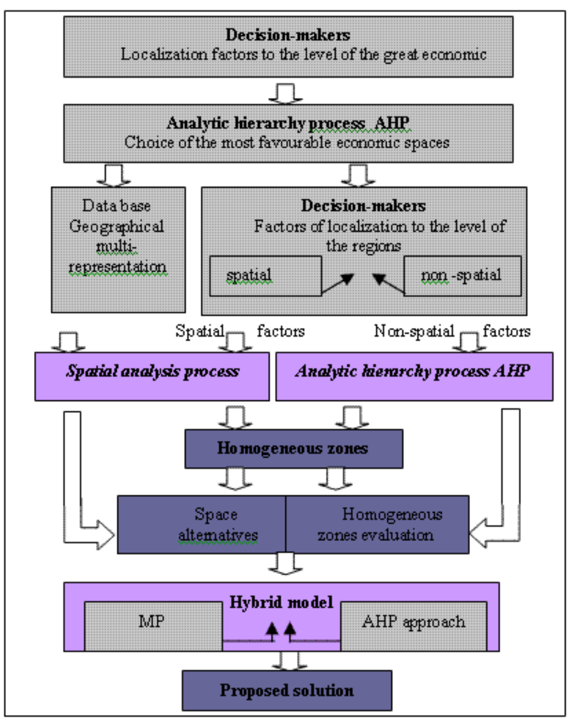


**Figure 1. Proposed approach**

We use mainly four processes:

– The first process is charged to describe the perimeter of study using a geographical data base managed by a GISMR;

– The second process makes it possible to build the homogeneous zones, through factors related to the level of localities. It is also used for the multi-representation of these zones that makes it possible to present the spatial alternatives in the most reliable way;

– The third is the analytic hierarchy process AHP. It is charged to evaluate homogeneous zones resulting from the space analysis process;

The fourth is based on MP; its objective is to maximize the total utility of choice.

## 2.2 Spatial analysis process

The spatial criteria specified by the decision-makers to establish favourable choices are taken into account by the spatial analysis process. This process uses the possibilities of spatial combinations offered by the GISMR [4]. And it builds the homogeneous zones and makes the choice of the candidate sites according to the aspects meeting the specific needs for the actors.

To evaluate the various criteria, the GISMR explores the same geographical area according to several representations where each one reflects an interpretation, a point of view or a quite precise scale. The result of this evaluation is the combination of answers for each criterion according to the corresponding representation. This result is generally translated into term of chart in a multi-scale chart and multi-topics.

## 2.3 Analytic hierarchy process AHP

The objective of this process is to overcome the complexity of the problem of industrial localization by the hierarchical decomposition, and the evaluation of the various actions considered during the decision-making process. This process offers a methodology to rank alternative courses of action based on the decision's judgments concerning the importance of the criteria and the extent to which they are met by each alternative.

## 2.4 Mathematical programming process

The goal of this model is to select one or more space alternatives who maximize an objective function subject to a set of constraints related basically to the industrial localization. Our model takes into account the factors of localization and different criteria of decision.

## 3 Illustration of the proposed approach

To illustrate the proposed approach, we present, in this section, an example of localization of a logistic base.

### 3.1 Stage I

In the first, we choose the most favourable great spaces, by questioning the decision-makers and by considering the factors able to influence the localization. This choice is based on consultations of several concerned actors. Thus we retained three Factors which appear to intervene in a consequent way in the decision-making process: the **market**, the **comparative advantages** and the **governmental settlement**. On this level we chose only one country "**Morocco**".

### 3.2 Stage II

In the continuation, the question is the choice of the localization factors related to the second level. These factors are used to build the homogeneous zones. The choice must be precise for discriminating between the zones, and not to be redundant to avoid raising the importance allotted to an unspecified dimension. The selected factors are: market, workforce, geographical situation and transport.

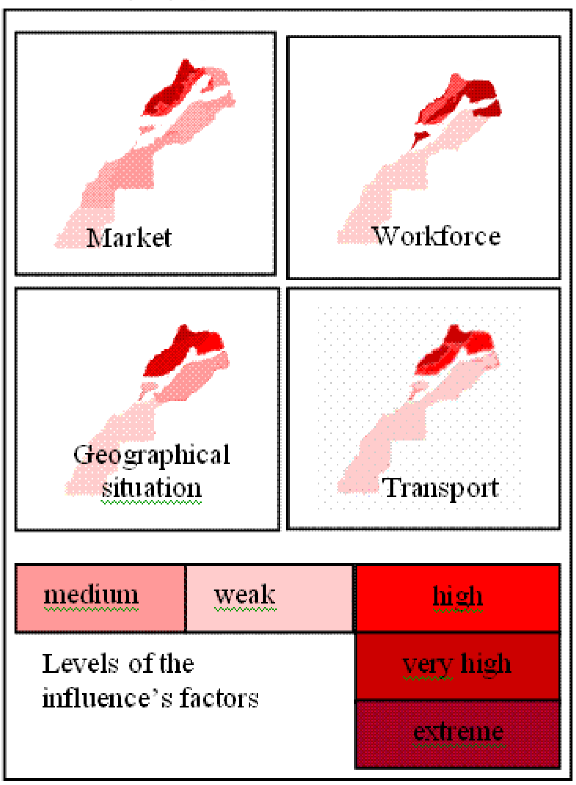


**Fig. 2**. Layer evaluation compared to the localization factors

To build homogeneous zones, the process of space analysis starts with the determination of the charts of the homogeneous zones reflecting each one a space factor. The space alternatives of the same zone are similar compared to the maximal and minimal values of indifference fixed by the decision-makers.

Thereafter, through the technique of layers superposition, the process of space analysis determines the intersection zones. These zones are constituted of the regroupings of the space alternatives belonging to homogeneous zones on the level of all the layers. These zones are homogeneous compared to all the space factors. For the non-space factors we don't need a great territorial data analysis to determine the homogeneous zones.

The space analysis process explores the same chart in several representations (figure 2). Each representation reflects a qualification of the importance of a factor. These qualifications are as: **weak**, **medium**, **high**, **very high** and **extreme**.

Following this stage we determined 12 homogeneous zones. To evaluate them we use the AHP method.

### 3.3 Stage III

The construction of the hierarchy is the first step in the problem solving process of the AHP method. The hierarchical decomposition of our problem is in two levels. The first level represents the localization criteria taken into account. The last level of the hierarchy represents the alternates: zones A, B, C, D, E, F, G, H, I, J, K and L.

The pairwise comparisons for the above evaluation of the importance of the zones are shown below:

Pairwise comparisons of evaluation criteria (Highest level elements in the hierarchy) (table 1):

| | Market | Workf. | Geog.l situation | Transp. | Total | Weights |
|---|---|---|---|---|---|---|
| Market | 1 | 6 | 2 | 2 | 11.00 | 0.46 |
| Workforce | 1/6 | 1 | 1/3 | 1/3 | 1.82 | 0.07 |
| Geographical situation | 1/2 | 3 | 1 | 1 | 5.50 | 0.23 |
| Transportation | 1/2 | 3 | 1 | 1 | 5.50 | 0.23 |
| | | | | Total | 23.82 | 1.00 |

**Tab. 1.** Pairwise comparisons of evaluation criteria

The vector of weights is obtained by the normalization of the vector of the sums of the comparisons values corresponding to each criterion.

Table 2 present the results of pairwise comparisons of zones A, B, C, D, E, F, G, H, I, J, K and L (Elements of the lowest level in the hierarchy with respect to criterion dimensions) :

| Criteria \ ZONE | A | B | C | D | E | F | G | H | I | J | K | L |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Market | 0.0728 | 0.1460 | 0.0344 | 0.1093 | 0.0364 | 0.0364 | 0.0727 | 0.1093 | 0.1443 | 0.0727 | 0.0727 | 0.0910 |
| Workforce | 0.1095 | 0.1097 | 0.0821 | 0.0546 | 0.0993 | 0.0993 | 0.0993 | 0.0546 | 0.0840 | 0.0993 | 0.0546 | 0.0546 |
| Geographical situation | 0.1818 | 0.1363 | 0.0911 | 0.0454 | 0.1818 | 0.0911 | 0.0454 | 0.0454 | 0.0454 | 0.0454 | 0.0454 | 0.0454 |
| Transportation | 0.1000 | 0.1000 | 0.0670 | 0.0828 | 0.0828 | 0.0828 | 0.0670 | 0.0670 | 0.1025 | 0.0828 | 0.0828 | 0.0828 |

**Tab. 2.** Results of pairwise comparisons of Zones

Once the normalized are computed for all levels of the hierarchy, they are combined by moving through the hierarchy, starting at the lowest level. The table below (Table 3) illustrates this procedure.

| Zones | Global utility |
|---|---|
| A | $u_1 = 0.46 \times 0.0728 + 0.07 \times 0.1095 + 0.23 \times 0.1818 + 0.23 \times 0.1000 = 0.1059$ |
| B | $u_2 = 0.46 \times 0.1460 + 0.07 \times 0.1097 + 0.23 \times 0.1363 + 0.23 \times 0.1000 = 0.1291$ |
| C | $u_3 = 0.46 \times 0.0344 + 0.07 \times 0.0821 + 0.23 \times 0.0911 + 0.23 \times 0.0670 = 0.0579$ |
| D | $u_4 = 0.46 \times 0.1093 + 0.07 \times 0.0546 + 0.23 \times 0.0454 + 0.23 \times 0.0828 = 0.0835$ |
| E | $u_5 = 0.46 \times 0.0364 + 0.07 \times 0.0993 + 0.23 \times 0.1818 + 0.23 \times 0.0828 = 0.0845$ |
| F | $u_6 = 0.46 \times 0.0364 + 0.07 \times 0.0993 + 0.23 \times 0.0911 + 0.23 \times 0.0828 = 0.0636$ |
| G | $u_7 = 0.46 \times 0.0727 + 0.07 \times 0.0993 + 0.23 \times 0.0454 + 0.23 \times 0.0670 = 0.0662$ |
| H | $u_8 = 0.46 \times 0.1093 + 0.07 \times 0.0546 + 0.23 \times 0.0454 + 0.23 \times 0.0670 = 0.0799$ |
| I | $u_9 = 0.46 \times 0.1443 + 0.07 \times 0.0840 + 0.23 \times 0.0454 + 0.23 \times 0.1025 = 0.1062$ |
| J | $u_{10} = 0.46 \times 0.0727 + 0.07 \times 0.0993 + 0.23 \times 0.0454 + 0.23 \times 0.0828 = 0.0698$ |
| K | $u_{11} = 0.46 \times 0.0727 + 0.07 \times 0.0546 + 0.23 \times 0.0454 + 0.23 \times 0.0828 = 0.0667$ |
| L | $u_{12} = 0.46 \times 0.0910 + 0.07 \times 0.0546 + 0.23 \times 0.0454 + 0.23 \times 0.0828 = 0.0751$ |

**Tab. 3.** Global utilities of Zones

### 3.4 Stage IV (Choice of the candidates places)

This choice is based on the data generated by the process of space analysis. By using a multi-criterion methodology the decision-makers considered a set of space alternatives. During this analysis the decision-makers use the multiple representation of the chart to have all information necessary to make the most relevant possible decision.

To make the choice in the most interesting way the GISMR offers to the decision-makers, through the possibility of incorporating the most useful and relevant information and data, to express the preferences in the most objective and most convincing way.

Thus the decision makers determined 16 sites candidates in various cities (table 4); ***Casa1***, ***Casa2***, ***Rabat***, ***Sale***, ***Marrakech***, ***Beni Mellal***, ***Fes***, ***Meknes***, ***Agadir***, ***Essaouira***, ***Nador***, ***Elhouceima***, ***Eljadida***, ***Settat***, ***Safi*** and ***Tanger***.

| Zones | A | B | C | D | E | F |
|---|---|---|---|---|---|---|
| Sites | Casa 2 | Casa 1, Rabat | Salé | Elhouceima, Tanger | Nador | Eljadida, Settat |
| Uility | 0.1059 | 0.1291 | 0.0579 | 0.0835 | 0.0845 | 0.0636 |
| Zones | G | H | I | J | K | L |
| Sites | Essaouira, Safi | Agadir | Marrakech | Beni Mellal | Fès | Meknès |
| Uility | 0.0662 | 0.0799 | 0.1062 | 0.0698 | 0.0667 | 0.0751 |

**Tab. 4.** Global utilities of Sites

### 3.5 Stage V

Mathematical model: The distribution strategy of the industrialist in question neglects the costs of distribution and subdivides the territory in eight areas of distribution in which we must not install more than one unity of logistic base for each one.

| Areas | Large Casablanca | Center - North | Center - South | East - South |
|---|---|---|---|---|
| Candidates sites | $Su_1$ - Casa1<br>$Su_2$ - Casa2 | $Su_3$ - Rabat<br>$Su_4$ - Salé | $Su_5$ - Marrakech<br>$Su_6$ - Béni-Méllal | $Su_7$ - Fès<br>$Su_8$ - Meknès |
| **Areas** | **South** | **East** | **Center** | **North** |
| Candidates sites | $Su_9$ - Agadir<br>$Su_{10}$ - Essaouira | $Su_{11}$ - Nador<br>$Su_{12}$ - El Houceima | $Su_{13}$ - El Jadidda<br>$Su_{14}$ - Settat<br>$Su_{15}$ - Safi | $Su_{16}$ - Tanger |

**Tab. 5.** Areas of sites

The choice of only one site per area satisfies the constraints related to the minimal activity. Thus the mathematical model relating to this case and the final solution are presented as follows:

| Mathematical model | Final solution |
|---|---|
| $\text{Max } U = \sum_{i=1}^{16} Su_i \times u_i$<br>*subject to* : $\begin{cases} Su_1 + Su_2 = 1, \\ Su_3 + Su_4 = 1, \\ Su_5 + Su_6 = 1, \\ Su_7 + Su_8 = 1, \\ Su_9 + Su_{10} = 1, \\ Su_{11} + Su_{12} = 1, \\ Su_{13} + Su_{14} + Su_{15} = 1, \\ Su_{16} = 1, Su_i \in \{0,1\}, \forall i \in \{1,\ldots,16\}. \end{cases}$ | $U = \mathbf{0.7536}$<br>$\mathbf{Su_1} = 1, Su2 = 0,$<br>$\mathbf{Su_3} = 1, Su_4 = 0,$<br>$\mathbf{Su_5} = 1, Su_6 = 0,$<br>$Su_7 = 0, \mathbf{Su_8} = 1$<br>$\mathbf{Su_9} = 1, Su_{10} = 0$<br>$\mathbf{Su_{11}} = 1, Su_{12} = 0,$<br>$Su_{13} = 0, Su_{14} = 0, \mathbf{Su_{15}} = 1,$<br>$\mathbf{Su_{16}} = 1.$ |

**Tab. 6.** Final result

By the application of our approach for this case we obtained a very convincing total utility (U=0.7536) by choosing only 8 sites among 16. These sites are in the following cities: Casa1, Rabat, Marrakech, Meknès, Agadir, Nador, Safi and Tanger.

## 4 Conclusion

In conclusion, we can summarize the interest of our approach in two main points:

– The first point is the use of the GISMR as a tool for the centralization of all spatial and non spatial data necessary to the decision-making. It is an enough recent alternative. Its first interest , in the domain of the decision-making aid, is the great easiness of spatial analysis on geographical data to multiple representations,

– The second point is the combination of two techniques of the decision-making aid which are AHP and MP. We first showed the utility of this combination and after we have used them concretely for the localization of a logistic base.

The perspectives of our work are directed on two important axes:

– The first axis is the widening of our model to support new techniques of Multi Criteria Decision Making as the fuzzy analysis,

– The second axis concerns the GISMR and more especially the functionality of decision in an open GIS context.

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